Research Article

Stochastic optimization and modeling of high-velocity impact tests on high-temperature carbon–carbon composites

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Abstract
In this study, it is intended to optimize a high-velocity impact case of a composite plate. The case selected from literature focused on the failure response of advanced carbon–carbon (C/C) composites under high-velocity impacts. Based on the stochastic optimization method, three unique models are introduced within the present study's scope as dimensionless damage areas of front and back sides and the composite impact energy response. The difference between the equations found in the present study and the base study is the number of variables. Obtained prediction models consist of only the tests' input variables; thus, these models can be considered the essential prediction functions of high-velocity impact response of C/C composites under high temperatures. Multiple nonlinear regression method is used for objective functions of the optimization problem. Since the determination coefficient values have been found quite similar to the ones in the literature, the presented models can be considered successful in predicting the results. By utilizing the novel regression functions presented in this study, the damaged areas are minimized. Without the necessity of experimental research, further predictions can be made by operating the models found in the present study.

Keywords Ballistic impact · Multiple nonlinear regression · Optimization · Carbon–carbon composites · Failure

1 Introduction

Carbon–carbon composite systems are materials composed of carbon fiber reinforcement and a carbon matrix phase. There are two different forms of C/C composites as advanced and reinforced. These composite systems are preferred for their extraordinary properties in mechanical and thermal points of view [1–3].

C/C composites are known for their unique behaviors. These relatively new developed engineering materials show ceramic behavior in nature but behave in a range from pseudo-plastic to brittle. These composite materials are all-carbon composites with a reinforcing fiber phase and a matrix phase; both made up of carbon. These materials are also known as inverse materials. C/C composites are used in several industrial and medical applications, especially in re-entry nose tips, rocket nozzles, and aircraft brake discs, where their excellent thermo-structural properties become crucial [4].

Failure investigation on composite materials and any material is a long-term research field in material science. Even this research field has been focused on for decades still a trending topic because of the importance of application areas it is needed to be used and the topics availability of multidisciplinary working.

Varas et al. [5] studied the high-velocity impact response of carbon-reinforced epoxy woven laminates, and to predict these responses; they produced a numerical model. They predicted inter lamina failure using cohesive elements. Experimental and numerical results

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of failure area on the plate and residual velocity of the projectile were compared in this study. After their model is validated, another study was conducted to investigate the high-velocity impact behavior due to projectile slenderness. With the model they developed, the number of experimental tests can be reduced considerably for future studies investigates the high-velocity impact response of aeronautic composite structures.

Mohotti et al. [6] studied the high-velocity impact effects of projectiles on aluminum-polyurea composite plates. They proposed an analytical model to predict the residual velocity of projectiles impacted on aluminum–polyurea composite plates. Then, they verified the results with numerical and experimental studies. They represent stress–strain response under the composite system's high-velocity impact in the numerical analysis with Mooney–Rivlin and Johnson–Cook material models. It is concluded that polyurea content in composite plates reduces the residual velocity of the projectile.

Shear thickening fluids are exciting materials considered in ballistic applications lately because of their non-Newtonian nature. Park et al. [7] investigated the advantages of shear thickening fluids over traditional materials. They studied ballistic impact energy (IE) absorption behaviors of neat and shear thickening fluid impregnated Kevlar fabric by conducting a numerical study using LS-DYNA code. After comparing the results of both numerical and experimental, they were validated. Despite insufficiency in entirely indicating the effect of shear thickening impregnated fabric under the high-velocity impact, it is found that the significant factor in the energy absorption process is friction between the projectile, fabrics, and yarns.

In most of the studies from the literature, it is seen that the studies focused on highly specialized and narrow aims. However, the value of investigating a wide-angle of a specific topic at a time cannot be denied. One such study was conducted by Ansari and Chakrabarti [8]. Their study is about the progressive damage patterns and damage evolution types besides the effects of many ballistic properties such as thickness to span ratio, the thickness of the Kevlar-29/epoxy plate, and span of the target plate on the residual velocity of the projectile, depth of penetration, contact force, and radius of the damaged area. The numerical investigation was done using AUTODYN hydrocode and, the results are validated with those available in the literature. As a contribution, they generated many new results to understand the nature of three-dimensional progressive damage propagations and failure behavior of composite plates under the high-velocity impact.

Ballistic responses of C/C composites are among the trending topics in ballistic studies because of their exceptional thermal and mechanical properties. Xue et al. [9] studied hypervelocity impact on determining the effects of hypervelocity impact on ablation response of SiC coated C/C composites. Their motivation was the lack of studies about the hypervelocity impact property of C/C composites. The extraordinary properties in an ultra-thermal environment make C/C composites significant materials in the aerospace industry. The study investigated the phenomena experimentally and concluded that SiC–C/C composites experience fewer ablation rates than neat C/C composites, and impact velocity affects ablation rates and ablation areas. This study's results can help increase aircraft service times by improving coated C/C composites' anti-ablation properties.

Xue et al. [10] studied another case again with C/C composites. They used an experimental method to obtain the impact direction interactions, residual flexural strength and, damage propagation. The conclusions showed impact velocity affects impact resistance of C/C plates and exact positions of failure propagations; also, increasing impact velocity leads to fracture type becoming more plastic than brittle. These results are also significant because of the real-life applications of C/C composite plates in aerospace engineering.

Cunniff [11] approached the optimization of ballistic structures differently than the traditional ways. While ultimate mechanical properties define the ballistic appropriacy, dimensionless groups' indication success was investigated in his study both analytically and experimentally. The dimensionless groups established in the study allow a directed optimization process for ballistic textiles.

Patel et al. [12] introduced an innovative stochastic design method to determine composite beams' ballistic impact behavior. They used fiber failure initiation models with sensitivity-based design optimization method and 3D stochastic finite element method. It is concluded that the probability of failure of the anti-symmetric cross-ply arrangement is lower than the other ply lay-up arrangements. Their results can be used for design optimization to obtain better ballistic properties and lighter composite beams.

Jolly and Williamsen [13] regressed relationship functions to define the ballistic limit of dual-wall protection systems used in NASA's Space Station Freedom project. A regression method of stepwise linear least squares and multiple nonlinear regression analysis methods were used for the optimization process. As a result, a set of expressions was presented for use in computations of the probability of no critical failure for the space station.

According to Schonberg's report [14], if Holly and Williamsen's approach were used to reevolve the ballistic limit equations used in Bumper 3 software to provide estimate predictions, statistics based ballistic limit equations for wall structure and configurations would have obtained. Also, their statistics-based uncertainty information for determining how accurate the results could be used with this approach's help.

In the application areas of decision-making problems, such as banking, signal processing, and aerospace, the problems consist of minimizing or maximizing mathematical functions; the stochastic optimization process is the researchers' preferred method. The stochastic optimization process provides the extremum values of dependent parameters on random variables of these mathematical functions. [15]

Ozturk et al. [16] studied stochastic algorithms to minimize surface roughness in a systematic cutting process problem. To avoid characteristic scattering of the stochastic methods, they conducted four different optimization algorithms (Nelder–Mead, Simulated Annealing, Differential Evolution, and Random Search) on the problem. Their study found that minimizing Si wafers' surface roughness can be successfully achieved by employing different rational regression models with stochastic optimization approaches.

Deveci et al. [17] studied the effect of fiber angle domains on maximum buckling resistance in laminated composites. They introduced an optimization approach to determine the optimum stacking sequence for this purpose. Their approach consists trust-region reflective algorithm and genetic algorithm to establish higher accuracy of the results. Obtained results showed that when the Puck failure theory is under consideration for the buckling optimization, the approach presents dependable stacking sequence layouts.

Ozturk et al. [18] conducted an optimization study on the lapping processes of wire-sawn silicon wafers using lapping and polishing machine. Rotation speed and lapping time were considered as the design variables of the optimization problem in their study. They concluded the importance of the combined experimental and stochastic optimization approach to surface roughness investigation on the silicon wafer lapping process.

Xie et al. [19] studied both experimentally and analytically on high-velocity impact effects on C/C composites at high temperatures. The ranges for projectile velocity and experimental temperatures are 1600–4600 m s^{-1} and 25–1429 °C, respectively. A fast-electric heating system was used to obtain high temperatures in this study. Their study is taken as the base study for the present paper. Figure 1 shows the schematic drawing of the two-stage light gas gun and the impact chamber. In the base study's experimental part, the spherical projectiles were accelerated by a two-stage light-gas gun. The fast-electric heating system comprises copper electrodes, a cooling water tank, and a voltage transformer. Heating the C/C composite samples and measuring the temperature was done by the fast-electric heating system.

In the high-velocity impact case from the base study, C/C composite plates were impacted by projectiles at elevated temperatures. The base study's analytical part was conducted to obtain mathematical models of objective functions using the experimental results. Further predictions can be made by operating the models without the necessity of any experimental procedure.

In the present study, to improve the base study's mentioned models, the optimization process has been done using different regression models on Wolfram Mathematica 11.0 software. Their convergence of determination coefficient compares the results. Objective functions are constructed with the design variables, which are system inputs of the physical phenomena. The design variables are the temperature of the composite plate (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p).

Optimization study is applied to objective functions, which are system outputs of the physical phenomena. Two different models have been introduced in the present study using a second-order multiple nonlinear regression method to predict the dimensionless damaged areas easier than and yet equally accurate to the models presented in the base study. Another output, the impact energy (IE), is studied to be predicted, and another second-order multiple nonlinear regression model is developed for this purpose.

The present paper is organized as follows. Section 2 describes and analyses the base study experiments used

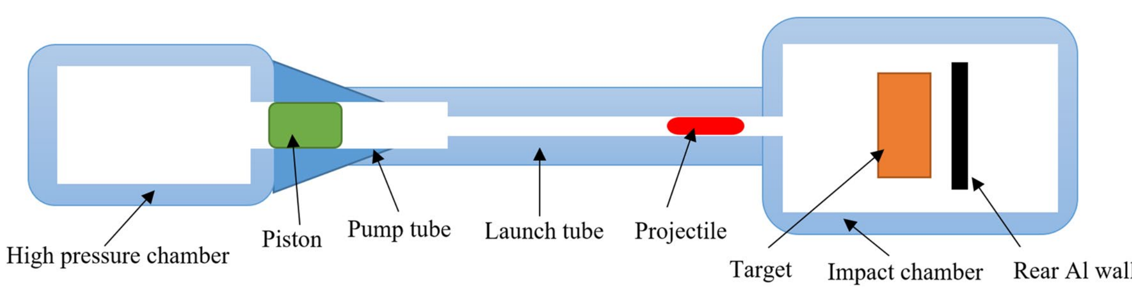

Fig. 1 Schematic drawing of a two-stage light gas gun

to investigate the high-velocity impact response of C/C composites and includes discussing the present paper's analytical approach. Section 2.1 covers the multiple non-linear regression analysis and the nonlinear regression model's decision progress. In Sect. 2.2, the present study's optimization algorithm is introduced, and the algorithm options are discussed. Section 2.3 explains Mathematica's implementation of the optimization process and introduces the commands briefly. The main problem, the motive, and the solution approach are expressed in Sect. 3. In Sect. 4, regression model options are compared by their accuracies, the regression models of the phenomena are constructed, and results of the optimization study are presented by comparing the results with the results of base study. Also, by comparing the minimized output results, the most appropriate optimization method is decided, and at the end of the section, overall results are discussed. Finally, Sect. 5 presents our conclusions.

2 Materials and methods

In the base study, seventeen high-velocity impact tests were executed experimentally. As a result of the high-velocity impact tests, damages were observed on the front and back surfaces of the C/C composite samples. The damages were measured quantitatively by calculating the failure zones' areas on the composite samples' front and back surfaces. Table 1 demonstrates the input and output variables of the high-velocity impact test results and dimensionless damaged areas [19].

The experimental results show the input and output variables of each experimental case. T indicates the temperature of the composite samples during the impact tests. A fast-electric heating system controlled the test specimen's temperature, and thus extreme environmental conditions were simulated. The ratio t_w/d_p is between the C/C laminate thickness and diameter of the Si_3N_4 projectile. Taking these two dimension-variables into consideration provided the effects of composite sample thickness and the spherical projectile diameter altogether in a rational form. A_{front}/d_p^2 and A_{back}/d_p^2 ratios are between front and back damage areas of the C/C composite plate and the projectile squared diameter. The high-velocity impact of the projectile causes the damaged areas. Frontal damage area occurs by a direct impact, while the direct effect of the penetration and the trauma caused by the impact can provoke the back damage area's development. IE is one of the experiment outputs and defines the energy developed by impact and transferred from the projectile to the specimen plate.

The regression analysis method is used to obtain more accurate models for the objective functions. The regression analysis method can accurately determine which variables have how much impact on the relevant phenomena. Operating this method gives crucial information about the process of phenomena. It allows us to understand which factors matter most, which factors can be neglected, and how they affect each other.

Depending on the problem, different types of regression models can be used. The most used regression function types are linear, multiple, and polynomial regressions. These methods can be combined in order to approach the

Table 1 Impact test results [19] ("1" represents perforation while "0" represents no perforation)

T (°C)	v_0 (km/s)	Test result	t_w/d_p	A_{front}/d_p^2	A_{back}/d_p^2	IE (J)
1205	1.690	1	1.667	3.537	4.207	69.449
1206	1.680	1	1.667	1.781	2.498	68.629
1225	1.710	1	2.5	1.268	2.198	21.067
1230	1.700	0	5	2.95	0	2.603
25	1.690	1	1.667	3.501	5.519	69.449
1212	1.980	1	1	3.100	3.225	441.334
1218	1.676	0	3	3.42	0	2.530
1270	2.000	0	5	3.82	0	3.602
1230	1.704	1	1	3.376	3.999	326.871
1193	1.760	1	1.667	2.928	3.799	131.374
1205	1.700	1	1	2.640	3.303	70.273
900	1.700	1	3.333	2.090	3.591	70.273
819	1.886	1	1.667	8.998	10.956	86.492
1429	1.608	1	1.667	4.143	10.754	62.873
1220	4.600	1	1.667	21.32	25.169	514.525
1205	3.090	1	1.667	11.133	11.364	232.171
1234	3.900	1	5	12.554	14.52	13.698

problem properly. Linear regression provides a connection between a dependent variable and an independent variable with the help of a regression line. This line can be recognized as the best fit straight line of the relationship graph [20]. Multiple linear regression is the appropriate method to investigate the relationship between one continuous dependent variable and multiple independent variables. In the polynomial regression technique, the regression line is a curved line that fits into the data points than a straight line. The power of some independent variables can be more than 1 for a polynomial regression [21].

2.1 Multiple nonlinear regression analysis

Regression analysis is a convenient statistical tool to determine the process parameters' connections in an engineering process. Regression models are used in literature in different forms such as Linear, Logistic, Nonlinear, and Stepwise. These forms of regression analysis are used to characterize the phenomena, and some forms of different linear and nonlinear regression models are shown in Table 2. Linear regression models can describe some engineering processes well, but not all of them. When linear regression models become insufficient, different nonlinear modeling approaches with general functional classes are appropriate. Logarithmic, trigonometric, rational, and power functions are examples of the nonlinear functions in the literature [16].

One of the regression models is the second-order polynomial nonlinear regression function. This method's difference from the other methods is modeling the dependent or criterion variables as a nonlinear function of model parameters and independent variables. Second-order multiple nonlinear regression forms define more than one independent variable (for the present case, input types) in the objective function. The objective function simply indicates each variable's contribution to the value to be optimized. Basically, the objective function is a function of the design variables to be maximized or minimized. The multiple nonlinear regression analysis is chosen as the appropriate method to solve the present case. Considering the IE determination coefficient results, the linear rational model is chosen only for the IE optimization case.

The optimization process has been done by using Wolfram Mathematica 11.0 software. Design variables (system inputs) are defined, and for this case, the design variables are the temperature of the composite sample (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p). Any change in the experiment conditions means a change in design variables; thus, leading to a change in the objective function and outputs. For the present case, if any of the temperatures of a composite sample, impact velocity of the projectile, or the ratio between the laminate thickness and diameter of the projectile variables changes, the outputs are dimensionless damage areas the impact energy are going to change. The objective functions define the system outputs, and they are dimensionless frontal damage area ($A_{front}/{d_p}^2$), dimensionless back damage area ($A_{back}/{d_p}^2$), and impact energy (J), for the present case.

In the modeling step, the determination coefficients (R^2) adjusted coefficient of determination (${R_{adj}}^2$), Akaike information criterion (AIC), and Bayesian information criterion (BIC) have been calculated for each of the outputs. The model with the highest R^2 and ${R_{adj}}^2$ values and the lowest AIC and BIC values is preferred.

The system's global minimum values are determined for each of the two objective functions of damaged areas and IE's objective function. There are two solver functions in Mathematica to determine constrained optimal minimum and maximum outputs of a problem: NMinimize and NMaximize. Using these solvers, implementation of various algorithms to find constrained global optimum point is possible. Even non-differentiable functions can be dealt with by these methods [22]; NMinimize command is used to obtain the best scenario of the impact problem: minimum back and front damage areas of the C/C composite target plate and IE of the physical phenomena.

2.2 Optimization algorithm

There are various optimization algorithms to work with problems focused on extremum designs, and these algorithm types can be separated into traditional and non-traditional methods. In the mathematical optimization

Table 2 Forms of different linear and nonlinear regression models [16]

Model name	Nomenclature	Formula
Multiple linear	L	$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$
Multiple linear rational	LR	$Y = \frac{a_0+a_1x_1+a_2x_2+a_3x_3}{b_0+b_1x_1+b_2x_2+b_3x_3}$
Second-order multiple nonlinear	SON	$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_1x_2 + a_5x_2x_3 + a_6x_1x_3 + a_7{x_1}^2 + a_8{x_2}^2 + a_9{x_3}^2$

process of modeling ballistic impact response, traditional optimization methods become insufficient to solve these optimization problems because of the high amount of nonlinear terms content to define the problem. Stochastic methods are required for these situations, such as Differential Evolution, Simulated Annealing, Genetic Algorithms, and Nelder–Mead methods. Rao [23] discussed many different optimization algorithms and methods with a great extent of scope. In the present study, a modified version of the integer-based Differential Evolution algorithm has been used to solve the minimum dimensionless front and back damage area of the C/C composite target plate and IE of the projectile design-optimization problems.

2.2.1 Modified differential evolution algorithm

Complex composite design problems can be solved by using the differential evolution algorithm. This stochastic optimization method operates a population of solutions and solves design problems through its four main phases. These phases are initialization, mutation, crossover, and selection. Even differential evolution algorithm does not obtain global optimum results for all cases; it is accepted and operated as an efficient algorithm.

In Mathematica, differential evolution algorithm consists of a population of solutions called r points instead of a particular solution. The number of r points must be higher than the number of design variables.

For the iteration process, the differential evolution algorithm generates a mutant vector for each element of the considered population of r points. The algorithm scales two random vectors with a scaling factor, F, and adds it to another vector. In the crossover phase, the present vector and mutant vector are operated together, and a trial vector is generated. After comparing the trial vector with the current population element, the best case is selected, and the operation continues [24].

The differential evolution algorithm consists of 4 main phases: initialization, mutation, crossover, and selection. These phases are shown in Fig. 2 as in the algorithm process.

2.2.2 Random search algorithm

One of the Mathematica commands and a stochastic optimization algorithm used in the present case for comparison is Random Search. The Random Search approach determines random starting points during the solution's working process on the optimization problem and collects a population combined with the chosen random starting points. The final step of the algorithm process is the evaluation of convergence. In this step, the algorithm occupies one of the local search approaches, FindMinimum, to determine the merging quality of selected starting points to the local minimum. The Random Search algorithm has four sub-process options. The first option, SearchPoints, implement the amount of starting points to the algorithm process. The other option, RandomSeed, assigns the initial value for the random sub-process. Whether the optimization problem has constraints, the FindMinimum command minimizes the objective function by using the appropriate process. The final option, PostProcess, fine-tunes the solution by using an appropriate combination of solution methods [25].

2.2.3 Nelder Mead algorithm

Nelder Mead is an optimization method that does not process with derivatives, and initially, it is developed to solve unconstrained optimization problems. For a given function of n variables, the Nelder Mead method generates an array of n + 1 points assigned for a polytope's vertices, an n-dimensional geometric object with flat sides.

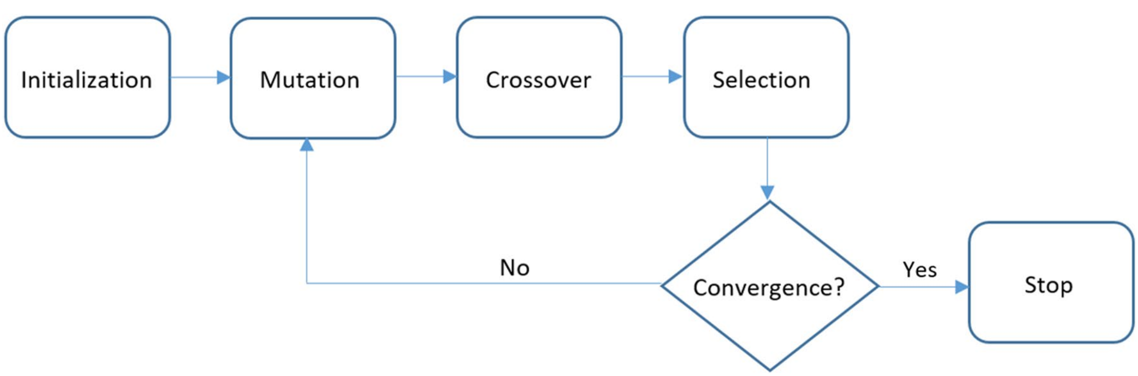

Fig. 2 Flowchart of the differential evolution algorithm

The algorithm operates the iteration process of assigned points by producing and replacing the previous worst point. Once a new point better than the previous best one is obtained, that means the reflection of the trial point is successfully obtained on the centroid. Nelder Mead algorithm gives better global optima results compared to Random Search algorithm, but Differential Evolution outperforms it in our algorithm comparison study [25].

2.3 Mathematica implementation

The modeling and optimization process has been done by using Wolfram Mathematica 11.0 software. The FindFit command is an appropriate selection obtaining the optimal fit between the parameters and the expression. It calculates the best fitting parameters to its data in numerical forms. The data with the equal number of coordinates (x, y, …) and variables can be expressed in

$$\left\{\left\{x_1, y_1, \ldots, f_1\right\}, \left\{x_2, y_2, \ldots, f_2\right\}, \ldots\right\}$$

The data with one coordinate system can also be expressed in the form of

$$\left\{f_1, f_2, \ldots\right\}$$

The command FindFit by default, finds a least-squares fit. Possible Method command settings for being used with FindFit include ConjugateGradient, Gradient, Levenberg-Marquardt, Newton, NMinimize, and QuasiNewton. In the present study, the NMinimize solver has been selected at this step.

NMinimize command minimizes the function concerning the chosen variable. The capability of Mathematica commands FindMinimum, NMaximize and Nminimize, RandomSearch, NelderMead, and DifferentialEvolution are evaluated in finding the global minimum. The effect of options to find the global minimum is examined for each algorithm. Notably, changing the values of SearchPoints and RandomSeed options are more effective than altering the value of other options for these optimization algorithms. Mathematica functions to optimize complex problems in science and engineering with their certain characteristic features using search algorithms. Although these methods are efficient in finding global optima, it might be difficult to find optimum results even without constraints and boundary conditions.

Constraints might be either in the list form or in a rational combination of domain options, equalities, and inequalities. For example, if one needs to specify results in integer form, the unknown parameter z should be included as $z \in$ Integers in line. Then, this constraint restricts the possible solutions as being only integers. Besides, the NMinimize command requires a rectangular starting region to begin optimization. It means that each variable in the given function should have a finite upper and lower bound. Using the Method option to apply other types of search algorithms is a way to provide non-automatic set solutions.

The command DifferentialEvolution consists of specific adjustment options: CrossProbability, InitialPoints, PenaltyFunction, PostProcess, RandomSeed, ScalingFactor, SearchPoints, and Tolerance, whether none of them does guarantee to find global optima [22].

The Mathematica implementation process begins with creating a data table and defining its elements based on experimental results. The values of inputs and the outputs are defined for each variable as value groups. Then, the data table is constructed with the Table command as each row of the table contains the variables of each experiment. The FindFit command is used by defining a second-order polynomial expression format to be used with three variables as three input values, and the coefficients of the second-order polynomial model are obtained. The suitability of the generated model is checked by comparing the results of the model and the experimental results' output values. A table is created with the generated model results and experimental output results, and then the determination coefficient study is conducted over this table. If the generated model fits over the experimental findings properly, the determination coefficient value should be greater than 0.95. Finally, the NMinimize command is used to obtain minimum output values with specific input variables. The generated model of the physical phenomena and boundary conditions, which are the maximum and minimum variable values obtained from experimental results, is defined in the NMinimize command. After running the command, the interest's minimum output value with the specific input values is obtained from the command output. To specify the optimization algorithm, Method built-in option is used. Differential Evolution algorithm is operated by using Method option in NMinimize command.

3 Problem definition

The base study [19] aimed to determine the dimensionless front and back damage areas of C/C composites under high-velocity impacts and high-temperature conditions. In real-life applications of advanced C/C composites, they are being used under high-temperature conditions because of their material properties and the lack of equivalent opponents to stand under these conditions. The motivation was minimizing the need to do experiments, which are expensive and time-consuming, and to calculate the damage behaviors analytically instead. To reach this goal, they developed two different equations to solve and determine the dimensionless damaged areas.

In the present study, the motive is to develop the same equations presented in the base study, but in simpler forms and as accurate as possible. Simple prediction equations can give the results with less time and process power. Reaching this objective will provide a better approach to critical real-life applications of advanced C/C composites such as the aerospace industry and aeronautical structures under high-velocity impacts.

One of the stochastic optimization methods, differential evolution, is used to develop the prediction equations. Differential evolution algorithm permits to obtain global optimum results among the data, and the predictions belong to the data to perform the iteration process. Two-second order polynomial models and one linear rational model are obtained with appropriate regression analysis. After implementing appropriate minimization command, the final models were able to give optimized input values, which are the temperature of the composite plate (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p) concerning the minimized output value. To consider the models as successful, determination coefficients should be over 0.95, and this is the defined criteria of the success of the present study.

4 Results and discussion

An optimization study has been conducted for three different regression models. The inputs and outputs of the case are determined. The temperature of the composite plate (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p) values are used as inputs, and dimensionless frontal damage area (A_{front}/d_p^2) and dimensionless back damage area (A_{back}/d_p^2) are used as outputs. IE is also considered as an output of the problem as its values are given in the base study, but since its optimization study was not conducted in the base study, only its objective function and optimization results are given in the present case without comparison. The regression models and their accuracy criteria results are presented in Table 3.

The model with the highest (very close to 1) R^2 and R_{adj}^2 values and the lowest AIC and BIC values is preferred. Two second-order multiple nonlinear regression models and one linear rational model are selected as the objective function and the constraints of the present optimization problem considering each of the regression models' results. After defining input variables in Mathematica, three models have been constructed as

$$\begin{aligned} A_{front}/d_p^2 = & -157.812958 + 0.126571x_1 + 0.000002\left(x_1{}^2\right) \\ & + 95.199804x_2 - 0.078081x_1x_2 + 1.069159\left(x_2{}^2\right) \\ & - 1.450941x_3 + 0.001062x_1x_3 - 0.127985x_2x_3 + 0.089770\left(x_3{}^2\right) \end{aligned} \tag{1}$$

Table 3 Results of the regression models in terms of determination coefficients

Nomenclature	Outputs	Models	R^2	$R^2_{adjusted}$	AIC	BIC
L	A_{front}/d_p^2	$-4.56311-0.00104357\ x_1+5.76027\ x_2-0.443356\ x_3$	0.91	0.89	73.52	77.69
	A_{back}/d_p^2	$-3.01434-0.00173756\ x_1+6.78202\ x_2-1.37544\ x_3$	0.83	0.79	91.90	96.07
	IE	$23.1586+0.0519974\ x_1+101.062\ x_2-70.7383\ x_3$	0.62	0.54	213.00	217.17
LR	A_{front}/d_p^2	$(829.744+0.431929\ x_1+254.285\ x_2+15.6286\ x_3)/(-100.633+0.129407\ x_1-4.96005\ x_2+4.37702\ x_3)$	0.98	0.98	57.54	65.03
	A_{back}/d_p^2	$(219.432+0.00144065\ x_1+239.509\ x_2-132.813\ x_3)/(101.787+0.0243875\ x_1-16.6879\ x_2-6.62985\ x_3)$	0.85	0.81	99.79	107.29
	IE	$(52{,}447.1-169.241\ x_1+73{,}808\ x_2+38{,}657.4\ x_3)/(4163.59-4.06858\ x_1-93.768\ x_2+1027\ x_3)$	0.94	0.93	190.596	198.095
SON	A_{front}/d_p^2	$-157.813+0.126571\ x_1+2.25647\times10^{-6}\ x_1^2+95.1998\ x_2-0.0780818\ x_1\ x_2+1.06916\ x_2^2-1.45094\ x_3+0.00106288\ x_1\ x_3-0.127986\ x_2\ x_3+0.0897707\ x_3^2$	0.99	0.98	56.55	65.71
	A_{back}/d_p^2	$-329.558+0.26045\ x_1+0.0000176041\ x_1^2+183.391\ x_2-0.158082\ x_1\ x_2+2.48883\ x_2^2+9.52724\ x_3-0.0103908\ x_1\ x_3+0.828026\ x_2\ x_3+0.163975\ x_3^2$	0.99	0.98	64.52	73.69
	IE	$1642.21-1.17746\ x_1-0.0000711245\ x_1^2-948.709\ x_2+0.987922\ x_1\ x_2-6.78295\ x_2^2+35.7503\ x_3-0.222711\ x_1\ x_3-46.9808\ x_2\ x_3+43.5729\ x_3^2$	0.83	0.79	216.43	225.60

Table 4 Mean prediction confidence interval table for A_{front}/d_p^2 case

Observed	Predicted	Standard error	Confidence interval
3.537	2.51877	0.360997	{1.66515, 3.37239}
1.781	2.47637	0.364496	{1.61447, 3.33826}
1.268	2.62081	0.501272	{1.43549, 3.80613}
2.95	3.38245	0.699374	{1.7287, 5.03621}
3.501	3.50955	0.896577	{1.38948, 5.62962}
3.1	3.88931	0.512197	{2.67816, 5.10046}
3.42	2.55822	0.547193	{1.26432, 3.85213}
3.82	3.37953	0.644798	{1.85483, 4.90424}
3.376	2.68335	0.492589	{1.51857, 3.84814}
2.928	2.88386	0.347048	{2.06322, 3.7045}
2.64	2.66293	0.49989	{1.48088, 3.84498}
2.09	2.01776	0.883683	{−0.0718238, 4.10733}
8.998	8.98401	0.889929	{6.87966, 11.0884}
4.143	4.11178	0.882104	{2.02594, 6.19762}
21.32	21.321	0.892329	{19.211, 23.4311}
11.133	10.9306	0.839287	{8.94605, 12.9152}
12.554	12.6286	0.892537	{10.5181, 14.7391}

Table 5 Comparison of the determination coefficient results

Outputs	Present study	Base study [15]
A_{front}/d_p^2	0.987	0.998
A_{back}/d_p^2	0.987	0.944
IE	0.940	–

$$A_{back}/d_p^2 = -329.557776 + 0.260450x_1 + 0.000017\left(x_1{}^2\right) + 183.391072x_2 - 0.158082x_1x_2 + 2.488831\left(x_2{}^2\right) + 9.527239x_3 - 0.010390x_1x_3 + 0.828026x_2x_3 + 0.163974\left(x_3{}^2\right) \tag{2}$$

where x_1, x_2, and x_3 represent the temperature of the composite plate (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p), respectively. After all, considering one model function, a mean prediction confidence interval table is given in Table 4 to show differences between predicted and experimental results for each observation. It is evident that each of the selected model's standard error values is smaller than 0.9 for calculated confidence intervals. Therefore, it shows a reasonable prediction capability of the model.

The determination coefficients have been calculated for three models. A comparison of these results with those in the base study is shown in Table 5. Also, $R_{adj}{}^2$ values are found for ($A_{front}/d_p{}^2$) and ($A_{back}/d_p{}^2$) as 0.984. The IE model's determination coefficient is not shown in Table 5 due to the lack of the optimization study of IE in the base study. The determination coefficient of the objective function of IE is calculated as 0.940.

To minimize the dimensionless front and back damaged areas and impact velocity, NMinimize command is used with arbitrary constraints. The constraints are defined from the dataset presented in Table 1, as all values should be between the minimum and maximum values. After applying the dataset constraints, the output of the NMinimize command gives the results in Table 6.

x_1, x_2, and x_3 represent the temperature of the composite plate (°C), the impact velocity of the projectile (km/s), and the ratio between the laminate thickness and diameter of the projectile (t_w/d_p), respectively.

It is observed that the Nelder Mead method gave slightly close minimized output results to Differential Evolution results than the Random Search approach. Compared to similar temperature and projectile velocity values, the Nelder Mead approach predicts a 63.5% smaller ratio between the laminate thickness and diameter of projectile than the Random Search approach for dimensionless

$$IE = (52447.1 - 169.241x_1 + 73808x_2 + 38657.4x_3)/(4163.59 - 4.06858x_1 - 93.768x_2 + 1027x_3) \tag{3}$$

Table 6 Optimized input results and minimized dimensionless damage areas and impact energy values

Method	Output	Minimized value	x_1 (°C)	x_2 (km/s)	x_3 (t_w/d_p)
Nelder Mead	A_{front}/d_p^2	4.3×10^{-9}	1329	2.305	1.87
Differential Evolution		9.64×10^{-10}	471.38	1.666	2.905
Random Search		1.03×10^{-7}	1428.98	1.953	2.945
Nelder Mead	A_{back}/d_p^2	4.5×10^{-8}	1318.231	1.648	4.997
Differential evolution		2.13×10^{-8}	1407.59	1.727	3.881
Random Search		1.52×10^{-7}	1428.98	1.806	2.972
Differential Evolution	IE (J)	1	1386.74	1.848	1.175

frontal damage area case. In comparison, the same prediction is 68.14% greater for dimensionless back damage area case.

For instance, in the first row, the output shows that between the defined constraints, and the values of 1329 °C of composite plate temperature, 2.305 km/s of projectile velocity, and 1.870 ratios between the laminate thickness and diameter of the projectile, the dimensionless frontal damaged area will be 4.3×10^{-9}.

Optimization results show that temperature change did not substantially affect between front and back of dimensionless damage areas. On the other hand, the ratio between laminate thickness and the projectile diameter showed an astounding effect on the development's dimensionless damaged area. Despite more excellent optimized velocity value for the frontal damage area case, the lesser value of laminate thickness and projectile diameter ratio gave similar dimensionless damage area results with the back-damage area case. In other words, the dimensionless frontal damage area can be minimized by decreasing the ratio between the projectile's laminate thickness and diameter.

Unexpectedly, minimum impact energy data obtained for the high-velocity impact case depended not only on the impact velocity but also on the composite plate's temperature and the ratio between the laminate thickness and the diameter of the projectile. A specific temperature value and a laminate thickness—projectile diameter ratio, other than the minimum impact velocity value, concluded minimum impact energy result.

5 Conclusion

In the base study from the literature, impact tests were conducted on C/C composite plates under high-temperature conditions, and the aim was to determine connections between the ballistic resistance of the composites and various impact parameters. They presented two different equations to solve and determine dimensionless damage areas of C/C composites under high-temperature conditions. In the present study, three novel equations are presented to determine the dimensionless damage areas and predict the impact energy response. The optimization process has been done by using Wolfram Mathematica 11.0 software. The new models presented in this study show fair determination coefficient agreements with the base study ones. Also, our prediction model of dimensionless back damage area converges the experimental results by the rate of 98.7%, while the base model has a convergence rate of 94.4%. Finally, global minimum optima outputs are found with applied constraints. According to the global optima results, impact velocity is more effective than the temperature of the composite sample, the thickness of the composite sample, and the dimensions of the projectile on C/C composites' ballistic resistance. It should be noted that the procedure involving modeling-optimization outlined in this study is appropriate for similar tasks. However, the models obtained are valid only for the mentioned material, environmental conditions and, load cases. As future work, we are thinking of expanding the modeling studies on C/C impact behaviors using hybrid approaches such as neuro-regression and ANN-Fuzzy so that these behaviors can be modeled more accurately mathematically.

Availability of data and materials Data can be shared if requested.

Code availability Wolfram Mathematica.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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