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Is there democracy in leptonic sector of 3-3-1 model

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Abstract

Even though the standard model (SM) of particle physics aligns with many experimental results, the need to expand the SM arises to address important open questions. The most suitable and minimal extension of the SM is the local gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, which is known as the '3-3-1 model' in the literature. In this paper, a democratic mass matrix (DMM) approach is applied to the lepton sector of one of the anomaly free 3-3-1 model. It is demonstrated that the DMM parametrization can fit with experimental mass and 3×3 Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS) data. Also we have presented an extended PMNS matrix elements for the gauge bosons of $K^{\pm}$ (and $K^0, \bar{K}^0$) which mediate between new heavy neutrinos and leptons (known neutrinos).

1. Introduction

The local group of the standard model (SM) is $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, which agrees with experiments up to 1000 GeV [1]. However, many particle physicists feel that SM is not the ultimate theory, but rather a useful temporary theory that emerges from a more fundamental theory, and expansions of SM are inevitably welcomed.

In the case of these expansions, it is already well-known that the SM can be expanded either by adding new fermion fields to the existing model (a right-handed neutrino field is added in a minimal expansion), or the local group can be expanded by selecting multiple Higgs representations of the scalar sector, or both of the expansions can be applied [2]. The second method is generally preferred and the extension of the $SU(3)_L \otimes U(1)_X$ flavor group with possible fermion and Higgs-boson representations has been investigated by many authors [3]. Some have studied these extensions as indistinguishable duplicates of a family as in SM [4], while others have looked at them as a multi-family construct [5, 6], implying a natural solution to the fermion family number. Many models given in [4] cause gauge anomalies, flavor-changing neutral currents, right-handed currents at low energies, and violation of quark-lepton universality. This is not realistic because it contains physical inconsistencies, such as the model examined in [5]; meanwhile, this is in agreement with the SM with the three quark and three lepton families, and the anomaly of the model is eliminated by the addition of quarks carrying exotic electric charges. The model in [6] is also three-family and is in agreement with low energy phenomenology, but does not contain exotic electrically charged fermions.

A possibility that can be explored in the context of these expansions involves the democratic mass matrix (DMM) approach, which has been proposed mainly by H. Fritzsch and his colleagues [7] for the SM quarks in 1978. This approach can be extended into lepton sector with similar arguments [8]. In this approach, all fermions with the same quantum number behave equally under weak interaction in the up and down sectors (or neutral and charged leptons) and they are indistinguishable before the symmetry breaking. For example, one would have indistinguishable mass matrices for leptons before the symmetry breaking:

$$\mathcal{M}_\nu^0 = h_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1}$$

and

$$\mathcal{M}_\ell^0 = h_\ell \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \qquad (2)$$

Since there is only one Higgs field in the SM, $h_\nu = h_\ell$ is expected. In this case, it is naturally expected that $m_\tau \cong m_{\nu_\tau}$. However, the neutrino mass is very tiny in reality. The seesaw mechanism can generate tiny left handed neutrino mass while creating a sterile right handed neutrino with huge mass [9]. Therefore, one can use the seesaw mechanism to explain the great mismatch between neutrino mass and charged lepton mass. Another approach is applying DMM scheme to an extension of the SM with more fundamental fermions and breaking the democracy with very small amounts. Also, DMM scheme permits us to obtain correct Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrices.

In this study, the lepton sector of the 3-3-1 model is investigated in the light of DMM.

The structure of rest of the paper is as follows: the quark and lepton contents and new gauge bosons, neutral and charged currents of the chosen sub-model of the 331 model are given in section 2. After giving detailed information on DMM scheme, the new parameterization for the variant of 3-3-1's sub-model is presented in section 3. Masses of SM charged and neutral leptons and new heavy neutrinos obtained with the help of DMM parametrization are also given in this section. PMNS matrix and other mixing matrices corresponding to new gauge bosons, specific to the chosen sub-model, are given in section 4. Concluding remarks are made in section 5.

2. The 3-3-1 gauge model as extension of SM

One of the minimal extensions of SM is the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, where various sub-models studied earlier [10] contain no exotic electrically charged particles. A consistent model (in which extensions or particle additions to SM should not break SM symmetries at the quantum level) is expected to be free of anomalies coming from all possible vertices. These are the vertices of the triangle diagrams, which are the one loop corrections to a two-point function with a vector current or an axial current inserted. These anomalies are called triangle anomalies in the present study. Anomalies can be removed by satisfying four equations corresponding to vertices coming from $\left[SU(3)_c\right]^2 \otimes U(1)_X$, $[SU(3)_L]^2 \otimes U(1)_X$, $[\text{gravitation}]^2 \otimes U(1)_X$ and $[U(1)_X]^3$ for every model. Two of such models (models A and B) are sub-models of 3-3-1 model that triangle anomalies are canceled in one family, and the other two (Models C and D) are sub-models that have no triangle anomalies in three families. Additional fundamental fermions required for Models A, B, C and D are given below:

	Model A	Model B	Model C	Model D
Additional heavy quarks	D, S, B	U, C, T	U, C, B	D, S, T
Additional leptons for every family	1 charged and 4 neutral leptons	3 charged and 2 neutral leptons	1 charged lepton	1 neutral lepton

The quark and lepton contents of Model D are briefly given in [11]. However, there are also several sub-variations of the D model, and one of these is considered in the present paper. The quark content of this variation, which was previously studied in [11], is the same as in [10]:

$Q_L^\alpha = \begin{pmatrix} u_\alpha \\ d_\alpha \\ D_\alpha \end{pmatrix}_L$	$u_{\alpha L}^c$	$d_{\alpha L}^c$	$D_{\alpha L}^c$
$\{3,3,0\}$	$\{3^*,1,-\frac{2}{3}\}$	$\{3^*,1,\frac{1}{3}\}$	$\{3^*,1,\frac{1}{3}\}$

(3)

for $\alpha = 1,2$ corresponds to the quarks in two of the three families. Quarks of the third family are:

$Q_L^3 = \begin{pmatrix} d_3 \\ u_3 \\ U_3 \end{pmatrix}_L$	u_{3L}^c	d_{3L}^c	U_{3L}^c
$\{3,3^*,\frac{1}{3}\}$	$\{3^*,1,-\frac{2}{3}\}$	$\{3^*,1,\frac{1}{3}\}$	$\{3^*,1,-\frac{2}{3}\}$

(4)

Numbers in parenthesis denote $\{SU(3)_C, SU(3)_L, U(1)_X\}$ quantum numbers, where X arising in the electric charge generators of the gauge group is given as:

$$Q = \left(\frac{1}{2}\right)\lambda_{3L} + \left(\frac{1}{2\sqrt{3}}\right)\lambda_{8L} + XI_3, \tag{5}$$

where $\lambda_{iL}, i = 1, 2, \ldots, 8$ are Gell–Mann matrices for $SU(3)_L$ and I_3 is 3×3 identity matrix. The lepton content of this model is as follows [11]:

$\Psi_L^{\alpha} = \begin{pmatrix} \alpha^- \\ \nu_\alpha \\ N_\alpha \end{pmatrix}_L$	$\alpha_{\alpha L}^{+}$	$\nu_{\alpha L}^{c}$	$N_{\alpha L}^{c}$
$\{1, 3^*, -\frac{1}{3}\}$	$\{1,1,1\}$	$\{1,1,0\}$	$\{1,1,0\}$

(6)

for $\alpha = 1, 2, 3$ corresponds to the leptons of the three families and N_α are new heavy neutrinos not existed in the SM.

The four equations to remove anomalies in Model D, obtained from more general equations given in reference [10] as special case, are listed below:

$$\sum_{\alpha=1}^{3}\left(3X_{Q^\alpha} + X_{u_\alpha} + X_{d_\alpha} + X_{q_\alpha}\right) = 0, \tag{7a}$$

$$\sum_{\alpha=1}^{3}\left(3X_{Q^\alpha} + X_{\Psi^\alpha}\right) = 0, \tag{7b}$$

$$\sum_{\alpha=1}^{3}\left(9X_{Q^\alpha} + 3X_{u_\alpha} + 3X_{d_\alpha} + 3X_{q_\alpha} + 3X_{\Psi^\alpha} + \sum_{singlet} X_{\ell s_\alpha}\right) = 0, \tag{7c}$$

$$\sum_{\alpha=1}^{3}\left(9X_{Q^\alpha}^3 + 3X_{u_\alpha}^3 + 3X_{d_\alpha}^3 + 3X_{q_\alpha}^3 + 3X_{\Psi^\alpha}^3 + \sum_{singlet} X_{\ell s_\alpha}^3\right) = 0. \tag{7d}$$

Values of X quantum numbers for model D will be given in equations (3), (4) and (6). Subscript q_α is corresponding to D , S and T heavy isosinglet quarks for model D. In three-family models, one of the families has different quantum numbers from the other twos. Here the electroweak gauge group is supposed to be $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$. It is also assumed that left-handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two basic representations of $SU(3)_L$ (3 and 3*).

The new heavy neutrinos have potential as dark matter candidates [12]. Similar to $[SU(3)]^3$ and $[SU(3)]^4$ models with stable dark matter [13], this extension of the SM is able to accommodate a dark candidate via residual discrete Z_2 symmetry, which is a remnant of the spontaneous symmetry breaking of gauge groups at higher scale. The achieved model is an anomaly-free model, motivated by some important issues which cannot be answered by the SM, alongside the observed neutrino masses and mixings, the number of families, and the presence of an appropriate particle choice for a stable dark matter. In this respect, it stands out as one of the most potentially productive expansions of SM. Their constraints to observe on LHC and FCC are studied in details on [14].

This model also has three Higgs fields. They are $(\phi_1^-, \phi_1^0, \phi_1'^0)$, $(\phi_2^-, \phi_2^0, \phi_2'^0)$ and $(\phi_3^0, \phi_3^+, \phi_3'^+)$. Vacuum expectation values (VEV) of Higgs fields are following:

$$\langle\phi_1\rangle = (0, 0, M)^T, \tag{8a}$$

$$\langle\phi_2\rangle = \left(0, \frac{\eta}{\sqrt{2}}, 0\right)^T, \tag{8b}$$

$$\langle\phi_3\rangle = \left(\frac{\eta'}{\sqrt{2}}, 0, 0\right)^T, \tag{8c}$$

where $\eta \sim 250$ GeV ($\eta' = \eta$ can be taken for simplicity).

In addition, this model has a total of 17 gauge bosons. One of the gauge fields is the gauge boson associated with $U(1)_X$, eight of them are $SU(3)_C$ associated gauge bosons. The gauge fields of the electroweak sector can be listed as $W^{\pm}$, $K^{\pm}$, K^0 and $\bar{K}^0$ with mass, and Z and Z', which are also massive and uncharged. The masses of the new bosons are proportionate to the symmetry breaking scale of the model, in the order of a few TeV. The masses of the gauge bosons of the electroweak sector can be found using the following expressions:

$$m^2_{W^{\pm}} = \frac{g^2}{4}\left(\eta^2 + \eta'^2\right), \tag{9a}$$

$$m^2_Z = \frac{m^2_{W^{\pm}}}{C^2_W}, \tag{9b}$$

$$m^2_{K^{\pm}} = \frac{g^2}{4}\left(2M^2 + \eta'^2\right), \tag{9c}$$

$$m^2_{K^0(\bar{K}^0)} = \frac{g^2}{4}\left(2M^2 + \eta^2\right), \tag{9d}$$

$$m^2_{Z'} = \frac{g^2}{4\left(3 - 4S^2_W\right)}\left[8C^2_W M^2 + \frac{\eta^2}{C^2_W} + \frac{\eta^2\left(1 - 2S^2_W\right)^2}{C^2_W}\right], \tag{9e}$$

where C_W and S_W are the cosine and sine of the electroweak mixing angle respectively with experimental value of $S^2_W = 0.23122$. It should be emphasized that in addition to the SM, there are five new gauge bosons, which may lie within the detection limits of the Large Hadron Collider (LHC), as we assume their masses to be in the order of a few TeV. Limitations on the masses of these particles have been identified by the absence of certain types of expected events [15]. By using last ATLAS [16] and CMS data [17], a new and restrictive constraint on the mass of the Z' boson was found to be $M_{Z'} > 5.1$ TeV and $M_{Z'} > 4.6$ TeV at 95% CL, respectively.

In fact, the common feature of many models obtained by extending the SM is the participation of extra heavy gauge bosons [15], the charged ones usually denoted by W'. In the LHC, W' bosons would be observed through production of fermion or electroweak boson pairs resonantly. The most extensively considered signature contains a high-energy electron or muon and large lost transverse energy. Assuming that these new bosons couple with fermions in the SM, restrictive constraints on the mass of W' are obtained as $M_{W'} > 6$ TeV at 95% CL [18]. Although this limitation does not directly apply to our model, it gives a sense of the masses of the $K^{\pm}$ and K^0 bosons.

Charged currents in this model are:

$$\begin{aligned}\mathcal{L}_{CC}7 = -\frac{g}{\sqrt{2}}\Big[&\bar{\nu}^{\alpha}_L\gamma^{\mu}e^{\alpha}_L W^+_{\mu} + \bar{N}^{\alpha}_L\gamma^{\mu}e^{\alpha}_L K^+_{\mu} + \bar{\nu}^{\alpha}_L\gamma^{\mu}N^{\alpha}_L K^0_{\mu}\\ &+ (\bar{u}_{3L}\gamma^{\mu}d_{3L} + \bar{u}_{\alpha L}\gamma^{\mu}d_{\alpha L})W^+_{\mu}\\ &+ (\bar{U}_{3L}\gamma^{\mu}d_{3L} + \bar{u}_{\alpha L}\gamma^{\mu}D_{\alpha L})K^+_{\mu}\\ &+ (\bar{u}_{3L}\gamma^{\mu}U_{3L} - \bar{D}_{\alpha L}\gamma^{\mu}d_{\alpha L})K^0_{\mu} + h.c.\Big],\end{aligned} \tag{10}$$

neutral currents are:

$$\mathcal{L}^{NC} = -\frac{g}{2CW}\sum_f\left[\bar{f}\gamma^{\mu}(g'_V + g'_A\gamma^5)fZ'_{\mu}\right], \tag{11}$$

where f shows leptons and quarks; g, g'_V and g'_A are the coupling constants of $SU(3)_L$.

As can be seen from the above expressions, K^+ and K^- gauge bosons provide transitions between charged leptons and new heavy neutrinos, while K^0 and $\bar{K}^0$ gauge bosons mediate the interactions of SM neutrinos and new heavy neutrinos. The new heavy neutrinos, which are candidates for dark matter, annihilate each other via the Z' gauge boson and form the fermions we observed in the SM.

3. Democratic approach to the leptonic sector of 3-3-1 model

The DMM approach was developed by H. Harari and H. Fritzsch to solve the mass hierarchy and mixings problems [7], but was unsuccessful in predicting top quark's mass. To remedy this, a number of papers were published, in which DMM scheme was applied to four family SM [8, 19]. Later, the SM type fourth family fermions were excluded by ATLAS and CMS data [20]. As a consequence, if DMM approach is correct, it will be inevitably applied to an extension of the SM. DMM scheme assumes that Yukawa coupling constants should be approximately same in the weak interaction Lagrangian. When the mass eigenstates are turned on, fermions gain different masses [8, 19].

In this section, we applied DMM approach to the lepton sector of Model D. The Yukawa Lagrangian corresponding to the lepton sector is:

$$\mathcal{L}_Y^{\ell} = \sum_{i,j}^{3} \psi_L^i C \left(a_{\ell_i \ell_j} \phi_3 \ell_{jL}^{+} + a_{N_i N_j} \phi_1 N_{jL}^{C} + a_{N_i \nu_j} \phi_1 \nu_{jL}^{C} + a_{\nu_i \nu_j} \phi_2 \nu_{jL}^{C} + a_{\nu_i N_j} \phi_2 N_{jL}^{C} \right) + h.c., \tag{12}$$

where $a_{\ell_i\ell_j}$, $a_{N_iN_j}$, $a_{N_i\nu_j}$, $a_{\nu_i\nu_j}$ and $a_{\nu_iN_j}$ are Yukawa coupling constants and C is the charge conjugate operator.

For example, in the case of one family, $m_e = a_{ee}\eta'/\sqrt{2}$ ($\eta\prime = \eta$ can be taken for simplicity) would be for the charged lepton sector, and a mass matrix for the neutrino sector would be obtained as follows [10, 11]:

$$\mathcal{M}_{\nu_e N_e} = \begin{pmatrix} -a_{\nu_e\nu_e}\eta/\sqrt{2} & -a_{\nu_e N_e}\eta/\sqrt{2} \\ a_{N_e\nu_e}M & a_{N_e N_e}M \end{pmatrix}. \tag{13}$$

For the special case $a_{N_eN_e} = a_{N_e\nu_e} = a_{\nu_e\nu_e} = a_{\nu_eN_e} \equiv a$ the mass eigenvalues of the above mass matrix $m_{\nu_e} = 0$ and $m_{N_e} = a(M + \eta/2)$. To obtain the small-mass of ν_e neutrino, we need to deviate from this special case by a small amount. For example, $a_{\nu_e\nu_e} = a_{N_eN_e} = a$ and $a_{\nu_eN_e} = a_{N_e\nu_e} = \varepsilon a$ to achieve this, where ε is very close to one. Therefore the mass matrix for the neutrino sector becomes:

$$\mathcal{M}_{\nu_e N_e} = \begin{pmatrix} -a\eta/\sqrt{2} & -\varepsilon a\eta/\sqrt{2} \\ \varepsilon a M & aM \end{pmatrix}. \tag{14}$$

By diagonalizing this mass matrix, the mass eigenvalues are obtained:

$$m_{\nu_e} = \frac{a}{2}\left(M + \frac{\eta}{\sqrt{2}} - \sqrt{M^2 - \sqrt{2}M\eta + 2\sqrt{2}\varepsilon^2 M\eta + \frac{\eta^2}{2}} \right), \tag{15}$$

$$m_{N_e} = \frac{a}{2}\left(M + \frac{\eta}{\sqrt{2}} + \sqrt{M^2 - \sqrt{2}M\eta + 2\sqrt{2}\varepsilon^2 M\eta + \frac{\eta^2}{2}} \right). \tag{16}$$

If the neutrino sector mass matrix is generalized to the three-family case, one obtains a (6×6) matix:

$$\begin{pmatrix} \mathcal{M}_{\nu_\ell} & \varepsilon\mathcal{M}_{\nu_\ell} \\ \varepsilon\mathcal{M}_{N_\ell} & \mathcal{M}_{N_\ell} \end{pmatrix}, \tag{17}$$

where

$$\mathcal{M}_{\nu_\ell} = \begin{pmatrix} -\frac{a_\nu\eta}{\sqrt{2}}(1-4\xi_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1-\xi_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1-\xi_\nu+\varrho_\nu) \\ -\frac{a_\nu\eta}{\sqrt{2}}(1-\xi_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1+4\xi_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1+4\xi_\nu+\varrho_\nu) \\ -\frac{a_\nu\eta}{\sqrt{2}}(1-\xi_\nu+\varrho_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1+4\xi_\nu+\varrho_\nu) & -\frac{a_\nu\eta}{\sqrt{2}}(1+4\varrho_\nu) \end{pmatrix}, \tag{18}$$

and

$$\mathcal{M}_{N_\ell} = \begin{pmatrix} a_\nu M(1-4\xi_N) & a_\nu M(1-\xi_N) & a_\nu M(1-\xi_N+\varrho_N) \\ a_\nu M(1-\xi_N) & a_\nu M(1+4\xi_N) & a_\nu M(1+4\xi_N+\varrho_N) \\ a_\nu M(1-\xi_N+\varrho_N) & a_\nu M(1+4\xi_N+\varrho_N) & a_\nu M(1+4\varrho_N) \end{pmatrix}. \tag{19}$$

In a similar manner, the charged lepton sector (3×3) mass matrix becomes:

$$\begin{pmatrix} \frac{a_\ell\eta}{\sqrt{2}}(1-4\xi_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1-\xi_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1-\xi_\ell+\varrho_\ell) \\ \frac{a_\ell\eta}{\sqrt{2}}(1-\xi_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1+4\xi_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1+4\xi_\ell+\varrho_\ell) \\ \frac{a_\ell\eta}{\sqrt{2}}(1-\xi_\ell+\varrho_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1+4\xi_\ell+\varrho_\ell) & \frac{a_\ell\eta}{\sqrt{2}}(1+4\varrho_\ell) \end{pmatrix}. \tag{20}$$

First, let us assume that ξ_ν, ϱ_ν, ξ_N, ϱ_N, ξ_ℓ and ϱ_ℓ are zero. In this case, when the matrix elements are taken as equal in groups of (3×3) arrays as in the mass matrices given above, only two neutrinos have mass in the uncharged lepton sector, while the other four are obtained as massless. Similarly, in the charged lepton sector, the electron and muon are massless, while only the tau lepton has mass. In the DMM approach, small additions or subtractions can be made to differentiate the matrix elements in order to add mass to other fermions.

Then for $\frac{a_\ell \eta'}{\sqrt{2}} = 478$ MeV ($\eta' = \eta$ is taken), $\xi_\ell = 0.236$ and $\varrho_\ell = 0.001\,078\,34$ the charged lepton masses can be obtained as $m_e = 0.510\,99$ MeV, $m_\mu = 105.6$ MeV and $m_\tau = 1.77$ GeV. Similarly, by using $\frac{a_\nu \eta}{\sqrt{2}} = 25.929$ keV, $a_\nu M = 200$ TeV, $\xi_\nu = -0.008702$, $\varrho_\nu = -0.0068$, $\xi_N = 0.0099$, $\varrho_N = -0.0099$ and $\varepsilon = 0.999\,999\,638$, the masses of the neutrinos in the SM can be obtained as $m_{\nu_e} = 11.1236$ meV, $m_{\nu_\mu} = 14.0082$ meV, $m_{\nu_\tau} = 52.7798$ meV, and the masses of the new heavy neutrinos are $m_{N_e} = 2.29$ TeV was found as $m_{N_\mu} = 7.38$ TeV and $m_{N_\tau} = 603$ TeV.

The flavor change of neutrinos has been observed and confirmed by numerous experiments using neutrinos from the Sun, atmosphere, reactors and accelerators [21–23]. This phenomenon, also known as neutrino oscillations, states that at least two of the SM neutrinos have a nonzero mass, so that the first deviation from the SM occurs. Three massive neutrino models were very successful in explaining Solar neutrino data given as the difference of mass squares $\Delta m^2_{21} : (6.82\text{–}8.04)\times 10^{-5}$ eV^2 and the atmospheric neutrino data given as difference of mass squares $\Delta m^2_{31} : (2.43\text{–}2.60)\times 10^{-3}$ eV^2 [23]. By using the mass matrices above, the difference of the squares of the neutrino masses is found as $\Delta m^2_{21} = 7.24\times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.59\times 10^{-3}$ eV^2. As can be noticed, these values are consistent with the experimental data.

4. PMNS and other mixing matrices

Unitary matrices U_L^ℓ, U_L^ν and U_L^N are then obtained by diagonalizing the selected mass matrices for the lepton, neutrino and new heavy neutrino sectors. Using these unitary matrices, the PMNS matrix and the mixing matrices for $K^\pm$, K^0 and $\bar{K}^0$ are obtained as follows:

$$U_{PMNS} = (U_L^\nu)^\dagger U_L^\ell = \begin{pmatrix} 0.801 & 0.580 & 0.151 \\ 0.449 & 0.414 & 0.792 \\ 0.397 & 0.702 & 0.591 \end{pmatrix} \tag{21}$$

$$U_{K^\pm} = (U_L^N)^\dagger U_L^\ell = \begin{pmatrix} 0.982 & 0.062 & 0.179 \\ 0.055 & 0.997 & 0.046 \\ 0.181 & 0.035 & 0.983 \end{pmatrix} \tag{22}$$

$$U_{K^0} = (U_L^N)^\dagger U_L^\nu = \begin{pmatrix} 0.030 & 0.065 & 0.997 \\ 0.049 & 0.997 & 0.064 \\ 0.998 & 0.047 & 0.033 \end{pmatrix}. \tag{23}$$

Experimental limits of PMNS matrix are [23, 24]:

$$|U|_{3\sigma}^{w/o\ SK-atm} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.232 \to 0.507 & 0.459 \to 0.694 & 0.629 \to 0.779 \\ 0.260 \to 0.526 & 0.470 \to 0.702 & 0.609 \to 0.763 \end{pmatrix}, \tag{24}$$

$$|U|_{3\sigma}^{with\ SK-atm} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.144 \to 0.156 \\ 0.244 \to 0.499 & 0.505 \to 0.693 & 0.631 \to 0.768 \\ 0.272 \to 0.518 & 0.471 \to 0.669 & 0.623 \to 0.761 \end{pmatrix}. \tag{25}$$

If the PMNS matrix is compared with the latest experimental data, it can be seen that many values are within the 3σ limits, while a few are very close to the 3σ limits.

5. Conclusion

A wide variety of mass matrix schemes (mass matrix for Wolfenstein parametrization, various zero texture mass matrices, etc) have been developed to date. The most natural of these is the DMM scheme. However, this scheme is less useful for the 3-family SM because of the extreme differences between the masses of the third family quarks and charged/neutral leptons. In this paper, we applied DMM approach to the lepton sector of the 3-3-1 Model, which is one of the simplest extensions of the SM. We successfully derived lepton and neutrino masses. We compared the mixing matrix corresponding to these masses with the current experimental limits of the PMNS matrix and found that the values obtained are within acceptable limits. In addition, we found mass values for the new heavy neutrinos predicted by the 3-3-1 Model and also the mixing matrices for the mixture of new heavy neutrinos with SM leptons and neutrinos. In the literature, this type of heavy neutrinos is seen as one of the best candidates for dark matter, and the success of the DMM scheme for this model will lead to new directions for further study in the future.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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