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# An assignment and routing problem with time windows and capacity restriction

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**Abstract**

In under-developed or developing countries, the needs of indigent population are of a pressing nature most of the time. One of the objectives of municipal authorities is to collect donations and distribute the donated items and funds to indigent residents. This task requires a good matching of donations and needs for effectively meeting the needs of recipients. The distribution route is also important for time/cost-effective operations. This study presents a mixed integer programming model that integrates these matching and routing problems. Because of the humanitarian nature of the problem, the main objective is to maximize the utility of the assignment of the donated items. There are several criteria regarding donations and recipients that affect the utility of an assignment, such as the travel time between the location of the donation and the candidate recipient, income level, age, and the number of household of the candidate recipient, the recipient's previous usage of this service, and the age of donated item. Priorities among criteria are set by making a series of judgments based on pairwise comparisons as in the Analytical Hierarchy Process. We consider a real life problem for a district in Izmir, Turkey. The planning period is taken as a working day, and there are time windows in which donors and residents are willing to be served, i.e., the donor must be visited on a specified time interval during the working day. A single truck with a limited capacity is to distribute the donations. The output of the mathematical model is the optimal assignment of the items and the visiting sequence of individuals to maximize total utility. We present our model along with numerical examples and results.



*Keywords*: Assignment Problem; Vehicle Routing Problem; Pick-up and Delivery; Time Windows; Utility Maximization

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## 1. Introduction

In developing countries, the population of indigent residents is considerable and the needs of these residents are of a pressing nature most of the time. One of the responsibilities of municipal authorities is to collect donations and funds from donors and to distribute them to indigent residents fairly. Generally, donation centers that work as departments of municipalities fulfill these activities. The problem actually involves two stages; the assignment of the donated items to the indigent residents and the distribution of these items to their recipients. A fair assignment is crucial for effectively meeting residents' needs, while optimizing the distribution activity is important for timely satisfaction of needs and efficient utilization of public resources**.**

In this study, we consider a real life problem at a donation center in Izmir, Turkey. We propose a novel mixed integer programming model that integrates both stages of the problem; the matching of the items and residents, and the distribution of these items. People in the district of the donation center donate their old household goods or clothing by making phone calls, or sometimes by directly bringing in the items. The donations are collected and distributed daily; hence the planning period is taken as a working day. Furthermore, there are time windows, in which donors and residents are willing to be served, i.e., the donor/resident must be visited on a specified time interval during the working day for the pick-up/delivery of the donated items. There is a single depot of limited capacity for storing the donated items overnight, and a single vehicle for distributing the items during the working day. The mission of the center is to distribute the items to the indigent residents considering their needs and economical situation.

During a day, a donated item is either picked up from the donor's residence and directly transported to the needy resident by the vehicle (if there is need for that item), or it may be transported to the depot and stored there until needed. In the current situation, the planning of the assignments and the distribution schedule is done manually, usually in a first-come-first-serve basis. However, this assignment scheme may lead to inappropriate/unsatisfactory assignments as well as inefficient utilization of the vehicle during a day and high transportation costs.

In this study, we formulate an integrated assignment and routing problem for solving the problem of the donation center. Because of the humanitarian nature of our problem, we identify the main objective as maximizing the total utility obtained from the assignments. We include several criteria regarding donations and recipients for determining the utility of an assignment. The criteria include the travel time (as an indicator of the transportation cost) between the location of the donation and the candidate recipient, income level and the age of the candidate recipient, the recipient's previous usage of the donation service and the age of the donated item.

As to the best of our knowledge, our study is the first to handle such a problem of simultaneous assignment and routing of a single vehicle involving pick-ups and deliveries, and time windows. Previously studied dial-a-ride problem carries some similarities with the routing stage of our problem in terms of the objective function and the constraints. The dial-a-ride problem mainly aims at designing vehicle routes and schedules for *n* users for pick-up and drop off activities. Applications of the problem can be seen in door-to-door transportation services for the elderly and the disabled people [1]. The same user usually has two requests during the same day; the outbound request (from home to destination) and the inbound request (for return). The aim is to accommodate as many as requests possible. Based on the demand structure, the problem is analyzed in two main categories; dynamic and static. In the dynamic setting, the requests are gradually obtained through the day whereas all requests are known in advance in the static setting. The problem may also be classified based on the number of vehicles in the fleet as single or multi-vehicle. The static single vehicle problem is considered by Psaraftis [2], and Sexton and Bodin [3][4], whereas Gendreau et al. [5] study on the dynamic version.

One of the heuristics for the static multi-vehicle dial-a-ride problem including service quality constraints is developed by Jaw et al. [6]. Cordeau and Laporte [7] propose a tabu search for the static multi-vehicle problem with time windows. Ioachim et al. [8] present an approximate method for mini-clustering that involves solving a multi-vehicle pickup and delivery problem with time windows and known customer requests. Madsen et al. [9]

describe a method for the solution of a static dial-a-ride routing and scheduling problem with time windows, and propose an improved and generalized version of this method where new customer requests are dynamically inserted in the vehicle routes. A comprehensive review on the dial-a-ride problem is provided by Cordeau and Laporte [1].

The routing stage of our problem is closer to the static single vehicle dial-a-ride problem with time windows than other routing problems. Both problems deal with the pick-up and delivery activities during the same routing schedule. From a humanitarian perspective, both the dial-a-ride problem and our problem have customer-based objectives; minimizing total customer inconvenience defined by excess ride time/delivery time deviation and maximizing utility, respectively. However, some significant differences exist between the two problems. For example in our problem, the depot is used also for storing the unassigned items. In the dial-a-ride problem, a customer is picked up from his home and transported to a destination. After a while he is picked up from that point and delivered to his home again. On the other hand, in our problem an item is picked up from a donor's home and delivered to either the assigned resident's home or the depot. Hence, there is no return flow for any item. The most important difference between the two problems is that our problem involves simultaneous assignment and routing decisions while the dial-a-ride problem deals with routing for a given assignment scheme.

In the next section, we define our problem in detail. The mathematical model is presented in Section 3 while Section 4 summarizes some preliminary numerical results. We conclude with future research directions in Section 5.

## 2. Problem Definition

There are $n$ items to be distributed to $m$ indigent residents, where $n$ is not necessarily equal to $m$. At the end of the day, picked-up items that are not distributed (not needed) are taken to the depot of known capacity and stored there until needed. As it was stated previously, the planning horizon is taken as a working day. Many types of donations are collected by the donation center. The collected items differ in terms of volume, age and brand. Initially, eight main types of items are identified (though the list may be easily expanded), and are shown in Table 1.

While assigning the donated items to indigent residents we consider the utility of the assignment as a measure for estimating the importance/priority. In order to estimate this utility value, several criteria are taken into consideration. Table 2 summarizes the criteria and scores used in calculating the utility value of each assignment, which will in turn from the objective function coefficients for our model.

The first criterion considers the travel time between the donated item's location (the donor's address) and the indigent's residence. This criterion is important for achieving cost-efficient operations at the center; we assume that a higher travel time increases the transportation cost, therefore decreasing the overall utility. We define three levels of priority for all criteria, and classify all incoming requests accordingly. For instance, there are three income levels and whenever there is an incoming call from a needy resident of income level 3, it is classified as level 3 upon receipt, and receives a score $s_{23}$ from this criterion. Criteria 2, 3 and 4 are indicators of the priority regarding the indigent's social and economical status. However, criteria 3 and 4 are not used at the same time, since the residents requiring this service are usually either senior citizens living alone or low-income families with many children. If the resident lives alone, the authorities state that seniority determines the priority of need. Otherwise, the number of household becomes important. Hence, we consider the age if the indigent resides alone, otherwise we take criterion 4 into consideration. Criterion 5 is significant for achieving a fair distribution of the items to the needy residents from the center's perspective. And finally, the age of donated items is assumed to decrease the potential utility obtained by the indigent. Assigning scores to the three criteria levels renders it possible to classify an incoming request easily.

Table 1- Item Types

| Item No | Items |
|---|---|
| 1 | White Appliances |
| 2 | Furniture |
| 3 | Clothing and Shoes |
| 4 | Kitchen Equipment |
| 5 | Pillow, Quilt or Blanket |
| 6 | Carpet |
| 7 | Curtain |
| 8 | Dry Food Packages |

Table 2 - The criteria used in the model and respective scores according to levels

| Criterion No | Criteria | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| 1 | Travel time | $s_{11}$ | $s_{12}$ | $s_{13}$ |
| 2 | Income | $s_{21}$ | $s_{22}$ | $s_{23}$ |
| 3 | Age | $s_{31}$ | $s_{32}$ | $s_{33}$ |
| 4 | Number of household | $s_{41}$ | $s_{42}$ | $s_{43}$ |
| 5 | Received services | $s_{51}$ | $s_{52}$ | $s_{53}$ |
| 6 | Item age | $s_{61}$ | $s_{62}$ | $s_{63}$ |

Next, we establish priorities among the criteria by making a series of judgments based on pairwise comparisons as in the Analytical Hierarchy Process (AHP) [10], which is a systematic method for determining the relative importance of the factors that have effect in decisions. After this analysis, a relative importance matrix is formed and the normalized weights of the criteria are set as $w_k$, $k = 1, \ldots, 6$, where $w_3 = w_4$, as these two criteria are disjunctive. Then, the utility value of each potential assignment is obtained.

Let us apply the process of determining the assignment utilities on an example consisting of 20 incoming requests for 20 donated items. Items from 1 to 15 represent new donations while the remaining ones are already stored in depot. The income level, number of household, age of the resident, number of received services, age of items, the distances and travel times between any two points are known. The levels and their respective scores are shown in Table 3. Item requests of all residents are known; hence the scores according to the priority levels of all criteria for all possible assignments can be computed. Suppose that resident 7 (a senior resident living alone) requires furniture, and items 5, 14, 16 and 18 are all furniture. In this case, 5-7, 14-7, 16-7 and 18-7 are possible assignments for resident 7, and the associated scores for all related criteria are shown in Table 4. For each possible assignment, the needy resident is to be scored in terms of criteria 2, 3 or 4, and 5 while the donated item is scored in terms of criterion 6.

Next, a relative importance matrix for all criteria is formed as in Table 5 based on pairwise comparisons by the decision maker. The authorities at the donation center are the decision makers in our problem. The relative importance values form the basis for the utility values of the assignments.

After the normalization process identical to that of AHP [10], the normalized weight of these criteria are set and shown in Table 6. Finally, the utility values for a potential assignment are calculated by multiplying the weight of each criterion with the score of the assignment in terms of that criterion, and taking the summation of these multiplications. The computed utility value can conveniently be inserted into the objective function. The determination of the values for the scores for each criterion is subject to experimentation for identifying the best combination.

Table 3- Respective scores of criteria for the example

| **Criteria / Score** | $s_{i1} = 2^0$ | $s_{i1} = 2^1$ | $s_{i3} = 2^2$ |
|---|---|---|---|
| Distance / Travel time | 26-40 km / > 1 hr | 11-25 km / btw.15-30 min | 2-10 km / < 15 min |
| Income | 1001-… TL | 501-1000 TL | 0-500 TL |
| Age of resident | 60-70 | 71-80 | 80+ |
| Number of household | 1 | 2-5 | 5+ |
| Number of received service | 6 | 4-6 | 0-3 |
| Item age | 16+ | 11-15 | 0-10 |

Table 4 - Example for a residents' request

| **Assignment** | **Distance/Time** | **Income level** | **Age** | **Number of service** | **Item age** |
|---|---|---|---|---|---|
| 5-7 | 2 | 1 | 4 | 4 | 4 |
| 14-7 | 2 | 1 | 4 | 4 | 1 |
| 16-7 | 2 | 1 | 4 | 4 | 1 |
| 18-7 | 2 | 1 | 4 | 4 | 4 |

Table 5 – Example importance matrix for the criteria

| **Criterion** | **Distance/Time** | **Income level** | **Age/ num. of hh** | **Number of** | **Item age** |
|---|---|---|---|---|---|
| Distance/Time | 1 | 1/7 | 1/5 | 1/9 | 3 |
| Income level | 7 | 1 | 7 | 3 | 9 |
| Age/ num. of hh. | 5 | 1/7 | 1 | 1/5 | 5 |
| Number of service | 9 | 1/3 | 5 | 1 | 7 |
| Item age | 1/3 | 1/9 | 1/5 | 1/7 | 1 |

Table 6 - Weight of criteria for the example

| **Criteria** | **Weight** |
|---|---|
| Distance/Time | 0.0574 |
| Income level | 0.4895 |
| Age/ Num.of hh. | 0.1252 |
| Num.of service | 0.2947 |
| Item age | 0.0332 |
| Sum | 1.0000 |

## 3. Mathematical Model

In this section, we present a novel mathematical model for simultaneous decisions of the assignment of donated items and the routing of a single vehicle in the presence of pick-ups and deliveries, time windows and capacity restriction. The parameters of the model are as follows:

$N$ : the number of nodes (including the depot)
$i, j$ : the indices for all nodes, $i, j = 1, \ldots, N$
$d$ : the indices for the donor nodes, $d = 1, \ldots, D$, where $D < N$
$v_i$ : volume at node $i$, $i = 1, \ldots, N$
$u_{ij}$ : the utility of assigning node $i$ to node $j$
$t_{ij}$ : travel time of the vehicle between node $i$ and node $j$
$a_i$ : start of time window for node $i$, $i = 1, \ldots, N$-1
$b_i$ : end of time window for node $i$, $i = 1, \ldots, N$-1
$C$ : total volume capacity of the vehicle

The following denote our decision variables:

$x_{ij}$ : takes the value of 1 if node (item) $i$ is assigned to node $j$, and 0 otherwise
$y_{ij}$ : takes the value of 1 if the vehicle visits node $i$ immediately before node $j$, and 0 otherwise
$s_i$ : the start time of service at node $i$
$l_i$ : load (in volume) of the vehicle after visiting node $i$

Node indices from 1 to $D$ represent the donor nodes where indices from $d$+1 to $N$-1 stand for the resident nodes. The last index $N$ symbolizes the depot.

Each request of an indigent resident and each donated item are taken as separate records, since it may receive different scores from the criteria according to the type of the item requested. The model therefore keeps separate node indices for each request and each donation, although they may belong to the same actual location. Dummy nodes are created as needed. Items at the depot at the start of the day are also treated as separate donor nodes. Our mixed integer programming model is given below.

Maximize $\sum_{i=1}^{d} \sum_{j=d+1}^{N} u_{ij} x_{ij}$
subject to:

$$\sum_{j=D+1}^{N} x_{ij} = 1 \qquad i = 1, \ldots, D \qquad (1)$$

$$\sum_{i=1}^{D} x_{ij} \leq 1 \qquad j = D+1, \ldots, N-1 \qquad (2)$$

$$\sum_{h \in N \backslash h \neq i} y_{ih} + \sum_{h \in N \backslash h \neq j} y_{hj} - x_{ij} \geq 1 \qquad i = 1, \ldots, D,\ j = d+1, \ldots, N-1 \qquad (3)$$

$$\sum_{j \in N \backslash i \neq j} y_{ij} = 1 \qquad \forall i \qquad (4)$$

$$\sum_{j \in N \backslash i \neq j} y_{ji} = 1 \qquad \forall i \qquad (5)$$

$$\sum_{j=1}^{N-1} y_{Nj} \leq 1 \qquad (6)$$

$$\sum_{i=1}^{N-1} y_{iN} = 1 \qquad (7)$$

$$a_i \le s_i \le b_i \qquad \forall i \qquad (8)$$

$$s_j - s_i + b_i\left(1 - x_{ij}\right) \ge 0 \qquad i = 1,...,D, j = d+1,...,N-1 \qquad (9)$$

$$s_j - s_i - t_{ij} + \left(b_i + t_{ij}\right)\left(1 - y_{ij}\right) \ge 0 \quad \forall i, j = 1,...,N-1, i \ne j \qquad (10)$$

$$l_j - l_i - v_j + \left(C + v_j\right)\left(1 - y_{ij}\right) \ge 0 \quad \forall i, j = 1,...,N-1, i \ne j \qquad (11)$$

$$l_i \le C \qquad \forall i \qquad (12)$$

$$s_N = 0 \qquad (13)$$

$$l_N = 0 \qquad (14)$$

$$x_{ij}, y_{ij} \in \{0,1\} \qquad \forall i, j \qquad (15)$$

$$s_i, l_i \ge 0 \qquad \forall i \qquad (16)$$

The objective function maximizes the total utility of the daily assignments. Constraint set (1) ensures that each item must be assigned to either an indigent or the depot. Constraint set (2) allows that an indigent may not receive an item. Constraint set (3) ensures that an indigent cannot be visited before its related donor is visited. Constraint sets (4) to (7) sets the start and end of the tour as the depot. Also, they handle that each node is visited exactly once except the depot. Constraint set (8) establishes time windows while constraint sets (9) and (10) define the service start time for each node. Constraint set (11) and (12) together handle the capacity constraint of the vehicle. Constraint sets (13) and (14) state that the vehicle is initially empty and it starts its tour at the depot. Finally, constraint sets (15) and (16) impose sign restrictions on the decision variables.

## 4. Numerical Results

Small problem instances of different scenarios are generated in accordance with real data in order to test the validity of the developed mathematical model. Volumes at the nodes are generated from discrete uniform distribution between 1 and 5. Lower and upper bound of time windows are also generated from discrete uniform distribution, as $a_i \sim$ U[0,10] and $b_i \sim$ U[6,40] except for depot. All utility values are assumed identical to obtain feasible assignments for scenarios 1 to 3, and for scenario 4 they are computed using the procedure explained in Section 2. The instances are solved with CPLEX 10 solver in GAMS 22.5 using an AMD II P820, 1.8 GHz computer with 3.74 GB RAM. The scenarios are described below.

**Scenario 1: Redundant capacity constraint**

In this scenario, the total volume of all items to be collected does not exceed the capacity of the vehicle. Therefore, the vehicle can visit donors and residents in any order while meeting the time window constraints. That is, the vehicle is allowed to visit all donors first and then distribute the collected items to the indigent residents. The vehicle is initially empty after leaving the depot. In the example below there are seven nodes; nodes 1 to 3 are donors, nodes 4 to 6 are residents and node 7 is the depot. All residents need different types of items. Table 7 shows the request of each resident and the donation of each donor.

From the table 7, it can be seen that there is no alternative assignment for any resident. Also, the volume of each donated item is given. The vehicle capacity is 30 (redundant). Assignments based on residents' needs and the routing sequence are shown in Fig. 1. Since there is no capacity problem, the vehicle first visits all donors, then it visits all residents to distribute the collected items.

Table 7 – Data for scenario 1

| Resident Node | Item | Donor Node | Item | Volume |
|---|---|---|---|---|
| 4 | White App. | 1 | Clothing | 2 |
| 5 | Clothing | 2 | Furniture | 4 |
| 6 | Furniture | 3 | White App. | 5 |

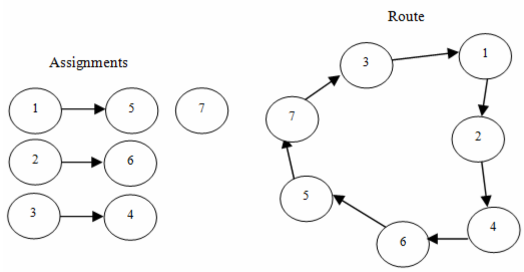


Fig. 1. Output of scenario 1

**<u>Scenario 2: Initial load of the vehicle not empty, active capacity constraint</u>**

The setting is similar to the one in Scenario 1 with the addition that the vehicle takes some items from the depot at the start of its tour, so it is not initially empty. Also, the total volume of collected items exceeds vehicle capacity. Again, there are seven nodes; nodes 1 to 3 are the donors, nodes 4 to 6 represent residents, and nodes 1 and 7 are the depot. Since all donations, requests and the depot are represented by separate nodes, node 1 represents both a donor and the depot node. Assignments and the routing sequence are shown in Fig. 2. After leaving the depot with one item, the vehicle directly visits node 5 and drops this item. Then, it picks another from node 3 and drops it at node 4. Next, it picks another donation from node 2. Finally, the remaining item on the vehicle is dropped at node 6 and the vehicle returns to the depot.

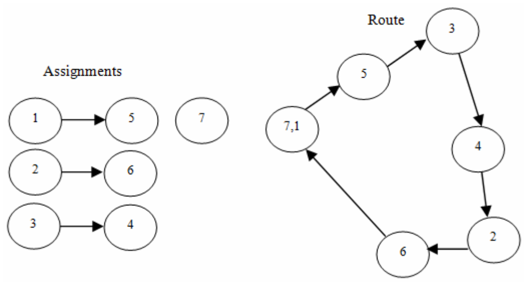


Fig. 2. Output of scenario 2

**<u>Scenario 3: Active capacity constraint, depot visited more than once</u>**

The setting is similar to the one in Scenario 2 with the addition that the vehicle has to visit the depot more than once within its tour due to overload. Again, there are seven nodes, and node 1 represents both a donor and the depot node. Assignments and the route are shown in Fig. 3. The tour now consists of two sub-tours as Depot-3-4-Depot, and Depot-2-6-5-Depot.

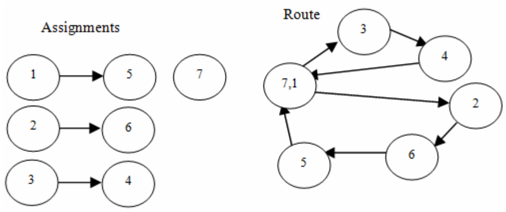


Fig. 3. Output of scenario 3

**Scenario 4: Utilities**

In this scenario, each resident has alternative assignment options, and the assignment is done in such a way to maximize overall utility. The data is shown in Table 8. Node 16 is the depot. The vehicle capacity is 200. Assignments and the route are shown in Fig. 4.

Table 8 – Data for scenario 4

| Resident Node | Item | Donor Node | Item | Volume |
|---|---|---|---|---|
| 9 | White App. | 1 | White App. | 5 |
| 10 | White App. | 2 | White App. | 5 |
| 11 | Furniture | 3 | Furniture | 10 |
| 12 | Furniture | 4 | Furniture | 10 |
| 13 | Curtain | 5 | Curtain | 15 |
| 14 | Curtain | 6 | Curtain | 15 |
| 15 | Carpet | 7 | Carpet | 20 |
| 16 | Carpet | 8 | Carpet | 20 |

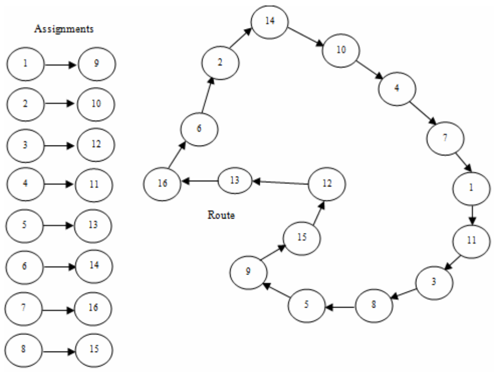


Fig. 4. Output of scenario 4

To see the time performance of the mathematical model, two instances with different node sizes (30 and 40 nodes) are generated. The instance with 30 nodes can be solved by GAMS within 420 seconds, whereas the instance with 40 nodes cannot be solved within a 3600-second time limit. This result indicates the need for different solution procedures for larger instances.

## 5. Conclusion and Future Work

In this study, a novel mathematical model is proposed to solve the integrated problem of assignment and routing at a donation center with the objective of utility maximization. We present the mathematical model with several numerical examples.

The highly combinatorial nature of the problem seems to necessitate heuristic approaches for obtaining near-optimal solutions for larger instances. Our future work will consist of developing heuristic or meta-heuristic algorithms for the problem, and an extensive computational experimentation to evaluate the performance of the algorithms as compared to the optimal solutions.

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