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A two-objective mathematical model without cutting patterns for one-dimensional assortment problems

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ABSTRACT

This paper considers a one-dimensional cutting stock and assortment problem. One of the main difficulties in formulating and solving these kinds of problems is the use of the set of cutting patterns as a parameter set in the mathematical model. Since the total number of cutting patterns to be generated may be very huge, both the generation and the use of such a set lead to computational difficulties in solution process. The purpose of this paper is therefore to develop a mathematical model without the use of cutting patterns as model parameters. We propose a new, two-objective linear integer programming model in the form of simultaneous minimization of two contradicting objectives related to the total trim loss amount and the total number of different lengths of stock rolls to be maintained as inventory, in order to fulfill a given set of cutting orders. The model does not require pre-specification of cutting patterns. We suggest a special heuristic algorithm for solving the presented model. The superiority of both the mathematical model and the solution approach is demonstrated on test problems.

1. Introduction

Problems of cutting large standard sizes into smaller order sizes demanded by customers or required for in-plant processing are found in various industries. They arise, e.g. in the production of paper rolls, steel bars, aluminum or wooden profiles. A large number of different types of cutting problems can be distinguished.

A typology on cutting and packing problems was investigated in [1,2]. One of the most important characteristics used for classification of such problems, is their dimensionality. The dimensionality is the minimum number of dimensions of real numbers necessary to describe the geometry of the patterns.

Especially, the one-dimensional cutting stock problem where smaller lengths (pieces, items) are to be cut from a (minimum) number of identical or different stock pieces is widely considered in the literature.

Two-dimensional problems appear in situations where a flat material has to be divided into products of smaller rectangular measures. Two-dimensional problems appear in situations where a flat material has to be divided into products of smaller rectangular measures.

One-and-a-half-dimensional cutting stock problem is a particular case of the two-dimensional problem in which the length of a sheet is sufficiently large (infinite for practical purposes). These kinds of problems arise when rectangular pieces are laid out on a very long roll of material.

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Usually, it is more economical to produce or procure only a few different types of standard sizes at first, to keep these sizes as inventory, and then to cut the stock sizes into the demanded sizes, than to produce or procure the required order sizes directly [3]. Therefore, two related problems arise: the cutting stock problem and the assortment problem [4].

The cutting stock problem is to determine how the available stock materials (particularly, paper rolls) should be cut into the required sizes.

The one-dimensional cutting stock problems with a single and multiple standard sizes have widely been studied in the literature; see for example, [5–9].

Many mathematical programming approaches for solving the cutting stock problems, assume the existence of the full set of cutting patterns, and the corresponding mathematical models use them as model parameters. Since the total number of cutting patterns to be generated may be very huge for such kinds of problems, both the generation and the use of the set of cutting patterns as a parameter set lead to computational difficulties in solving these problems. Therefore special solution procedures are required for finding satisfactory solutions.

The assortment problem, also referred to as a stock size selection problem, involves the choice of the best combination of different types of standard lengths (particularly, paper roll sizes) or briefly stock sizes to be maintained as inventory in order to minimize appropriate objective function(s). Under these conditions the efficiency of solutions obtained is closely related to the stock lengths selected.

Holthaus [3] considered the one-dimensional assortment problem and evaluated the possible savings in the total material cost which can be realized by using an assortment with two or more types of stock lengths, compared to an assortment with a single type of stock lengths. By investigating a large number of problem instances of different classes of the one-dimensional assortment problem, it is shown that there exist problem classes for which it is possible to realize substantial savings in the total material cost by using an assortment with two, three or four types of standard lengths.

Gasimov et al. [10] studied a 1.5-dimensional cutting stock and assortment problem. They proposed a new two-objective mixed integer linear programming model for solving this problem.

All the approaches presented in the above-mentioned works, use a set of cutting patterns as a parameter set in the mathematical model where one of the main characteristics is minimization of a total trim loss. It is well known that to produce a whole set of cutting patterns for problems with more than one stock material, is very difficult.

On the other hand, the trim loss can be reduced by increasing a number of different widths of roll stocks (see, for example [3,10]). Therefore, besides the minimization of a total trim loss amount, the minimization of the total number of different lengths of roll stocks to be maintained as inventory, is also of great importance.

In this paper, the two-objective linear integer programming mathematical model is developed for solving the one-dimensional cutting and assortment problem. The model has been constructed in the form of simultaneous minimization of two contradicting objectives, related to the total trim loss amount and the total number of different lengths of roll stocks to be maintained as inventory, in order to fulfill a given set of cutting orders. The model does not require pre-specification of cutting patterns.

A special heuristic algorithm for solving the presented model is presented. This algorithm uses special features of the problem under consideration and therefore is expected to be more efficient and faster than existing metaheuristics. The performances of both the mathematical model and the solution approach are demonstrated on test problems.

To the best of our knowledge, there are no previously considered models without the use of cutting patterns as a model parameter set, for solving the one-dimensional cutting stock and assortment problem.

The paper is organized as follows. The next section presents a detailed description of the problem. The heuristic algorithm for solving the mathematical model is explained in Section 3. The design of computational experiments and solution results are reported in Section 4. For comparison, an additional single-objective (on trim loss minimization only) mathematical model with cutting patterns is solved for every test problem. We present a comprehensive explanation on the solution results obtained. Finally, Section 5 draws some conclusions from this work.

2. Problem formulation

During a planning period, n order pieces are to be fulfilled. The sizes and the number of pieces are given. Given the set of all possible types or sizes of standard lengths which can be produced or procured from a supplier, the assortment problem involves the choice of the best combination of different types of standard lengths to be maintained as inventory and to be used for cutting the order pieces. In this case, the problem is to select the optimal (minimal) number of roll sizes that have to be stocked and to find the corresponding cutting patterns in order to produce the required order pieces which has to be cut from the selected rolls by simultaneously minimizing the total trim loss amount.

For formulating the mathematical model of this problem the following notations are introduced.

Sets and parameters

Let

- m be the number of all roll sizes (standard lengths), which are available for the producer,
- n be the number of all order pieces (products) with different widths to be fulfilled during the planning period,
- $I = \{1, \ldots, m\}$ be a set of roll sizes,
- $J = \{1, \ldots, n\}$ be a set of order pieces,

- c_j be the length of order piece j,
- d_j be the demand for order piece j,
- L_i be the length of roll i,
- K_i be the maximal number of times the roll of type i can be used in the production process.

Decision variables

Let

- x_{ik} be the binary variable indicating whether the roll of type i will be used for kth time:

$$x_{ik} = \begin{cases} 1 & \text{if roll } i \text{ is used for } k\text{th time,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

- z_i be the binary variable indicating whether the roll of type i will be used or not:

$$z_i = \begin{cases} 1 & \text{if roll } i \text{ is used,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

- y_{ijk} be the number of times the order piece j is involved in the roll of type i when this roll is used for kth time.

Objective functions

We have two objective functions:

- The total trim loss amount:

$$f_1(x, y, z) = \sum_{i=1}^{m} \sum_{k=1}^{K_i} \left[L_i x_{ik} - \sum_{j=1}^{n} c_j y_{ijk} \right]. \tag{3}$$

- The total number of roll types used:

$$f_2(x, y, z) = \sum_{i=1}^{m} z_i. \tag{4}$$

Under these notifications we can formulate a multi-objective IP model (P) for the one-dimensional cutting stock and assortment problem described above, in the following form:

$$(P) \ \min[f_1(x, y, z), f_2(x, y, z)] \tag{5}$$

subject to

$$\sum_{i=1}^{m} \sum_{k=1}^{K_i} y_{ijk} = d_j \quad \text{for all } j = 1, \dots, n, \tag{6}$$

$$\sum_{j=1}^{n} c_j y_{ijk} \le L_i x_{ik} \quad \text{for all } i = 1, \dots, m; \ k = 1, \dots, K_i, \tag{7}$$

$$\sum_{k=1}^{K_i} x_{ik} \le K_i z_i \quad i = 1, \dots, m. \tag{8}$$

x_{ik} and z_i binary, y_{ijk} nonnegative integer for all

$i = 1, \dots, m, \quad j = 1, \dots, n,$ and $k = 1, \dots, K_i$.

Constraint set (6) ensures that the demand for any order piece has to be met. Constraint set (7) are called the knapsack constraints and ensures that the length of the cutting pattern k generated for the roll of type i cannot exceed the lengths L_i of this roll. If any cutting pattern is assigned to a roll of type i, then constraint set (8) forces $z_i = 1$. If no cutting pattern is assigned to a roll of type i, then the left-hand sides of the inequality in constraint sets (8) are zero, and hence, the constraint set (8) permits a choice between $z_i = 0$ and $z_i = 1$. $z_i = 0$ must yield a smaller value of f_2 than $z_i = 1$. Therefore, because the objective is to minimize f_2, an algorithm yielding an optimal solution would always choose $z_i = 0$ when $\sum_{k=1}^{K_i} x_{ik} = 0$. Finally, the integrality constraints are added to functional constraints.

Note that problem (P) defined by relations (5)–(8) together with the integrality constraints may involve a huge number of binary and integer decision variables which causes difficulties in the solution process. In the next section the heuristic solution algorithm is presented for solving this mathematical model.

3. Solution method

The idea of the heuristic solution procedure developed for solving the mathematical model (P) is as follows. First, all the order pieces are ranked with respect to their lengths beginning from the largest one. Let $J = \{1, \dots, n\}$ be the ranked set of all order pieces.

Then, since the available roll type of shortest length is the cheapest one, all the available roll types are ranked with respect to their lengths beginning with one having minimal length. Let $I = \{1, \ldots, m\}$ be the ranked set of all roll types available.

We define the initial allowable trim loss length t (for example beginning with $t = 0$) and set $COUNT = 0$. Choose the first roll type, set $i = 1$.

The algorithm begins with generating cutting patterns for implementing on roll type i with trim loss t as follows. Choose the first order piece in J and set $j = 1$. The order piece j is placed on the roll of type i for $h_1 - COUNT$ times, where h_1 is a greatest number satisfying the inequalities $h_1 \leq d_j$, and $h_1 \times c_j \leq L_i$, and c_j and L_i are the lengths of order piece j and roll type i, respectively. Similarly, the next order piece (that is order piece $j = 2$) can be placed on the same roll i, and so be included in the same cutting pattern, if the remaining part of L_i is available (that is if $L_i - ((h_1 - COUNT) \times c_1) \geq c_2$), and so on. That is, the cutting pattern of the form $(h_1 - COUNT) \times c_1 + h_2 \times c_2 + h_3 \times c_3 + \cdots$ is generated so that the trim loss $L_i - ((h_1 - COUNT) \times c_1 + h_2 \times c_2 + h_3 \times c_3 + \cdots) \leq t$ is satisfied.

We denote this cutting pattern by C. The cutting pattern C is then implemented on the roll i by taking into account the demand amount for every product involved in that cutting pattern. The number of times that the cutting pattern will be implemented, is determined such that the total amount for every order piece involved in this cutting pattern does not exceed the demand amounts of the corresponding order pieces. Once the demand of some order piece is fulfilled, this product is excluded from the set of all products. After, the algorithm updates the set of order pieces J, and the demand amounts for all the products involved in J, the procedure is repeated for the new set of products beginning with the same roll type for which C is generated.

If the demands for all order pieces are fulfilled, then the first feasible solution has been obtained. The order of elements in the set J is changed so that the element $j + 1$ becomes the first one and the whole process is repeated for the updated set J. This leads to the second feasible solution and so on, the process continues for n times, until the nth element in J becomes the first one. Then, a feasible solution providing the minimal trim loss amount with a minimal roll type number is selected among all these n feasible solutions. This becomes a solution of the problem and the algorithm is terminated.

If no cutting pattern satisfying the condition "trim loss" $\leq t$ is generated, algorithm consider the next roll type by setting $i = i + 1$ and so on.

If the cutting pattern with "trim loss" $\leq t$ cannot be generated for all roll types, algorithm sets $COUNT = COUNT + 1$ and repeats the cutting pattern generating process, until $COUNT$ will become equal to h_1.

If the cutting pattern with "trim loss" $\leq t$ cannot be generated for all roll types, and for all available values of the parameter $COUNT$, algorithm increases the allowable trim loss length by setting $t = t + a$ and repeats the above steps. The number a can be chosen by taking into account the lengths of order pieces and roll types, for example one possible value for a may be $a = 1$ cm.

The use of parameter $COUNT$ allows us to generate different cutting patterns beginning with the different number of the same order piece.

To present the essential steps of this solution procedure, we define the following notations.

Let

- $I = \{1, \ldots, m\}$ be the set of all roll sizes, ranked with respect to their lengths beginning from the shortest one (or from the largest one, in dependence on desire of decision maker);
- $J = \{1, \ldots, n\}$ be a set of order pieces, ranked with respect to their lengths beginning from the largest one;
- t be the allowable trim loss length;
- $COUNT$ be the parameter that is used to determine the number of the first order piece in a cutting pattern.
- s be the number of feasible solutions.

With these notations, the algorithm for selecting the minimal number of stock sizes and for generating cutting patterns to minimize the total trim loss amount and satisfying demand constraints works as follows.

Algorithm. *Initial Step.* Set $I = \{1, \ldots, m\}, J = \{1, \ldots, n\}, i = 1, j = 1, k = 1, s = 1, t = 0$ (allowable trim loss length), $COUNT = 0$ and choose a suitable value for the parameter a.

Step 1. Generate all cutting patterns for roll size i, beginning with the first order piece s in J, whose trim loss do not exceed t, by the following way.

The order piece s is placed on the roll of type i for $h_s - COUNT$ times, where h_s is a greatest number satisfying the inequalities $h_s \leq d_s$, and $h_s \times c_s \leq L_i$, and c_s and L_i are the lengths of the order piece s and the roll type i, respectively. Similarly, the next order piece (that is the second element of J) can be placed on the same roll i, and so be included in the same cutting pattern, if the remaining part of L_i is available, that is if $L_i - ((h_s - COUNT) \times c_s) \geq c_2$, and so on. Thus, the cutting pattern of the form $(h_s - COUNT) \times c_s + h_2 \times c_2 + h_3 \times c_3 + \cdots$ is generated such that, the trim loss $L_i - ((h_s - COUNT) \times c_s + h_2 \times c_2 + h_3 \times c_3 + \cdots) \leq t$ is satisfied.

If such a cutting pattern is generated go to *Step* 2, if no cutting pattern is generated, set $i = i + 1$ and consider the next stock size and so on.

If the cutting pattern with "trim loss" $\leq t$ cannot be generated for all roll types, set $COUNT = COUNT + 1$ and repeat the cutting pattern generating process, until $COUNT$ will become equal to h_1.

If the cutting pattern satisfying "trim loss $\leq t$" could not be generated for all roll sizes, increase the allowable trim loss length by setting $t = t + a$ and try again.

Step 2. Let C_k be the cutting pattern generated in Step 1, and let i be the roll type for which C_k is generated. Implement the cutting pattern C_k on the roll i for d times, where d is determined such that the total amount for every order piece included in this cutting pattern, will not exceed its demand amount.

Step 3. Update the demand amounts for the order pieces involved in the cutting pattern C_k. Update the set J of all order pieces, by excluding the ones whose demands are fulfilled. If $J = \emptyset$ (that is the demands for all order pieces are fulfilled), then sth feasible solution has been calculated: go to *Step* 4. If $J \neq \emptyset$ then update the set I by putting the stock size i to the first place, set $t = 0$, $k = k + 1$ and go to *Step* 1.

Step 4. Set $s = s + 1$. If $s \leq n$ then update the set J as $J = \{s, 1, \ldots, s-1, s+1, \ldots, n\}$, set $i = 1$ and go to *Step* 1.

If $s > n$ then choose the feasible solution with a minimal trim loss amount and a minimal roll type number and STOP: the solution is found.

4. Computational results

In this section, the superiority of both the developed mathematical model and the performance of the proposed algorithm is demonstrated on nine test problems with different values of the following model parameters:

- the number of available roll size types with their lengths,
- the number of order pieces with their lengths, and
- demand amounts.

Nine test problems have been generated randomly. The Visual Basic code has been written for implementing the proposed algorithm.

For comparison, an additional single-objective (on trim loss minimization only) mathematical model with cutting patterns is solved for every test problem. This mathematical model is solved by applying GAMS/CPLEX solver. Documentation and information about GAMS are available via the World Wide Web at the URL: www.gams.com. Computations were carried out on Intel Core 2 Duo computer with 2.00 GHz processor, 1.87 GB RAM.

In what follows we explain each test problem separately and present their solutions obtained for both mathematical models.

First we present the additional mathematical model.

The mathematical model with cutting patterns

Sets and parameters

Let

- m be the number of all roll sizes (standard lengths), which are available for the producer,
- n be the number of all order pieces (products) with different widths to be fulfilled during the planning period,
- p be the number of all cutting patterns,
- $I = \{1, \ldots, m\}$ be a set of roll sizes,
- $J = \{1, \ldots, n\}$ be a set of order pieces,
- $K = \{1, \ldots, p\}$ be a set of all cutting patterns,
- c_j be the length of product j,
- d_j be the demand for product j,
- a_{jk} be the total number of order piece j contained in cutting pattern k, $j \in J$, $k \in K$,
- f_{ik} be the trim loss amount from roll i and cutting pattern k, (in cm), $i \in I$, $k \in K$,
- S be the number of different types of standard lengths to be selected from m possible roll sizes,
- U be a large positive constant.

Decision variables

Let

- x_{ik} be the integer variable indicating the number of times that the roll of type i will be cut using the cutting pattern k, $i \in I$, $k \in K$.
- z_i be the binary variable indicating whether the roll of type i will be used or not:

$$z_i = \begin{cases} 1 & \text{if roll } i \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

For simplification, we consider the situation when the producer is interested in choosing only a few number of types among all the possible m roll sizes. In this case the problem becomes to determine which S roll sizes and how many of each size should be stocked in order to minimize the total trim loss and to fulfill the order pieces during the planning period. For this model the number of different roll sizes to be kept as inventory is treated as a parameter S.

Under these notifications we can formulate the single-objective integer programming model ($P1$) with cutting patterns, for the one-dimensional cutting stock and assortment problem as

$$(P1)\ \min \sum_{i=1}^{m} \sum_{k=1}^{p} f_{ik} x_{ik} \tag{9}$$

subject to

$$\sum_{i=1}^{m}\sum_{k=1}^{p} a_{jk}x_{ik} = d_j \quad \text{for all } j = 1, \ldots, n, \tag{10}$$

$$\sum_{k=1}^{p} x_{ik} \le Uz_i \quad \text{for all } i = 1, \ldots, m. \tag{11}$$

$$\sum_{i=1}^{m} z_i \le S, \tag{12}$$

$$x_{ik} \quad \text{nonnegative integer for all } i \in I,\ k \in K. \tag{13}$$

4.1. Experiments

The following notations and parameters are used to summarize the test problems and solution results.

- $L = (L_1, \ldots, L_m)$ is the vector of different roll sizes, where L_i is the length of roll type i.
- $c = (c_1, \ldots, c_n)$ is the vector of different order pieces, where c_j is the length of order piece j.
- $d = (d_1, \ldots, d_n)$ is the vector of demand amounts for order pieces, where d_j is the demand amount for order piece j.
- K is the available number of rolls of every type which can be used in the production process, that is in all experiments we have taken $K_i = K$ for all roll types i.
- I_{opt} denotes the set of roll types selected by a solution method.
- The set $I_{\text{opt}} \times N = \{(i, N_i)\}_{i \in I_{\text{opt}}}$ is used to denote the pairs of solution results, where i indicates the type of the roll size i selected, and N_i denotes the total amount of roll size i used in the production process to fulfill the demand.

Experiment 1.

$$m = 2, \qquad n = 5, \qquad L = (100, 110),$$
$$c = (10, 20, 30, 40, 60), \qquad d = (6, 11, 4, 20, 15), \qquad K = 21.$$

Experiment 2.

$$m = 3, \qquad n = 10, \qquad L = (100, 110, 120),$$
$$c = (10, 20, 30, 40, 60, 15, 25, 35, 45, 65), \qquad d = (7, 11, 3, 20, 15, 5, 10, 13, 20, 15),$$
$$K = 32.$$

Experiment 3.

$$m = 4, \qquad n = 20, \qquad L = (100, 110, 120, 130),$$
$$c = (10, 20, 30, 40, 60, 15, 25, 35, 45, 65, 11, 12, 13, 14, 21, 22, 23, 24, 31, 32),$$
$$d = (16, 11, 13, 20, 15, 15, 10, 13, 20, 15, 15, 11, 13, 20, 15, 15, 10, 13, 2, 15),$$
$$K = 42.$$

Experiment 4.

$$m = 4, \qquad n = 30, \qquad L = (200, 220, 240, 280),$$
$$c = (10, 20, 30, 40, 60, 15, 25, 35, 45, 65, 11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34, 41, 42, 43, 44, 51, 52, 53, 54),$$
$$d = (5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15),$$
$$K = 34.$$

Experiment 5.

$$m = 4, \qquad n = 40, \qquad L = (100, 110, 120, 130),$$
$$c = (10, 20, 30, 40, 60, 15, 25, 35, 45, 65, 11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34, 41, 42, 43, 44, 51, 52, 53, 54, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71),$$
$$d = (5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15),$$
$$K = 191.$$

Experiment 6.

$$m = 5, \qquad n = 40, \qquad L = (10000, 9400, 5200, 8900, 6800),$$

$$\begin{aligned} c = {} & (732, 1746, 1210, 290, 1212, 715, 1471, 1405, 1974, 344, 1699, 172, 351, 1227, 1739, 272, \\ & 1903, 1121, 1326, 107, 726, 1917, 1116, 501, 1599, 439, 821, 485, 361, 860, 1252, 562, 1131, 271, \\ & 1075, 987, 1171, 1979, 228, 1370), \\ d = {} & (217, 232, 265, 249, 266, 269, 215, 215, 213, 267, 299, 259, 287, 284, 277, 223, 200, 255, 269, 226, 240, \\ & 209, 266, 254, 241, 264, 229, 257, 285, 204, 255, 257, 283, 222, 218, 289, 244, 214, 223, 290), \\ K = {} & 547. \end{aligned}$$

Experiment 7.

$$m = 5, \qquad n = 100, \qquad L = (400, 500, 600, 700, 800),$$

$$\begin{aligned} c = {} & (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, \\ & 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, \\ & 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, \\ & 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100), \\ d = {} & (5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, \\ & 3, 20, 15, 5, 10, 13, 20, 15, 13, 13, 13, 13, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, \\ & 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, \\ & 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 17, 17), \\ K = {} & 107. \end{aligned}$$

Experiment 8.

$$m = 10, \qquad n = 20, \qquad L = (101, 102, 103, 104, 105, 106, 107, 108, 109, 110),$$

$$c = (50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 32, 33, 42, 44, 27, 19, 10, 40, 20, 30),$$

$$d = (273, 20, 27, 19, 32, 28, 100, 82, 55, 42, 48, 35, 29, 50, 35, 40, 23, 42, 51, 32),$$

$$K = 101.$$

Experiment 9.

$$m = 10, \qquad n = 200, \qquad L = (500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, 1400),$$

$$\begin{aligned} c = {} & (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, \\ & 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, \\ & 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, \\ & 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, \\ & 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, \\ & 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, \\ & 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, \\ & 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, \\ & 193, 194, 195, 196, 197, 198, 199, 200), \\ d = {} & (5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, 3, 20, 15, 5, 10, 13, 20, 15, 5, 11, \\ & 3, 20, 15, 5, 10, 13, 20, 15, 13, 13, 13, 13, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, \\ & 14, 14, 14, 14, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 16, 16, \\ & 16, 16, 16, 16, 16, 16, 16, 16, 17, 17, 16, 16, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, \\ & 17, 17, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 19, 19, 19, 19, 19, 19, 19, \\ & 19, 19, 19, 19, 19, 19, 19, 19, 19, 19, 19, 19, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, \\ & 20, 20, 20, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 22, 22, 22), \\ K = {} & 233. \end{aligned}$$

4.2. Solution results

In this subsection we present solution results obtained for test problems described in the previous subsection. We first present the solution results for problem (P) obtained by using the heuristic algorithm developed. Tables 1–4 present solution results for Experiments 1–4, respectively, where the numbers of roll types selected by the algorithm, along with the cutting patterns generated for implementing on that roll, the total number of every roll type used and the trim loss amount are depicted.

Table 1
Solution results for Experiment 1 obtained by the heuristic.

Cutting pattern no.	Roll type selected	Number of rolls used	Cutting patterns generated	Trim loss
1	100	15	$(1 \times 60) + (1 \times 40)$	0
2	100	2	$(2 \times 40) + (1 \times 20)$	0
3	100	1	$(1 \times 40) + (2 \times 30)$	0
4	100	1	$(2 \times 30) + (2 \times 20)$	0
5	100	1	(5×20)	0
6	100	1	$(2 \times 20) + (6 \times 10)$	0

Table 2
Solution results for Experiment 2 obtained by the heuristic.

Cutting pattern no.	Roll type	Number of rolls used	Cutting pattern	Trim loss
1	100	10	$(2 \times 40) + (1 \times 20)$	0
2	100	13	$(1 \times 65) + (1 \times 35)$	0
3	110	2	$(1 \times 65) + (1 \times 45)$	0
4	120	7	(2×60)	0
5	120	1	$(1 \times 60) + (1 \times 45) + (1 \times 15)$	0
6	120	3	$(2 \times 45) + (1 \times 30)$	0
7	100	5	$(2 \times 45) + (1 \times 10)$	0
8	100	2	(4×25)	0
9	100	1	$(1 \times 45) + (2 \times 25)$	5
10	100	1	$(1 \times 20) + (4 \times 15) + (2 \times 10)$	0

Table 3
Solution results for Experiment 3 obtained by the heuristic.

Cutting pattern no.	Roll type	Number of rolls used	Cutting pattern	Trim loss
1	100	10	$(2 \times 40) + (1 \times 20)$	0
2	100	13	$(1 \times 65) + (1 \times 35)$	0
3	110	2	$(1 \times 65) + (1 \times 45)$	0
4	120	7	(2×60)	0
5	120	1	$(1 \times 60) + (1 \times 45) + (1 \times 15)$	0
6	120	8	$(2 \times 45) + (1 \times 30)$	0
7	120	1	$(1 \times 45) + (2 \times 32) + (1 \times 11)$	0
8	120	4	$(3 \times 32) + (1 \times 24) + (1 \times 11)$	0
9	100	1	$(3 \times 30) + (1 \times 10)$	0
10	100	2	(4×25)	0
11	110	1	$(4 \times 24) + (1 \times 14)$	0
12	110	1	$(3 \times 24) + (1 \times 23) + (1 \times 15)$	0
13	130	1	$(5 \times 23) + (1 \times 15)$	0
14	100	1	$(1 \times 23) + (3 \times 22) + (1 \times 11)$	0
15	100	2	$(4 \times 22) + (1 \times 12)$	0
16	100	1	$(1 \times 22) + (3 \times 21) + (1 \times 15)$	0
17	120	1	$(5 \times 21) + (1 \times 15)$	0
18	120	1	$(3 \times 21) + (3 \times 15) + (1 \times 12)$	0
19	100	1	$(2 \times 15) + (5 \times 14)$	0
20	110	1	$(7 \times 14) + (1 \times 12)$	0
21	130	1	$(2 \times 14) + (7 \times 13) + (1 \times 11)$	0
22	100	1	$(1 \times 32) + (2 \times 31)$	6
23	100	1	$(2 \times 30) + (1 \times 25) + (1 \times 15)$	0
24	100	1	$(1 \times 25) + (2 \times 24) + (1 \times 23)$	4
25	100	1	$(2 \times 23) + (2 \times 22) + (1 \times 10)$	0
26	100	1	$(1 \times 22) + (3 \times 21) + (1 \times 15)$	0
27	100	1	$(1 \times 21) + (1 \times 20) + (3 \times 15) + (1 \times 14)$	0
28	100	1	$(4 \times 14) + (3 \times 13)$	5
29	100	1	$(3 \times 13) + (5 \times 12)$	1
30	100	1	$(2 \times 12) + (6 \times 11) + (1 \times 10)$	0
31	100	1	$(6 \times 11) + (3 \times 10)$	4
32	100	1	(10×10)	0

Table 1 demonstrates that for solving the cutting and assortment problem whose data is given in Experiment 1, the roll of size 100 is the only one which has been selected by the algorithm, and this roll is used for totally $21 = 15+2+1+1+1+1$ times. The heuristic algorithm implemented for solving the mathematical model (P) for this problem, has generated six cutting patterns. The first cutting pattern which consists of two order pieces – one of size 60 and the other one of size 40, has been implemented for 15 times. The second cutting pattern consists of tree order pieces – two of size of 40 and one of size 20, and so on. The total trim loss is zero.

The solution results obtained by the heuristic for Experiment 2, are presented in Table 2. The obtained results demonstrate that the demand constraints for all order pieces are satisfied as equality. For example, the order piece of size 65 is contained in the cutting patterns presented on second and third rows of Table 2. Each of these cutting patterns contain this order piece for only one time. The first cutting pattern, that is $(1 \times 65) + (1 \times 35)$, is implemented on the roll of size 100 for 13 times, and the second one $(1 \times 65) + (1 \times 45)$ is implemented on the roll of size 110 for 2 times. Thus, $13 + 2 = 15$ which equals the demand amount for this order piece (see the description of Experiment 2 presented in the previous section). Table 2 demonstrates that totally 10 cutting patterns have been generated by the heuristic for solving Experiment 2, among which only ninth cutting pattern has a trim loss of size $5 : 100-[(1\times45)+(2\times25)] = 5$. All the available (tree) roll types have been used in the production process. The roll of size 100 has been used by 6 cutting patterns totally for $10+13+5+2+1+1 = 32$ times. Similarly, the rolls of sizes 110 and 120 are used for 2 and 11 times, respectively (see also Table 6).

The solution results obtained by the heuristic for Experiment 3, are presented in Table 3. Let us explain the steps of the heuristic algorithm on this table. It seems that a feasible solution beginning with a cutting pattern which uses order piece of size 40 first, has been selected as the best solution by the heuristic algorithm. The set of products J for this solution corresponds to the order of products in the form of 40, 65, 60, 45, 35, 32, and so on. The order of order pieces in the first cutting pattern begins by the product of length 40. This cutting pattern has been generated for implementing on roll of the shortest length 100. The value of the parameter h_1 for this product and roll type pair, equals 2 (because $100 - \mathbf{2} \times 40 = 20 < 40$). For the remaining part of this roll (of length 20,) the most suitable product is the order piece of length 20. Thus, the first cutting pattern $(2 \times 40) + (1 \times 20)$ is generated for roll of length 100 with zero trim loss. This cutting pattern is implemented on the roll of size 100 for 10 times. Therefore, the demand amount of 20 for the order piece of size 40 has been entirely satisfied. Therefore, the product of length 40 is excluded from the set of order pieces. On the other hand, since the demand for product of size 20, equals 11, in the demand amount for this product will be made an adjustment such that, a new demand amount for this product equals 1 ($=11 - 10$) after this step.

To generate the next cutting pattern, algorithm tries the second element in the updated set J (that is the product of length 65) and the same roll of size 100. For this selection, the most suitable (with zero trim loss) cutting pattern is $(1\times65)+(1\times35)$. The demand amounts for the products of lengths 65 and 35, are 15 and 13, respectively. Therefore, algorithm implements this cutting pattern for 13 times and excludes the order piece of length 35 from the set J. At the same time, in the demand amount for product of length 65 is made an adjustment, and it has been put equal to 2 ($= 15 - 13$).

All trials for the next cutting pattern on the roll of size 100 having the product of size 65 at the first place, leads to trim loss amount greater than $t = 0$. Therefore, algorithm considers the next roll, that is the roll of size 110. For this roll, the first cutting pattern that should be considered, is the pair of products of lengths 65 and 45, because it is not possible to cut the cutting pattern consisting of pair of products with lengths of 65 and 60 (which is the next to the product of length 65 in the set J) from the roll of length 110. Hence, the cutting pattern $(1 \times 65) + (1 \times 45)$ (with zero trim loss) is generated, and implemented for 2 times, and so on.

Table 4 presents another interesting interpretation for the heuristic algorithm implemented for Experiment 4. Algorithm has recognized that the feasible solution whose cutting patterns list begins with the product of length 51, is the best one and this cutting pattern has been implemented on the largest roll size. For this solution, the set of product indices J corresponds to the sequence of products of the form 51, 65, 60, 54, 53, 52, 45, 44, and so on. This situation can obviously be viewed from the sequence of cutting patterns presented in Table 4.

The total number of best cutting patterns generated by the heuristic and the total number of roll sizes used in the solution process of problem (P) for all experiments are depicted in Table 5. For comparison, the total number of cutting patterns used in the parameter set of problem ($P1$) along with the total number of rolls used in the solution process, are given in the same table.

The Visual Basic code has been written for generating cutting patterns for solving the mathematical model ($P1$). The set of cutting patterns were generated in two stages. In the first stage cutting patterns involving single order piece are generated. In the second stage, cutting patterns with more than one order piece were allowed. Since the number of all possible cutting patterns can be very huge, only cutting patterns with a trim loss not greater than a 20% of a corresponding roll type were generated. For solving problem ($P1$), CPLEX solver of GAMS software is implemented. Solution results have been obtained for only first, second and third experiments (see Tables 5 and 6).

Table 6 demonstrates superiority of both the proposed mathematical model and the developed heuristic for its solution over the mathematical model with cutting patterns. A reasonable number of cutting patterns could have been generated as a parameter set only for first three experiments. The solution results of ($P1$) obtained for all these experiments are worse than those obtained by the heuristic for (P). For example, the trim loss amounts obtained for model ($P1$) by GAMS, are greater than those obtained for (P) by the heuristic. The solution time for Experiment 3 is 1523.6 s for ($P1$), while it is only 2 s for (P). This solution time does not include the time for generating cutting patterns. Unfortunately, the program code for generating only a restricted number of cutting patterns could not be terminated within the reasonable time period and therefore we could not solve problem ($P1$) for Experiments 4–9.

The results presented for Experiments 5 and 8 in Table 6 show also that, the heuristic minimizes the number of roll types used in production process. Although the total numbers of different roll types available for these experiments are 5 and 10, there has been chosen only 2 and 8 roll types, respectively.

After generating the all n feasible solutions, decision maker can use her/his own priorities in selecting the best solution. If the number of roll types has a greater priority, then the corresponding solution can be chosen.

Table 4
Solution results for Experiment 4 obtained by the heuristic.

Cutting pattern no.	Roll type	Number of rolls used	Cutting pattern	Trim loss
1	280	2	$(5 \times 51) + (1 \times 25)$	0
2	280	3	$(4 \times 65) + (1 \times 20)$	0
3	280	1	$(3 \times 65) + (1 \times 60) + (1 \times 25)$	0
4	280	3	$(4 \times 60) + (1 \times 40)$	0
5	280	1	$(2 \times 60) + (2 \times 54) + (1 \times 52)$	0
6	280	2	$(5 \times 54) + (1 \times 10)$	0
7	280	1	$(3 \times 54) + (2 \times 53) + (1 \times 12)$	0
8	280	3	$(5 \times 53) + (1 \times 15)$	0
9	200	1	$(2 \times 53) + (1 \times 52) + (1 \times 42)$	0
10	200	3	$(3 \times 52) + (1 \times 44)$	0
11	200	1	$(1 \times 52) + (3 \times 45) + (1 \times 13)$	0
12	200	4	$(4 \times 45) + (1 \times 20)$	0
13	240	1	$(1 \times 44) + (4 \times 43) + (1 \times 24)$	0
14	240	2	$(5 \times 43) + (1 \times 25) + (1 \times 11)$	0
15	240	2	$(5 \times 42) + (1 \times 30)$	0
16	200	1	$(4 \times 42) + (1 \times 32)$	0
17	200	1	$(3 \times 42) + (1 \times 41) + (1 \times 33)$	0
18	200	3	(5×40)	0
19	200	2	$(5 \times 35) + (1 \times 25)$	0
20	280	1	$(6 \times 34) + (2 \times 32) + (1 \times 12)$	0
21	200	1	$(1 \times 53) + (1 \times 52) + (1 \times 45) + (1 \times 44)$	6
22	200	1	$(1 \times 43) + (2 \times 42) + (1 \times 41) + (1 \times 32)$	0
23	200	1	$(1 \times 41) + (2 \times 40) + (2 \times 35)$	9
24	200	1	$(1 \times 35) + (4 \times 34) + (1 \times 25)$	4
25	200	1	$(1 \times 34) + (4 \times 33) + (1 \times 32)$	2
26	200	1	(6×32)	8
27	200	1	$(4 \times 32) + (2 \times 31) + (1 \times 10)$	0
28	200	3	$(6 \times 31) + (1 \times 14)$	0
29	200	1	$(1 \times 30) + (2 \times 25) + (5 \times 24)$	0
30	200	1	$(7 \times 24) + (1 \times 23)$	9
31	200	1	$(8 \times 23) + (1 \times 15)$	1
32	200	1	$(1 \times 23) + (5 \times 22) + (3 \times 21)$	4
33	200	1	$(9 \times 21) + (1 \times 11)$	0
34	200	1	$(3 \times 21) + (4 \times 20) + (1 \times 15) + (3 \times 14)$	0
35	200	1	(14×14)	4
36	200	1	$(2 \times 13) + (9 \times 12) + (4 \times 11) + (2 \times 10)$	2

Table 5
Number of (optimal) cutting patterns and used rolls for problems (P) and ($P1$).

Exp. no.	Results for (P)		Results for ($P1$)	
	Number of cutting patterns generated	Total number of rolls used in solution process	Number of cutting patterns used	Total number of rolls used in solution process
1	6	21	42	21
2	10	45	95	44
3	32	72	424	69
4	36	56	1153	–
5	36	208	399	–
6	125	1371	454	–
7	95	161	17,568	–
8	32	473	118	–
9	231	453	43,174	–

Finally, we have added all the best cutting patterns generated by the heuristic, to the parameter set of problem ($P1$) and solved it again for Experiments 1–3 using GAMS. The results are presented in Table 7.

Table 7 demonstrates the strength of the proposed approach. The difference between the results obtained for problem ($P1$) depicted in Tables 6 and 7 show the effect of the cutting patterns generated by the heuristic algorithm. After adding best cutting patterns generated by the heuristic, to the parameter set of problem ($P1$), the trim loss amounts for all three experiments have been reduced. These trim loss amounts were 20, 25 and 50 for Experiments 1, 2 and 3, respectively (see Table 6). After including the cutting patterns calculated by the heuristic algorithm, these amounts have became 0, 0 and 1. The important ingredient here maybe the solution time. For example, Experiment 3 has 4 available roll types and 20 order pieces. Totally 424 cutting patterns have originally been generated by the Visual Basic code for the parameter set for solving this experiment (see Table 5). The solution time is 1523.6 s and trim loss equals 50 (see Table 6). The same problem,

Table 6
Results of numerical experiments for all experiments using the heuristic and GAMS software.

Exp. no.	m	n	Results for (P)			Results for $(P1)$		
			Trim loss	$I_{\text{opt}} \times N$	Time (s)	Trim loss	$I_{\text{opt}} \times N$	Time (s)
1	2	5	0	(1, 21)	<1	20	(1, 19), (2, 2)	<1
2	3	10	5	(1, 32), (2, 2) (3, 11)	<1	25	(1, 17), (2, 18) (3, 9)	<1
3	4	20	20	(1, 42), (2, 5), (3, 23), (4, 2)	2	50	(1, 16), (2, 28), (3, 13), (4, 12)	1523.6
4	4	30	49	(1, 34), (3, 5) (4, 17)	2	–	–	–
5	4	40	436	(1, 191), (2, 17)	2	–	–	–
6	5	40	7870	(1, 139), (2, 238) (3, 547), (4, 144) (5, 303)	2	–	–	–
7	5	100	80	(1, 107), (2, 35) (3, 2), (4, 16), (5, 1)	2	–	–	–
8	10	20	223	(1, 64), (2, 7) (3, 12), (4, 32) (6, 80), (7, 101) (9, 81), (10, 76)	2	–	–	–
9	10	200	38	(1, 33), (2, 37) (3, 233), (4, 97) (5, 8), (6, 1) (7, 33), (8, 4) (9, 23), (10, 4)	2	–	–	–

Table 7
Solution results obtained for problem $(P1)$ after including best cutting patterns generated by the heuristic to the parameter set of Experiments 1–3.

Exp. no.	Trim loss	Total number of rolls used	Numbers of roll types used	$I_{\text{opt}} \times N$	Time (s)
1	0	21	1	(1, 21)	1
2	0	45	3	(1, 29), (2, 10) (3, 6)	1
3	1	71	4	(1, 30), (2, 9) (3, 14), (4, 18)	50000.44

after adding the best cutting patterns obtained by the heuristic, has been solved by the same GAMS solver in 50000.44 s (see Table 7) with a trim loss amount of 1. Note that problem (P) for the same experiment has been solved by the heuristic only in 2 s with trim loss 20. These results show that in some situations the heuristic can be used for obtaining some "better" cutting patterns first, then these cutting patterns may be embedded into the parameter set of the corresponding mathematical model. But it should be remembered that it becomes unavailable to solve such a model for a huge data set.

5. Conclusions

In this paper, the two-objective linear integer programming mathematical model is developed for solving the one-dimensional cutting and assortment problem. The model has been constructed in the form of simultaneous minimization of two contradicting objectives, related to the total trim loss amount and the total number of different lengths of roll stocks to be maintained as inventory, in order to fulfill a given set of cutting orders. The model does not require pre-specification of cutting patterns.

To the best of our knowledge, such a model has not been treated in the literature before.

A special heuristic algorithm for solving the presented model, is presented. The performances of both the mathematical model and the solution approach are demonstrated on nine test problems.

For comparison, an additional single-objective mathematical model with cutting patterns is solved for every test problem. A comprehensive explanations on the solution results are presented.

These explanations demonstrate the superiority of both the constructed mathematical model and the solution method.

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