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Operational extended model formulations for Advanced Planning and Scheduling systems

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ABSTRACT

Since the basic reasoning of Manufacturing Resources Planning (MRPII) systems is flawed, a new breed of concepts called Advanced Planning and Scheduling systems (APS) have recently emerged to overcome the problems occurring on the shop floor. In this study, we develop improved and extended mixed integer programming formulations for APS systems at the factory planning level. First, we develop a basic model which explicitly considers capacity constraints, operation sequences, processing times, and due dates in a multi-machine, multi-order, multi-item environment where an item can be processed on a given set of eligible machines. The extensions to the basic model include sequence dependent setups, and transfer times between machines. We also show that our model with a little modification could be used to quote delivery times for customer orders in case due dates are not specified. We provide numerical examples and our conclusions along with future research directions.

1. Background and motivation

The popular and widely used production planning systems, such as Manufacturing Resources Planning (MRPII), require to have the right part, at the right place, at the right time and in the right quantity with minimum cost. Unfortunately, the fundamental reasoning of those systems is flawed. In other words, production scheduling of parts, components, subassemblies, end items is based on fixed lead times with infinite capacity and backward scheduling logic [1].

Since Material Requirements Planning (MRP) function of MRPII systems cannot provide capacity feasible production plans, this unavoidably causes serious problems on the shop floor, such as varying workloads, changing bottlenecks, high Work-in-Process (WIP) levels, lower machine utilisation, less throughput, late deliveries that cannot be resolved easily in the short term. That is, MRPII is unable to prevent capacity problems occurring on the shop floor. Hence, this leads to the conclusion that capacity problems must be solved and prevented at the higher levels [2,3]. Unquestionably, MRP and production scheduling is closely related, and they should be integrated together to generate realistic production schedules for the shop floor, which leads to the problem of Advanced Planning and Scheduling (APS) [4].

In the 1990s, a new breed of concepts called APS systems emerged. APS systems are equipped with a range of capabilities, including finite capacity planning at the floor level through constraint based planning as well as the latest applications of advanced logic for Supply Chain Management (SCM) [5]. Recent APS sytems tend to take a holistic and collaborative approach to provide global optimisation [6]. This collaborative approach extends an e-plant chain beyond a production site, where an e-plant chain is an extension of the integration beyond a production site by means of improved distribution management, electronic data interchange, and coordination of multiple plants [7]. At present, the tendency from the classical

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MRPII philosophy towards APS is evident as many industrial companies employ APS for solving different problems arising in the field of production and logistics [8].

Since the whole problem of planning and scheduling is rather complicated involving many elements and factors, it is not practical to solve the problem as a whole. For this reason, an APS system has a hierarchical planning framework that combines MRP with Capacity Requirements Planning (CRP) to allow feasible production plans to be created. Hence, a complete APS system has four major modules [9,10]: (i) strategic planning, (ii) demand planning, (iii) master planning and (iv) factory planning. Factory planning (FP) schedules customer requirements and dispatches manufacturing orders to the shop floor according to the master plan. In recent years, APS systems have become decision support tools including several capabilities, from finite capacity scheduling to constraint based planning [11]. These APS tools provide companies with capacity feasible production plans at different decision levels of the hierarchical planning framework.

In this paper, we do not intend to delve into the details of APS systems, rather develop mathematical models, namely mixed integer programming models (MIP) to show how optimisation models could be used in this context at the FP level. Interested readers can refer to the references for more detail.

There are also heuristic approaches to create capacity feasible schedules that take into account realistic constraints, such as sequence and machine dependent setups, parallel machines and multiple objectives [12,9,13]. These algorithms try to develop production schedules to balance demand with the resources of the factory. The best balance is achieved when all demands are satisfied and manufacturing resources are fully utilised at the same time [9,14,15]. In addition, APS systems are increasingly being used to make strategic decisions where the selection of outsourcing machine/operation, meeting the customers (single/multiple) due dates, minimising the makespan are the main objectives while satisfying several technological constraints [16].

The rest of the paper is organised as follows. In the next section, we provide the manufacturing environment which is a typical job shop, and define our problem. Then we present our basic model with multi-machine alternative where an item can be processed on a given set of eligible machines, and develop its various extensions (see [17] on extensions). The extensions to the basic model include sequence dependent setups, and transfer times between machines. The basic model without multi-machine alternative was initially proposed by Chen and Ji [4], and multi-machine alternative by Ornek et al. [18]. However, here we develop different formulations to solve the same type of problems in much less computing times. In the basic model and its extensions, it is assumed that customers dictate their due dates, and the factory tries to generate capacity feasible schedules under conflicting objectives, i.e., machine idle cost, earliness and tardiness costs. We also show that our model with a little modification could be used to quote delivery times for customer orders in case due dates are not specified. Since all APS systems are supposed to provide an efficient link between the MRP and scheduling system, the problem of estimating due dates under the condition of fulfilling customer requirements on the assumption that new orders arrive is of prime importance [19], because MRP systems assume fixed manufacturing lead times based on past data without considering the work-in-process (WIP), the number of planned and firmed production orders issued to the shop floor with associated lot sizes, alternative machine assignments, etc. Using unique fixed lead times, the workshop could be overloaded or idle. In Section 3, we present numerical examples and discuss the results. The conclusions and suggestions for future research are stated in the final section of the paper.

2. Problem definition and model development

In this paper we consider the same manufacturing setting as given in two recent papers by Chen and Ji [4] and Ornek et al. [18], which have real world applications, particularly in job shops. There are products with multi-level structures. A simple example is given in Fig. 1.

The root node F1 is the final product which consists of subassembly S1 and component C1. Subassembly S1 is made up of components C2 and C3. The numbers on the components indicate how many units of that component will be required for one unit of the parent item. Though this type of product structures are typical in industry, they are much more complicated

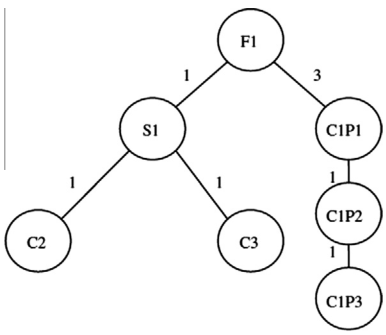

Fig. 1. A simple example of a product structure.

than this example. The items of the products have precedence relations among them. That is, child items have to be processed before parent items. Each item may require different operations on eligible machines, which are continuously reliable. We assume lot-for-lot strategy for component items. In the basic model, the setup times and transfer times between machines are assumed to be negligible. Later we relax these assumptions in extensions. If an item requires several operations on different machines then this item is fragmented into sub-items to reflect the operations. For example, if item C1 has three processing operations, namely P1–P3, then C1 is further divided into three child items: C1P1–C1P3, where C1P3 is a child item of C1P2, which is a child item of C1P1. Furthermore, each operation can be performed on at most one machine at a given time and preemption is not allowed. A machine can perform one operation at a time, and it works only for eight hours a day. The orders for products as well as for items that make up the products are received from customers with known due dates.

Main goal of the problem has a multi-objective nature. As mentioned in Hegedus and Hopp [20] cost of completing a customer order later or earlier than the requested due date is very crucial. Because, meeting a customer order later than the requested due date may cause dissatisfied customer, potential loss of demand and/or sometimes contractual penalty costs. On the other hand producing the order earlier than the quoted date means extra inventory to be held in stock until it is delivered to the customer and may cause inventory holding costs which is in fact the time value of money [21]. Therefore, solution of the problem will be higly sensitive to the assumed cost coefficients (See [21–24] for further discusson on the earliness, tardiness costs and sensitivity analysis). Similar to holding finished inventories in stock, WIP on heavily utilised machines must be reduced to minimize inventory costs. Hence, overall utilisation of machines must be increased, which is ensured by minimizing the idle time of machines.

Therefore, the objective of the paper is to determine an optimal schedule for the manufacturing orders to meet customer requirements with minimum total cost which consists of cost of idle times of the machines and penalties on tardiness and earliness.

2.1. The basic APS model with multi-machine alternative

In this section, we first develop the multi-machine alternative of the basic model where it is possible to process an item on eligible set of machines. This formulation could easily be applied to the single machine case with a little data manipulation. This formulation is different from that of proposed in Ornek et al. [18], and as we show later, it requires much less computing time for comparable problem instances.

We first define the indices, parameters and variables of the model.

Indices

i, j	index of order $i, j = 1,\ldots,n$
p, q	index of item $p, q = 1,\ldots,b$
k	index of machine $k = 1,\ldots,m$

Parameters

n	number of orders
b	number of items
m	number of machines
O_p	order index of item p, $O_p \in \{1,\ldots,n\}$
P_i	final item of order i
Q_i	quantity of order i
G_{pq}	number of item p needed for one unit of item q
R	set of immediate predecessor-successor pairs of items (p, q) such that item q must be processed immediately before item p, i.e., $G_{pq} > 0$.
t_{pk}	processing time required by item p on machine k ($p = 1,\ldots,b$) which is adjusted based on the size of demand for item p itself and its successors
r_k	ready time of machine M_k
d_i	due date of order i
I	cost of idle time per hour
TC	cost of tardy orders per day per job
EC	cost of early orders per day per job
M	a large positive number
F_p	set of machines capable of processing item p

Variables

$C_{\max}$	production makespan
S_p	production start time of item p
C_i	production completion time of order i

(*continued on next page*)

L_i	number of tardy days (real number) for order *i*
E_i	number of early days (real number) for order *i*
L'_i	number of tardy days (integer) for order *i*
E'_i	number of early days (integer) for order *i*

$$Y_{pqk} = \begin{cases} 1 \ \textit{if item p precedes item q on machine k}, \\ 0 \ \textit{otherwise}, \end{cases}$$

$$Z_{pk} = \begin{cases} 1 \ \textit{if item p is assigned to machine k}, \\ 0 \ \textit{otherwise}. \end{cases}$$

The mathematical formulation is as follows.

Objective function

$$Min\left\{ I\left(m \times C_{max} - \sum_{p=1}^{b} \sum_{k \in F_p} t_{pk} \times Z_{pk} - \sum_{k=1}^{m} r_k \right) + \sum_{i=1}^{n} (TC \times L'_i + EC \times E'_i) \right\}. \tag{1}$$

Subject to

$$C_i \leqslant C_{max} \quad \forall i, \tag{2}$$

$$S_p \geqslant \sum_{k \in F_p} r_k Z_{pk} \quad \forall_p, \tag{3}$$

$$S_q \geqslant S_p + \sum_{k \in F_p} t_{pk} \times Z_{pk} \qquad (p,q) \in R, \tag{4}$$

$$S_{P_i} + \sum_{k \in F_{P_i}} t_{P_i k} Z_{P_i k} = C_i \quad \forall i, \tag{5}$$

$$S_q \geqslant S_p + t_{pk} Z_{pk} - M(1 - Y_{pqk}) \quad \forall p,q,k | k \in F_q \cap F_p, \ p < q, \tag{6}$$

$$S_p \geqslant S_q + t_{qk} Z_{qk} - M(1 - Y_{qpk}) \quad \forall p,q,k | k \in F_q \cap F_p, \ p < q, \tag{7}$$

$$Z_{pk} + Z_{qk} \geqslant 2(Y_{pqk} + Y_{qpk}) \quad \forall p,q,k | k \in F_q \cap F_p, \ p < q, \tag{8}$$

$$Z_{pk} + Z_{qk} \leqslant Y_{pqk} + Y_{qpk} + 1 \quad \forall p,q,k | k \in F_q \cap F_p, \ p < q, \tag{9}$$

$$\sum_{k \in F_p} Z_{pk} = 1 \quad \forall p, \tag{10}$$

$$\frac{C_i}{8} - d_i \leqslant Li \quad \forall i, \tag{11}$$

$$d_i - \frac{C_i}{8} \leqslant E_i \quad \forall i, \tag{12}$$

$$L'_i \geqslant L_i \quad \forall i, \tag{13}$$

$$E'_i \geqslant E_i - 0.99 \quad \forall i, \tag{14}$$

$$\sum_{k \notin F_p} Z_{pk} \leqslant 0 \quad \forall p, \tag{15}$$

$$C_{\max} \geqslant 0, \tag{16}$$

$$S_{pk} \geqslant 0 \quad \forall p,k, \tag{17}$$

$$C_i, E_i, L_i \geqslant 0 \quad \forall i, \tag{18}$$

$$E'_i, L'_i \geqslant 0 \text{ and integer} \quad \forall i, \tag{19}$$

$$Y_{pqk}, Z_{pk} \in \{0,1\} \quad \forall p,q,k. \tag{20}$$

The objective function (1) minimizes the sum of the production idle time, tardiness and earliness costs. Note that tardiness and earliness costs can be order dependent. Constraints (2) ensure that the completion time of any order is less than or equal to production makespan ($C_{\max}$). Constraints (3) state that the start time of any item must be equal to or greater than the ready time of the machine it is assigned to. Constraints (4) guarantee that an item starts after its predecessor items are processed. Constraints (5) define the completion time of an order. Constraints (6)–(9) are disjunctive constraints, which provide that no two items can be processed on the same machine simultaneously if and only if both items are assigned to the same machine. If item p is scheduled before item q on machine k, ($Y_{pqk} = 1$), starting time of item q must be greater than or equal to the completion time of item p (6). Constraints (7) are the complementary of disjunctive constraints (6). If items p and q are scheduled on machine k, both items must have been assigned to that machine (8). If items p and q are assigned to the same machine, one of them must be scheduled before the other (9). In constraints (10), it is ensured that each item is assigned to only one machine in its eligible machine set. The tardiness and earliness of the orders are defined in constraints (11) and (12), respectively. Since the shift length is 8 h per day, the completion times of the orders are converted to days. In the constraints (13) and (14), the integer values of the tardiness and earliness are provided. Constraints (15) prevent assigning any item to non eligible machines. Constraints (16)–(20) define set constraints.

2.2. APS model formulation with sequence dependent setup time

Before an operation is performed on a machine it is assigned, a setup might be required in the form of tool changes, machine preparation, etc. If the setup is sequence independent, that is, the time it takes to set up the machine for the operation is not dependent on the order of the operations that are performed, then the processing times might be extended to accommodate the setup times. However, if the setup time is sequence dependent then it must be considered in the formulation. In this section, we develop a sequence dependent setup time extension of the APS model where setups are assumed to be detached, i.e. the machine can be setup before items arrive.

2.2.1. Redefined sets and indices

Sequence dependent extension of the problem requires formulation of immediate predecessor and successor items on a machine as in Travelling Salesman Problem (TSP). In the TSP problem, each city must be preceded and succeeded by a different city and a feasible tour ends at the starting city. As stated in Voß and Woodruf [25], if cities are replaced with jobs and the distances with production and changeover times, TSP returns the sequence dependent machine scheduling problem. Note that in multi-machine case, the TSP problem must be solved for each machine. As an example, let's assume that items assigned to a machine are "1", "2" and "3" and the processing order is 1-2-3. However, different from TSP problems, it is not possible to return to the starting item. Therefore, a starting dummy item (D1) and an end dummy item (D2) are required where there is no predecessor for D1 and no successor for D2. Hence, a feasible schedule for this small example could be D1-1-2-3-D2. Thus, in sequence dependent extension of the problem, dummy start and end items are defined for each machine, which belong to a dummy order. While dummy items "$b + 1$"…"$b + m$" are in the first position in the sequence of machines "1",…"m", "$b + m + 1$"…"$b + 2m$" are in the last position. Note that dummy items have processing times of "0" time unit.

i, j index of order i, $j = 1, \ldots, n+1$
k index of machine $k = 1, \ldots, m$
p, q index of item p, $q \in P = \{1, \ldots, b+2m\}$

2.2.2. New parameters

w_{pq} sequence dependent setup time required to changeover from item p to q.

2.2.3. Redefined variables

$$Y_{pqk} = \begin{cases} 1 \textit{ if item p immediately precedes item q on machine k}, \\ 0 \textit{ otherwise}. \end{cases}$$

2.2.4. Mathematical model

$$Min\left\{ I\left(m \times C_{\max} - \sum_{p=1}^{b}\sum_{k \in F_p} t_{pk} \times Z_{pk} - \sum_{k=1}^{m} r_k \right) + \sum_{i=1}^{n}(TC \times L'_i + EC \times E'_i) \right\}. \tag{21}$$

Subject to

$$C_i \leqslant C_{\max} \quad \forall i \leqslant n, \tag{22}$$

$$S_q \geqslant S_p + \sum_{k \in F_p} t_{pk} \times Z_{pk} \quad (p,q) \in R, \tag{23}$$

$$S_{P_i} + \sum_{k \in F_{P_i}} t_{P_i k} Z_{P_i k} = C_i \quad \forall i \leqslant n, \tag{24}$$

$$S_q \geqslant S_p + t_{pk} + Y_{pqk} w_{pq} - M(1 - Y_{pqk}) \quad \forall p,q,k | k \in F_q \cap F_p,\ p < q, \tag{25}$$

$$S_p \geqslant S_q + t_{qk} + Y_{qpk} w_{qp} - M(1 - Y_{qpk}) \quad \forall p,q,k | k \in F_q \cap F_p,\ p < q, \tag{26}$$

$$Z_{pk} + Z_{qk} \geqslant 2(Y_{pqk} + Y_{qpk}) \quad \forall p,q,k | k \in F_q \cap F_p,\ p < q, \tag{27}$$

$$\sum_{k \in F_p} Z_{pk} = 1 \quad \forall p \leqslant b, \tag{28}$$

$$\frac{C_i}{8} - d_i \leqslant Li \quad \forall i \leqslant n, \tag{29}$$

$$d_i - \frac{C_i}{8} \leqslant E_i \quad \forall i \leqslant n, \tag{30}$$

$$L'_i \geqslant L_i \quad \forall i \leqslant n, \tag{31}$$

$$E'_i \geqslant E_i - 0.99 \quad \forall i \leqslant n, \tag{32}$$

$$\sum_{k \notin F_p} Z_{pk} \leqslant 0 \quad \forall p \leqslant b, \tag{33}$$

$$C_{\max} \geqslant 0, \tag{34}$$

$$S_{pk} \geqslant 0 \quad \forall p,k, \tag{35}$$

$$C_i, E_i, L_i \geqslant 0 \quad \forall i, \tag{36}$$

$$E'_i, L'_i \geqslant 0 \text{ and integer} \quad \forall i, \tag{37}$$

$$Y_{pqk}, Z_{pk} \in \{0,1\} \quad \forall p,q,k, \tag{38}$$

$$\sum_{q \in P - \{p\}} \sum_{k=1}^{m} Y_{qpk} = 1 \quad \forall p | (p \leqslant b) \vee (p > b + m), \tag{39}$$

$$\sum_{q \in P - \{p\}} \sum_{k=1}^{m} Y_{pqk} = 1 \quad \forall p \leqslant b + m, \tag{40}$$

$$Z_{pk} = 1 \quad \forall p,k | (b + 1 \leqslant p \leqslant b + 2m) \wedge (1 \leqslant k \leqslant m), \tag{41}$$

$$S_p = r_k \quad \forall p | b + 1 \leqslant p \leqslant b + m, \tag{42}$$

$$S_p = C_{\max} \quad \forall p | b + m + 1 \leqslant p \leqslant b + 2m, \tag{43}$$

$$C_{n+1} = L_{n+1} = E_{n+1} = L'_{n+1} = E'_{n+1} = 0, \tag{44}$$

$$\sum_{q \in P - \{p\}} \sum_{k=1}^{m} Y_{qpk} = 0 \quad \forall p | b + 1 \leqslant p \leqslant b + m, \tag{45}$$

$$\sum_{q\in P-\{p\}}\sum_{k=1}^{m}Y_{pqk}=0 \quad \forall p|b+m+1\leqslant p\leqslant b+2m. \tag{46}$$

Objective function and constraints (21)–(38) are the same as constraints (1)–(20) except constraints (9). We rewrite them here because their domains are different. Although any two items are assigned to the same machine, they may not be their immediate predecessors; hence constraints (9) are not required. Since setups waste time and must be minimized for an efficient schedule, the objective function (21) is exactly the same as (1). In other words, total sequence dependent setup time is considered as an idle time and it is minimized also. While constraints (39) force that there must be an immediate predecessor for each item, constraints (40) guarantee that there must be an immediate successor for each of them. Note that constraints (39) and (40) are not formulated for the first position and the last position dummy items. Dummy items "$b+1$"…"$b+m$" and "$b+m+1$"…"$b+2m$" are assigned to machines "1"…"m" respectively (41). While starting times of the dummy items "$b+1$"…"$b+m$" are ready times of their corresponding machines (42), starting times of the dummy items "$b+m+1$"…"$b+2m$" are makespan of the schedule (43). $C_{n+1}, L_{n+1}, E_{n+1}, L'_{n+1}$ and E'_{n+1} variables are equal to zero for the dummy order (44). Finally, constraints (45) and (46) ensure that there is no item before dummy items "$b+1$"…"$b+m$" and after dummy items "$b+m+1$"…"$b+2m$" on each machine.

2.3. APS model formulation with sequence dependent setup and transfer times

In cases where operations can be performed on alternative machines, transfer times of the necessary parts between machines might depend on the locations of machines. Therefore, transfer or move times of parts between machines should also be taken into account in the formulations.

2.3.1. New parameters

k, r index of machine $k = 1,\ldots,m$
d_{kr} transfer time between machines k and r

2.3.2. New variables

$$Q_{pkqr}=\left\{\begin{matrix}1 \textit{ if item p is assigned to machine k and item q is assigned to machine r}\\ 0 \textit{ otherwise}\end{matrix}\right\}.$$

2.3.3. Mathematical model

Minimize (21)
Subject to (22), (24)–(40) and

$$S_q \geqslant S_p+\sum_{k\in F_p}t_{pk}Z_{pk}+\sum_{k=1}^{m}\sum_{r=1}^{m}Q_{pkqr}d_{kr} \quad (p,q)\in R, \tag{47}$$

$$Q_{pkqr}+1 \geqslant Z_{pk}+Z_{qr} \quad \forall p,k,q,r|(p\neq q)\wedge(k\neq r), \tag{48}$$

$$2\times Q_{pkqr}\leqslant Z_{pk}+Z_{qr} \quad \forall p,k,q,r|(p\neq q)\wedge(k\neq r), \tag{49}$$

$$Q_{pkqr}\leqslant 0 \quad \forall p,k,q,r|(p=q)\vee(k=r), \tag{50}$$

$$Q_{pkqr}\in\{0,1\} \quad \forall p,k,q,r. \tag{51}$$

Constraints (47) ensure that a job has to wait for its immediate predecessor to be moved to the machine it is on before its processing starts. Constraints (48)–(51) indicate the machines that items are assigned. If items p and q are assigned to machines k and r respectively, binary indicator variable has to be equal to 1 (48), zero otherwise (49). Constraints (50) provide valid upper bounds such that if any two items are assigned to the same machine or they are the same items, binary indicator variable is equal to zero.

3. Numerical examples and discussion of results

In this section, a set of examples are provided to illustrate the applications of the model formulations developed in the previous section. However, we note that we do not intend to solve the formulations to optimality but present feasible solutions. Our example data is based on the paper by Chen and Ji [4].

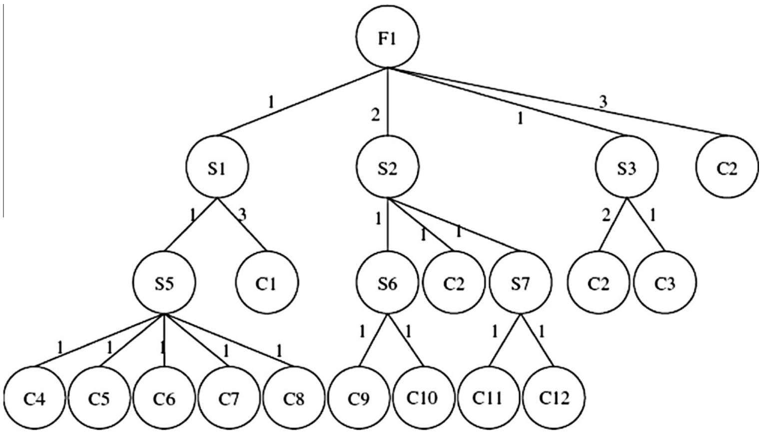

Fig. 2. BOM structure for order 1.

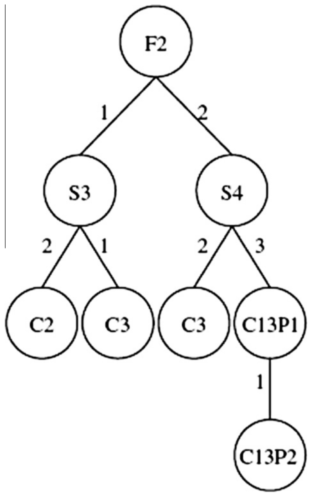

Fig. 3. BOM structure for order 2.

3.1. Parameters used for problem demonstration

3.1.1. Common parameters used by all models

Bill of Material (BOM) and the processing times of items on machines for the illustrative example are given in this section. Product structure is the same as Chen and Ji [4] where there are common subassemblies and components in different final products (see Figs. 2–6).

Sizes of the demands and due dates for the orders are given in the following table (see Table 1);

Idle time, tardiness and earliness costs are, \$50 per hour, \$250 and \$50 per day per order respectively.

There are 5 machines which are available during the regular shift length (8 h) in each day. The ready times of machines i.e. the times the machines are ready for processing, are given in Table 2 in hours. Item processing times on machines are also given in Table 2.

3.1.2. Sequence dependent setup times and transfer times

Sequence dependent setup times and transfer times between machines for model formulations presented in Sections 2.2 and 2.3 are given in the following tables. Note that while items DM1 through DM5 are starting dummy items, items DM6 through DM10 are ending dummy items on machines. Initial setup times are different since they differ for each machine. There is no requirement for changeover after the last item on each machine. Therefore sequence dependent setup times are zero from any item to the last dummy items (see Tables 3 and 4).

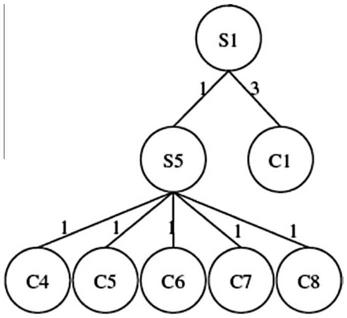

Fig. 4. BOM structure for order 3.

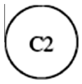

Fig. 5. BOM structure for order 4.

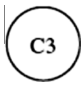

Fig. 6. BOM structure for order 5.

Table 1
Order data.

Order	Order size (units)	Customer due date (days)
O1F1	10	12
O2F2	5	14
O3S1	10	11
O4C2	30	2
O5C3	15	10

3.2. Numerical results for the models presented

In this section, the results of the computational studies for the models proposed in Section 2 are investigated by using parameters given in Section 3.1. All tests are performed on a PC with AMD Phenom II X4 955 3.20 GHz Processor, 4 GB RAM and Windows 7 32 Bit operating system. ILOG OPL Development Studio 6.1.1 with ILOG CPLEX 11.2.1 engine is used to solve the mixed integer programming models and ILOG script is used to run the preprocessing procedure.

3.2.1. Numerical result for the basic APS model

The best solution found in one hour runtime on test PC configuration is given in Fig. 7.

Notation on the Gantt chart is taken from Chen and Ji [4]. As an example, O1F1C2 refers to item C2 of parent F1 in the order 1 whereas O4C2 is the order 4 for item C2 with no parent item. The Gantt chart shows a production schedule which minimizes the total cost. Total cost of the illustrated schedule is $3,050 of which $1,850 is incurred due to the idle time cost, and the remaning $1,200 is the total earliness cost due to 24 days of earliness.

To further compare the efficiency of the proposed model with the one developed by Chen and Ji [4] we run our model by using the same data in their paper where alternative machines are not allowed. Resulting schedule is presented in Fig. 8.

Our model formulation and the model formulation given by Chen and Ji [4] find the optimal solution as $6,550 with the same makespan of 57 h. However, while the model formulation of Chen and Ji [4] proves optimality in 42726.78 s, our model formulation proves it in 40 s. Although the CPLEX version used by Chen and Ji [4] is not reported in their paper, dramatic decrease in the solution time is also ensured with the improvements in the model formulation. As a comparison, our model formulation generates 725 constraints and 532 variables, 477 of which are integer. Whereas these measures are 498, 337,

Table 2
Machine ready times and processing times of items (in hours). Empty cells indicate ineligibility of the machine for that item.

Item	Machine				
	1	2	3	4	5
F1				0.4	0.7
F2				0.3	0.6
S1				0.7	0.5
S2				0.3	0.5
S3				0.6	0.4
S4				0.5	0.6
S5				0.3	0.4
S6				0.4	0.2
S7				0.3	0.4
C1	0.1	0.2	0.2		
C2	0.1	0.2	0.3		
C3				0.4	0.3
C4	0.1	0.3			
C5	0.2	0.4			
C6	0.3	0.2			
C7	0.1	0.3			
C8	0.3	0.4			
C9	0.4	0.6			
C10	0.5	0.2			
C11	0.1		0.3		
C12	0.2	0.3			
C13P1	0.4	0.2	0.3		
C13P2	0.2	0.3			
Ready times	1	2	3	2	1

Table 3
Sequence dependent setup times between items (in hours).

ITEM	F1	F2	S1	S2	S3	S4	S5	S6	S7	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13P1	C13P2	DM1	DM2	DM3	DM4	DM5	DM6	DM7	DM8	DM9	DM10
F1		1	4	2	2	4	4	5	1	3	5	5	1	2	2	4	3	5	2	3	1	5	4										
F2	2		3	2	1	3	2	1	1	5	1	5	5	2	4	1	3	3	5	4	2	1	5										
S1	3	5		2	2	2	1	3	2	2	1	3	3	2	5	5	5	4	1	4	1	4	4										
S2	1	5	1		2	2	4	2	5	2	2	1	1	3	1	2	3	1	1	3	4	4	3										
S3	5	3	1	1		1	1	1	2	1	3	1	2	1	4	5	2	3	3	2	5	2	2										
S4	3	2	5	1	3		4	2	5	4	4	2	2	3	4	5	4	5	1	5	4	3	5										
S5	3	4	5	3	5	4		4	3	4	1	3	4	1	2	2	5	4	4	2	2	3	5										
S6	5	4	4	1	5	3	3		4	1	4	5	2	2	4	4	4	3	1	5	1	4	3										
S7	1	4	2	2	4	5	5	1		1	5	2	4	1	1	4	5	2	3	1	4	5	5										
C1	5	1	3	2	2	2	2	5	2		4	4	3	5	2	2	1	1	3	1	3	5	1										
C2	4	1	3	2	3	2	2	1	2	4		5	3	5	3	1	2	4	4	1	1	4	5										
C3	5	1	2	3	1	5	3	4	3	2	5		3	3	1	5	5	4	3	3	3	1	2										
C4	1	2	5	5	5	1	1	2	3	4	1	2		2	1	5	2	5	4	4	5	1	1										
C5	5	1	1	3	1	3	4	3	2	3	1	5	3		2	1	1	2	1	3	5	4	4										
C6	3	2	5	3	4	2	1	3	4	2	2	4	1	2		1	2	3	3	2	5	4	4										
C7	4	3	5	4	1	5	3	4	1	2	1	2	2	1	4		2	3	5	3	5	3	2										
C8	3	5	1	4	4	5	2	4	1	3	3	5	3	5	5	3		4	1	3	3	1	1										
C9	2	1	4	4	5	3	4	1	1	1	4	3	3	3	2	3	5		3	3	3	2	5										
C10	3	1	3	1	2	5	2	1	4	5	4	2	4	2	2	5	5	5		4	4	4	5										
C11	5	4	2	1	2	1	4	3	4	3	5	4	5	5	3	4	2	4	5		1	4	4										
C12	1	5	5	1	5	5	1	4	5	2	2	1	5	1	2	1	5	4	2	2		2	5										
C13P1	3	2	1	5	5	1	4	4	1	2	1	3	1	2	2	4	1	5	5	1	5		1										
C13P2	5	2	1	5	1	5	5	2	4	5	4	4	2	2	5	2	3	2	1	5	4	3											
DM1	5	3	3	2	4	4	5	5	5	4	4	4	4	5	1	1	5	5	3	4	4	5	2										
DM2	1	3	5	5	2	2	4	1	3	4	4	1	5	3	3	5	2	1	3	4	4	3	3										
DM3	3	5	3	2	4	3	2	4	4	1	2	3	1	3	2	5	3	2	5	2	3	2	2										
DM4	4	2	5	3	3	1	3	4	4	4	1	4	2	1	3	3	4	5	5	4	2	5	1										
DM5	3	3	3	4	1	1	1	5	5	3	5	4	1	1	4	3	4	4	1	5	5	4	3										
DM6																																	
DM7																																	
DM8																																	
DM9																																	
DM10																																	

Table 4
Transfer times between machines (in hours).

From	To				
	1	2	3	4	5
1		1	1.5	2.5	2.5
2	1.5		1	3	1
3	2	1		2.5	2.5
4	3	2	3		1
5	2	3	3	1.5	

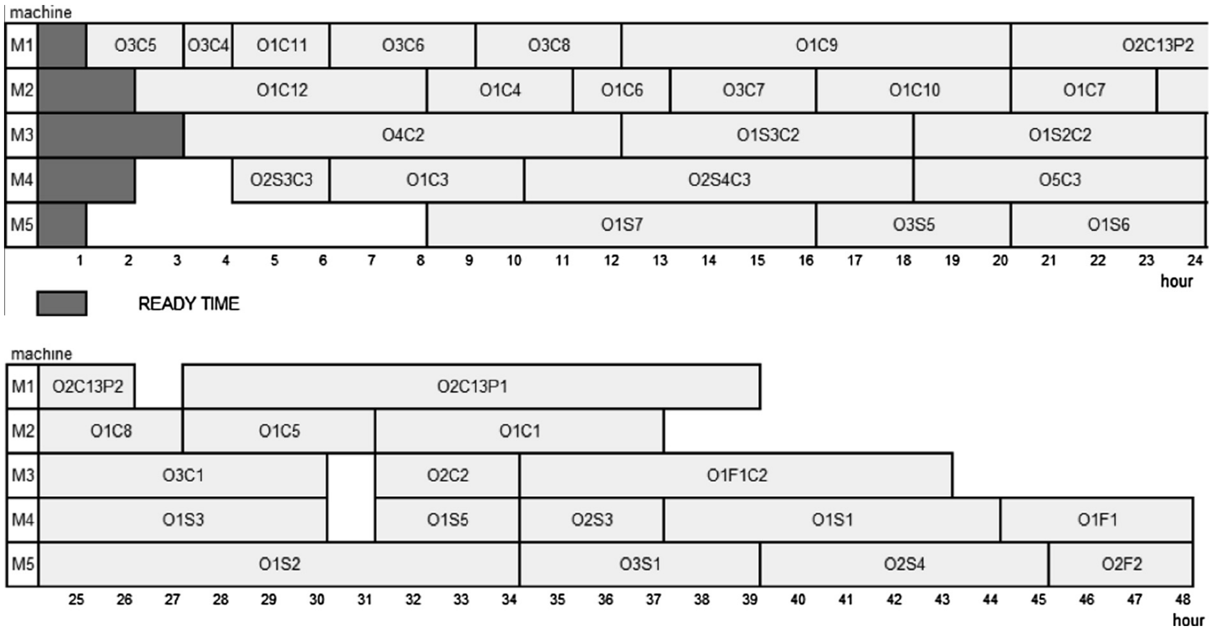

Fig. 7. Basic APS with alternative machines Gantt chart.

and 282, respectively, in the solution of Chen and Ji [4]. As a result, additional 227 constraints help to prune the feasible solution space, cause reduction in the solution time and show that our model formulation is better than others in the literature for practical size problems.

As mentioned in Section 1, the models developed in this study can be used to quote due dates for new customer orders. In this case, APS model is used by changing the objective function to minimise only latest completion time (i.e. makespan). By minimising only the makespan we find the best order due dates which maximise machine utilisation. In Fig. 9, we report the corresponding production schedule which minimises the makespan. The makespan is 46 h and it is less than the makespan of APS as expected (see Section 3.2.4 for further discussion on quoting customer due dates).

3.2.2. Numerical result for APS model with sequence dependent set-up times

Fig. 10 shows the resulting schedule with sequence dependent setup times given in Table 2

Presented schedule is the best solution found in 1 h run time. Cost of the schedule is $8,450 of which $550 is for totally 11 days earliness and $7,900 for total idle time. Note that setup times are assumed as non-productive lost times. Since the objective function minimises the total idle time, it also minimises setup cost. Total setup time is 77 h as 16, 12, 10, 19 and 20 h on machine 1–5, respectively. Total idle time is 81 h as 14, 29, 11, 2 and 25 h on machine 1–5 respectively. Hence, total idle (or non-productive) time is 158 h which corresponds to $7,900 idle time cost with 50$/hour unit idle time cost. In addition, including sequence dependent setup times increases the makespan of basic APS problem from 48 h to 75 h.

3.2.3. Numerical result for APS model with sequence dependent set-up and transfer times

Finally, effects of the non-zero transfer times between machines are investigated in Fig. 11.

The schedule shown in Fig. 11 is obtained in one hour run time and corresponds to $10,525 total cost of which $10,075 is non-productive time cost and $450 earliness cost for 9 days total earliness. The makespan for the APS model with sequence-dependent set-up and transfer times is 83.5 h. As an example, with setup times and transfer times, we take

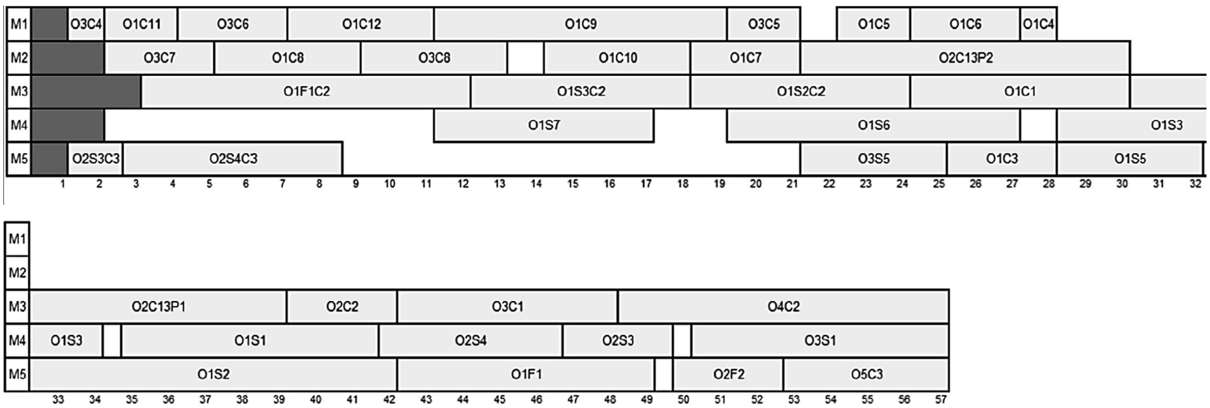

Fig. 8. APS Gantt chart without alternative machines.

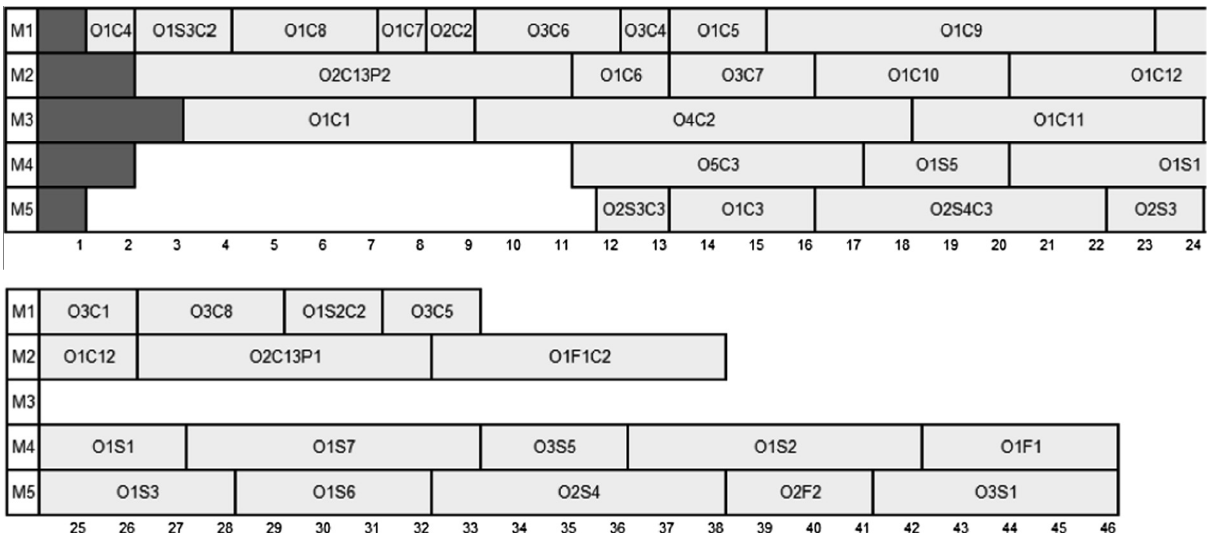

Fig. 9. APS with alternative machines and makespan objective Gantt chart.

O1C1 and O1S5 that go into O1S1. O1C1 is done on machine 3 (which terminates at 60th hour) and it takes 2.5 h to transfer to machine 4 where O1S1 is processed. Similarly, O1S5 is procesed on machine 5 (which terminates at 61th hour) and the transfer time is 1.5 h. Adding transfer times of 2.5 and 1.5 to terminating hours of O1C1 and O1S5, respectively, we conclude that O1S1 cannot start earlier than 62.5th hour. Since on machine 4, we switch from O1S2 (which terminates at 61.5th hour) to O1S1, the setup time is 1 h. This changeover also stipulates that O1S1 cannot start processing on machine 4 earlier than 62.5th hour. On the final resulting schedule, O1S1 starts on machine 4 at 62.5th hour as expected.

3.2.4. Due date and solution performance of the proposed models

In this section, first, we compare the realised due dates of each model with promised due dates in the following table where (1) : APS without alternative machines (Fig. 8), (2): APS with alternative machines (Fig. 7), (3) : APS with alternative machines and makespan objective (Fig. 9), (4) : APS with alternative machines and sequence-dependent setup times (Fig. 10), (5) : APS with alternative machines and sequence dependent setup and transfer times (Fig. 11).

Results in Table 5 reveal that all customer orders but O4C2 can be produced earlier than the promised due dates. In other words, promised due dates appear to be inflated. However this inflated due dates provide the company with the opportunity of handling uncertain or stochastic circumstances like machine failures and demand fluctuations in the upcoming days. In addition, including setup times and transfer times between machines, increases the realised due dates as expected.

Finally, for further researches on improving the performance of the proposed models, size of the problem instances and solution characteristics are given in the following table where (*) indicates the optimal solution. Note that, the computing time is set to maximum 3600 s. (See Table 6)

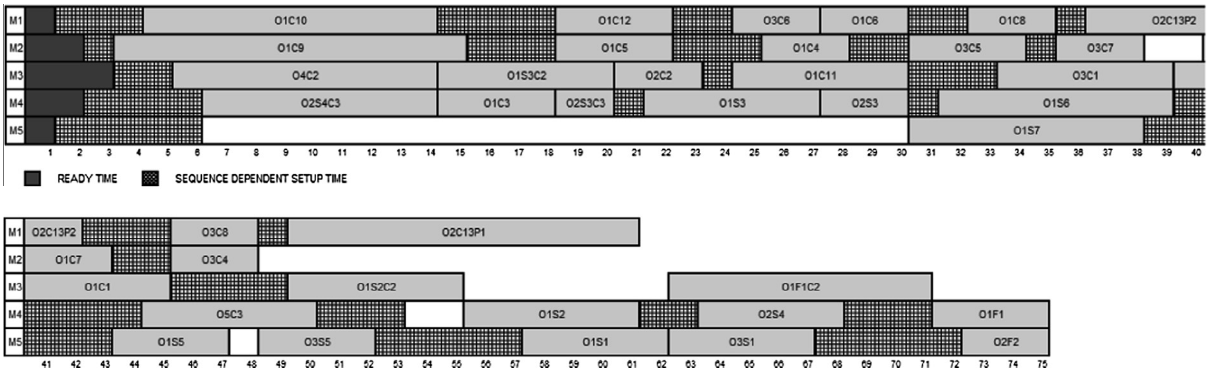

Fig. 10. APS with alternative machines and sequence-dependent setup times Gantt chart.

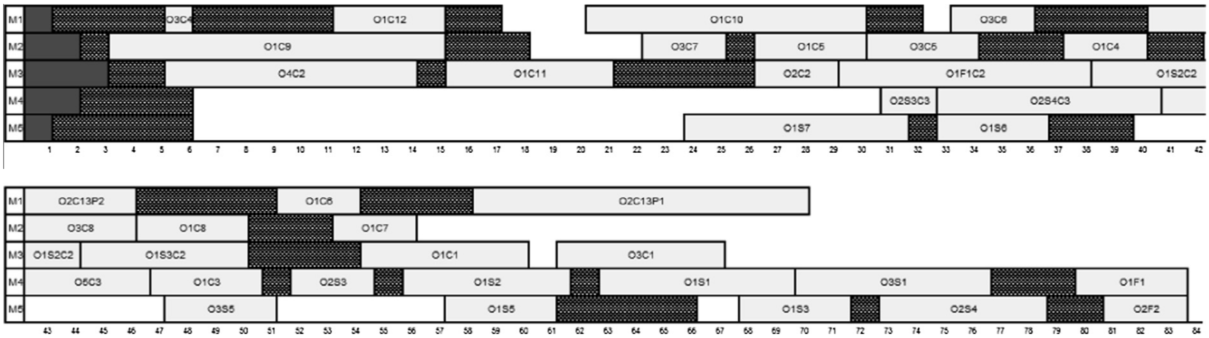

Fig. 11. APS with alternative machines and sequence-dependent setup and transfer times Gantt chart.

Table 5
Due date performance of the proposed models.

Order	Promised due date (in days)	Realised due date (in days)					Position (early/late/on time)					Difference (in days)				
		(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
O1F1	12	7	6	6	10	11	E	E	E	E	E	5	6	6	2	1
O2F2	14	7	6	6	10	11	E	E	E	E	E	7	8	8	4	3
O3S1	11	8	5	6	9	10	E	E	E	E	E	3	6	5	2	1
O4C2	2	8	2	3	2	2	L	O	L	O	O	6	0	1	0	0
O5C3	10	8	3	3	7	6	E	E	E	E	E	2	7	7	3	4

Table 6
Solution performance of the proposed models.

Problem	Number of constraints	Number of variables			Objective function	Solution time (s)
		Binary	Integer	Nonnegative		
APS without alternative machines (Fig. 8)	725	467	10	56	$6550*	40
APS with alternative machines (Fig. 7)	3221	1715	10	56	$3050	3600
APS with alternative machines and makespan objective (Fig. 9)	3221	1715	10	56	46 h	3600
APS with alternative machines and sequence-dependent setup times (Fig. 10)	18650	12250	12	69	$8450	3600
APS with alternative machines and sequence dependent setup and transfer times (Fig. 11)	125715	72275	12	69	$10,525	3600

4. Conclusions

In this paper, we develop and present mixed integer programming formulations for APS systems to be used at FP level in a typical job shop environment. The basic model is such that, an item can be processed on a given set of eligible machines, there are many products with multiple items at different levels and customers dictate due dates for their orders. The extensions to the basic model are sequence-dependent setups, and transfer times between machines. These formulations produce capacity feasible schedules which minimise total cost consisting of cost of idle times, cost of earliness and tardiness. The formulations could also be used to quote due dates for customer orders such that, in the order promising step, candidate orders can be fed into the model and more realistic and shop floor sensitive due dates can be returned to the customer using the model that minimizes the makespan (see Section 3.2.1). The tests on the models indicate that computing times are relatively shorter than those of found in literature for comparable problem sizes.

We note that our aim is not to provide optimal solutions to the formulations developed in this paper, rather to show how they can be applied in practice. Because, as stated in Pochet and Woolsey [22], the major characteristic of an APS system is optimisation or mathematical programming based algorithms. Hence, the mathematical models develoed can be used to gain insight on the system and instead of linear programming relaxation based optimisation methods, other methods like constraint programming, heuristics, etc., should be tried out that employ intelligent domain filtering and directed search facilities [26]. Furthermore, these methods can be adapted into an expert system.

A possible extension to the above models could be to convert static demands into dynamic demands. That is, at the beginning of every day new orders for the products are received and new schedules are prepared. In this case, either schedules so far prepared are assumed to be fixed and new orders are scheduled after them, or except for the items already completed or are in the process, the operations after the present day are rescheduled as new orders come in. Besides, reactive scheduling methods [21,27] and/or the cost of changes in the production schedule [25] can be considered as a resarch issue for handling dynamic demands.

In addition to changes in the demand pattern and preventive maintenance activities, there may be other unexpected but predictable circumstances in the manufacturing system like machine failures. Resarch on pro-active methods [28] and producing a schedule sensitive to machine failures is a challenging research area.

Furthermore, in this paper, it is assumed that items are produced on exactly one of the machines from the eligible machine set. However, if manufacturing technology allows, amount of production order of an item can be distributed to eligible machines (i.e., parallel machines, [29] and total cost of the schedule can be reduced more.

In this research, it is assumed that an item on a machine is transferred to the next machine once it is fully completed. However, in practice, it may be possible to transfer items to ne next machines with sublots. Hence, lot streaming in APS systems is a promising future research subject [30].

Finally, due to multiple goals of the APS problems such as minimizing earliness, tardiness and WIP costs, applying multi-objective programming methods [21] on proposed problems is a valuable research direction.

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