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Short Communication

A note on "A mixed integer programming model for advanced planning and scheduling (APS)"

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ABSTRACT

In a recent paper, Chen and Ji [Chen, K., Ji, P., 2007. A mixed integer programming model for advanced planning and scheduling (APS). European Journal of Operational Research 181, 515–522] develop a mixed integer programming model for advanced planning and scheduling problem that considers capacity constraints and precedence relations between the operations. The orders require processing of several operations on eligible machines. The model presented in the above paper works for the case where each operation can be processed on only one machine. However, machine eligibility means that only a subset of machines are capable of processing a job and this subset may include more than one machine. We provide a general model for advanced planning and scheduling problems with machine eligibility. Our model can be used for problems where there are alternative machines that an operation can be assigned to.

1. Mathematical model

In this section, we first define the parameters and variables used in the mathematical model which is based on that of Chen and Ji (2007). We also add new parameters that identify the eligible machines of the jobs and new variables for job-machine assignments. For similar type of problem definitions and model development see Liao et al. (2009) and Sawik (2004).

Objective function

$$Min\left\{I\left(m \times C_{\max} - \sum_{i=1}^{n}\sum_{p\in A(P_i)}\sum_{k=1}^{m}(t_{ipk} \times Q_i \times N_{ip} \times Z_{ipk}) - \sum_{k=1}^{m} r_k\right) + \sum_{i=1}^{n}(TC \times L'_i + EC \times E'_i)\right\} \quad (1)$$

Subject to

$$C_i \leqslant C_{\max} \quad \forall i \quad (2)$$

$$S_{ipk} + M(1 - Z_{ipk}) \geqslant r_k \quad \forall i,p,k \quad (3)$$

$$\sum_{k\in F_p} S_{ipk} \geqslant \sum_{l\in F_q} S_{iql} + \sum_{l\in F_q}(t_{iql} \times N_{iq} \times Q_i \times Z_{iql}) \quad \forall i,\ (q,p)\in R_i \quad (4)$$

$$\sum_{k\in F_{P_i}} S_{iP_ik} + \sum_{k\in F_{P_i}}(t_{iP_ik} \times Q_i \times N_{iP_i} \times Z_{iP_ik}) = C_i \quad \forall i \quad (5)$$

$$S_{jqk} \geqslant S_{ipk} + (t_{ipk} \times Q_i \times N_{ip}) - M(1 - Y_{ipjqk}) \quad \forall k,\ q\in A(P_j),\ p\in A(P_i),\ k\in F_q\cap F_p : p\neq q \quad (6)$$

$$S_{ipk} \geqslant S_{jqk} + (t_{jqk} \times Q_j \times N_{jq}) - M(Y_{ipjqk}) - M(1 - Z_{jqk}) - M(1 - Z_{ipk}) \quad \forall k,\ q\in A(P_j),\ p\in A(P_i),\ k\in F_q\cap F_p : p\neq q \quad (7)$$

$$Z_{ipk} + Z_{jqk} \geqslant 2(Y_{ipjqk} + Y_{jqipk}) \quad \forall k,\ q\in A(P_j),\ p\in A(P_i),\ k\in F_q\cap F_p : p\neq q \quad (8)$$

$$Z_{ipk} + Z_{jqk} \leqslant Y_{ipjqk} + Y_{jqipk} + 1 \quad \forall k,\ q\in A(P_j),\ p\in A(P_i),\ k\in F_q\cap F_p : p\neq q \quad (9)$$

$$S_{ipk} \leqslant M \times Z_{ipk} \quad \forall i,p,k \quad (10)$$

$$\sum_{k\in F_p} Z_{ipk} = 1 \quad \forall i,\ p\in A(P_i) \quad (11)$$

$$\frac{C_i}{8} - d_i \leqslant L_i \quad \forall i \quad (12)$$

$$d_i - \frac{C_i}{8} \leqslant E_i \quad \forall i \quad (13)$$

$$L'_i \geqslant L_i \quad \forall i \quad (14)$$

$$E'_i \geqslant E_i - 0.99 \quad \forall i \quad (15)$$

$$\sum_{k\in F_p} Z_{ipk} \leqslant 0 \quad \forall i,p,\ N_{ip} = 0 \quad (16)$$

$$C_{\max} \geqslant 0 \quad (17)$$

$$S_{ipk} \geqslant 0 \quad \forall i,p,k \quad (18)$$

$$C_i, E_i, L_i \geqslant 0 \quad \forall i \quad (19)$$

$$E'_i, L'_i \geqslant 0 \text{ and integer} \quad \forall i \quad (20)$$

$$Y_{ipjqk}, Z_{ipk} \in \{0,1\} \quad \forall i,p,j,q,k \quad (21)$$

The objective function (1) minimizes the total of production idle time and tardiness and earliness costs. Constraints (2) ensure that the completion time of any order is less than or equal to production makespan ($C_{\max}$). Constraints (3) state that if one item is assigned to a machine, item start time should be equal to or

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Nomenclature

Indices

i, j	index of order $i, j = 1, \ldots, n$
p, q	index of item $p, q = 1, \ldots, b$
k, l	index of machine $k, l = 1, \ldots, m$

Parameters

n	number of orders
b	number of items
m	number of machines
O_i	order index $(i = 1, \ldots, n)$
P_i	final item of order i
Q_i	quantity of order i
N_{ip}	number of item p needed for one unit of order i, $N_{iP_i} = 1$
t_{ipk}	processing time required by item p of order i on machine M_k $(p = 1, \ldots, b)$
r_k	ready time of machine M_k
d_i	due date of order i
I	cost of idle time per hour
TC	cost of tardy orders per day per job
EC	cost of early orders per day per job
M	a large positive number
$A(P_i)$	set of child items of item P_i
R_i	the set of immediate predecessor–successor pairs of items (q, p) for order i such that item q must be performed immediately before item p
F_p	the set of machines capable of performing item p

Variables

$C_{\max}$	production makespan
S_{ipk}	production start time of item p of order i on machine k
C_i	production completion time of order i
L_i	number of tardy days (real number) for order i
E_i	number of early days (real number) for order i
L'_i	number of tardy days (integer) for order i
E'_i	number of early days (integer) for order i
Y_{ipjqk}	1 if item p of order i precedes item q of order j on machine k; 0 otherwise
Z_{ipk}	1 if item p of order i is assigned to machine k; 0 otherwise

greater than machine ready time. Constraints (4) guarantee that a job starts after its predecessor jobs are processed. Constraints (5) define the completion time of an order. Constraints (6)–(9) are disjunctive constraints, which provide that no two items can be operated on the same machine simultaneously. If item p of order i is scheduled before item q of order j on machine k, $(Y_{ipjqk} = 1)$, starting time of item q must be later than the completion time of item p (6). Constraints (7) are the complementary of disjunctive constraints (6) if and only if both items are assigned to the same machine. If item p of order i and item q of order j are scheduled successively on machine k, both of the items must be pre-assigned to that machine (8). If item p of order i and item q of order j are assigned to the same machine, one of them must be scheduled before the other (9). Constraints (10) force the start time of an item to be equal to zero for the machines which it is not assigned to. In constraints (11), it is ensured that each item can be assigned to only one machine in its eligible machine set. The tardiness and earliness of the orders are defined in constraints (12) and (13), respectively. Since the shift length is 8 hours per day, the completion times of the orders are converted to days. In the constraints (14) and (15), the integer values of the tardiness and earliness are provided. In constraints (16), it is provided that if item p does not belong to order i, then Z_{ipk} is 0 for all machines. Constraints (17)–(21) define set constraints.

Constraints (2), (12)–(15) and (17)–(20) in our model are the same as constraints (2), (8)–(11) and (12)–(15) in the model developed by Chen and Ji (2007), respectively. Other constraints are required for this model to consider machine eligibility.

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