ANALYSIS OF COOPERATION IN SCHEDULING WITH OUTSOURCING

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ABSTRACT

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The need for outsourcing may increase with the enhanced product ranges. Outsourcing decision of the producers results in more complex problem of job scheduling especially for a system with high setup times. In this study, a cooperative scheduling for the outsourced capacity is proposed for maximizing the profit which is affected by the processing yield and tardiness. The aim of the cooperation on scheduling is to minimize setup requirements and therefore processing jobs earlier with higher processing yield and lower tardiness. The cooperation is designed to be performed by the companies that would outsource a part of their processes on a shared capacity. With the cooperative schedule, companies may process their identical jobs consecutively without setup and increase the utilization of the outsourced capacity. In order to demonstrate the proposed approach, an illustrative example from the leaf tobacco processing sector is introduced. For evaluating the effectiveness of the cooperative scheduling, a mathematical model is constructed and solved for both non-cooperative and cooperative scheduling with outsourcing approaches. In addition, Stackelberg Game and Nash Equilibrium are used for analysing the decisions of companies as non-cooperative games for both non-cooperative and cooperative outsourcing schedules. Results showed that the cooperative scheduling provides higher profit due to the minimization of setup times.

Keywords: Cooperative scheduling, Outsourcing, Game Theory, Stackelberg Game, Nash Equilibrium.

ÖZET

DIŞ KAYNAK KULLANIMI İLE ÇİZELGELEMEDE İŞBİRLİĞİ ANALİZİ

Tanıl, Mutlu İpek

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Dış kaynak kullanımı ihtiyacı, artan ürün çeşitliliği ile birlikte artmaktadır. Üreticilerin dış kaynak kullanımı kararı, özellikle kurulum sürelerinin yüksek olduğu sistemlerde çizelgeleme açısından karmaşık bir problem oluşturmaktadır. Bu çalışmada, üretim verimi ve gecikmeden etkilenmekte olan karlılığı en üst düzeye çıkarmak amacıyla işbirliği ile dış kaynak kullanımının çizelgelemesi önerilmektedir. Çizelgelemede işbirliğinin amacı, kurulum ihtiyaçlarını azaltarak işlerin daha erken, dolayısıyla daha yüksek üretim verimi ve daha az gecikme ile yapılmasını sağlamaktır. İşbirliği, işlerinin bir kısmını paylaşılan dış kaynak kullanımı ile yapacak olan şirketler araşında gerçekleşecek şekilde tasarlanmıştır. İşbirliği ile elde edilen çizelgeleme sayesinde şirketler özdeş işlerini kurulum ihtiyacı olmadan ardışık şekilde yapabilecek ve dış kaynağı daha verimli şekilde kullanabileceklerdir. Önerilen çizelgeleme yaklaşımı, yaprak tütün işleme sektöründen bir örnek üzerinde çalışılmıştır. İşbirliği ile çizelgelemenin etkisini değerlendirmek amacıyla matematiksel model oluşturulmuş ve hem işbirliksiz dış kaynak kullanımı hem de işbirliği ile dış kaynak kullanımı çizelgelemesi için çözülmüştür. Ayrıca, şirketlerin işbirliksiz ve işbirliği ile dış kaynak kullanımı çizelgelemesi kararları, işbirliksiz oyun olarak Stackelberg Oyunu ve Nash Dengesi ile analiz edilmiştir. Sonuç olarak, kurulum ihtiyacının azaltılması sebebiyle işbirliği ile dış kaynak kullanımı çizelgelemesinin karlılığı arttırdığı görülmüştür.

Anahtar Kelimeler: İşbirliği ile çizelgeleme, Dış kaynak kullanımı, Oyun teorisi, Stackelberg oyunu, Nash dengesi.

To my Halil...

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CHAPTER 1: INTRODUCTION

In this chapter, the motivation and purpose of the study are described. Then, the structure of the thesis is explained.

Due to the increase in product range, the need of outsourcing for producers may arise. With the increased resource availability and product varieties, the resource allocation becomes more complex in terms of job scheduling. In addition, outsourcing may increase the production costs. However for perishable products, the processing yield therefore the profitability is likely to raise with outsourcing since, more resources would be available for earlier processing. In order to keep the profitability high, the optimal schedule of jobs should be determined such that the outsourcing cost is minimized and the processing yield is maximized.

Scheduling practices have significant role for the profitability in a supply chain or a production environment. The main purpose of scheduling is ordering the processes in a timely and efficient way that the demand is satisfied on time and the costs are minimized. Besides the features of the operations or processes to be scheduled, the objective and resources are important for having an effective and efficient schedule. In real life applications, there may be numerous operations to be assigned to various resources with possible different objectives. This situation results in complex scheduling problems. In addition to the high amount of jobs or high number of resources, the processing characteristics, constraints or the objective increase the complexity. There are several algorithms used for constructing and improving feasible schedules.

Parallel machine scheduling with setup times is one of the complex settings in scheduling and the objective of minimizing tardiness on parallel machines is widely practiced. Setup time is an important parameter that affects the decisions on the schedule. Especially, high setup times may cause tardiness by extending the completion times of processes and decrease the utilization of the resources. As a result, tardiness is likely to be increased. It is previously shown by Du and Leung (1990) that minimizing total tardiness on single machine is NP-Hard. In addition, Azizoglu and Kirca (1998) have worked on the same objective with parallel machines which is consequently NP-Hard. As a consequence, several algorithms are developed for solving parallel machine scheduling problems with the objective of minimizing tardiness.

In this study, a cooperative outsourcing scheduling approach is proposed for minimizing tardiness and maximizing the processing yield with parallel machine setting where the sequence dependent setup times are high. There are two options covered for scheduling with outsourcing. First option is scheduling jobs at shared outsourcing capacity without cooperation, where the second option is scheduling the jobs at the shared outsource cooperatively. The cooperation is described to be carried out by the companies that have their own processes to be scheduled, and the cooperative schedule is designed to be performed on the shared capacity of outsourcing. The concept of the cooperation can be described as scheduling jobs at the shared outsource capacity commonly for companies such that there is a possibility for processing identical jobs of all companies consecutively for minimization of the setup requirements. When the number of setups are reduced, the jobs would be processed earlier and the processing yield would be higher with minimum tardiness.

In order to demonstrate the idea of cooperative scheduling with outsourcing, an example case of tobacco leaf processing is constructed and the impact of cooperation on schedule and the objective is analyzed. Tobacco leaf processing is performed on identical lines and each different job requires high setup. Recently, the tobacco varieties grown in Turkey are enhanced and therefore the diversity of the tobacco leaves are increased. Thus, the tobacco companies tend to outsource their processes in order to meet the deadlines and increase the processing yield. The example case includes processes to be scheduled with sequence dependent setup times and outsourced capacity acts as a parallel machine. The processing yield is designed to be higher at earlier processes and the objective is maximizing profit that is obtained by maximum yield and minimum tardiness. The presented cooperation approach is scheduling the jobs at outsourcing location commonly by the agents such that the setup requirements are minimized.

The purpose of this study is to examine the benefits of cooperative scheduling with outsourcing for companies and their approach to the cooperation. The example case is solved optimally by mathematical modelling in order to analyze the profitability of scheduling cooperation. In order to analyze the impact of cooperative scheduling, the profits are compared for both decentralized model that is a non-cooperative scheduling approach with outsourcing option for companies, and the centralized model which is a common solution of cooperative scheduling with the objective of maximizing total profit for all companies. Then, the analysis is performed with different parameters to show the profitability under various conditions.

Both non-cooperative and cooperative scheduling with outsourcing decisions of the companies are analyzed with Game Theory. Game Theory is one of the useful techniques for solving scheduling problems when the problem has multiple decision makers or multiple objectives. In the discussed problem, companies are decision makers and their decisions on the schedule are analyzed using Game Theory for both non-cooperative and non-cooperative scheduling approaches. In the non-cooperative scheduling, companies act as leader and follower and do not cooperate for scheduling with outsourcing. The outsourced capacity availability starts at different times for companies. The leader has the option of making the schedule first and follower makes decision after the leader. The decentralized model for non-cooperative case represents a leader and follower decision making process. For analyzing the decisions of the companies, the non-cooperative scheduling with outsourcing is converted to Stackelberg Game which is the leader and follower game and companies make decisions sequentially.

The centralized cooperative scheduling with outsourcing solution gives the highest total profit for all companies. However, this solution may not be preferred by companies since there is a solution with higher individual profit exists. Therefore, companies may have different preferences on cooperative scheduling with the decision of selecting jobs to be processed with cooperation. In order to analyze the cooperative scheduling decisions, the Nash Game is used. For cooperative scheduling with outsourcing, all companies act simultaneously without any leader or follower and make their decisions according to other companies' decisions until both companies decide on the same cooperative schedule. Thus, the cooperative scheduling is performed as a non-cooperative game for decision analysis.

The structure of the thesis is organized as follows. In Chapter 2, the previous related works on cooperative scheduling and scheduling with Game Theory are reviewed. Chapter 3 describes the problem and the motivation for cooperative scheduling with outsourcing. The methodology, comprising the mathematical model of the given case, and Game Theory applications are defined in Chapter 4. The example case is solved for decentralized non-cooperative scheduling with outsourcing and centralized cooperative scheduling with outsourcing optimally in Chapter 5. Then, profits with cooperation and without cooperation are compared with different parameters that are considered to increase the willingness of the companies for the cooperation. Thus, the main effects of the parameters such as costs and setup time, and the interaction between the parameters are analyzed. In addition, the Stackelberg and Nash Games are applied by generation of the payoff matrices for both decentralized non-cooperative schedules and cooperative schedules in Chapter 5. Lastly, the summary of the study and future recommendations are presented in Chapter 6.

CHAPTER 2: LITERATURE REVIEW

In this chapter, previous works on the subjects covered by this study are reviewed. The presented problem is designed on parallel machines with setup concept. Also the outsourcing decision with cooperation is analyzed by Game Theory applications. Therefore, the literature search is performed first on scheduling in general. Then, the review is focused on the parallel machine scheduling with setup, scheduling decisions on outsourcing, cooperative scheduling and Game Theory approaches for scheduling subjects. Lastly, the suggested approach of the study is presented.

Scheduling is a significant decision making process for supply chain and manufacturing systems. It is the organisation of the processes according to an objective to be optimized (Pinedo, 2012). The aim of the scheduling practices is allocation the jobs or processes to the resources in the most efficient way (Lian et al., 2006). There are numerous studies on scheduling since it is a critical operation and requires advanced practices. Even for a system including a small number of processes to be scheduled with a single objective, it may not be easy to obtain the optimal schedule that meets the objective perfectly. In other words, a system does not require vast amount of jobs, various constraints or a multiple objectives to be considered as complex to generate optimal schedule. Considering a small set of jobs to be processed in a job shop including small number of machines, the problem of minimizing the makespan is NP-hard (Kundakcı and Kulak, 2016). Minimizing maximum lateness in a system consisting of parallel machines is also NP-hard (Koulamas and Kyparisis, 2000). In order to solve these problems requiring big effort to solve, there are several heuristic methods developed. For example, Shortest Processing Time (SPT) rule is constructed for minimizing the flow time on parallel machines, whereas minimizing maximum lateness can be achieved with Earliest Due Date (EDD) rule (Pinedo and Hadavi, 1992). As the problem becomes more complicated with different additional parameters such as weights of jobs or setup times, and some constraints such as machine eligibility, advanced methods are required for obtaining a feasible schedule.

2.1 Parallel Machine Scheduling

Parallel machines setting consists of similar machines that conduct similar processes that is commonly used in several operations where it is also used for implementation of some processes such as scheduling as a special case of flow shops, single machine or gradatory systems (Pinedo, 2012). The concept of the parallel machine scheduling problems is to schedule n jobs on m parallel machines. In practice, the parallel machine scheduling can be easily NP-hard with objectives of minimizing makespan or total completion time with precedence constraints. The heuristic algorithms such as Longest Processing Time first (LPT) are developed for minimizing the makespan on parallel machines described in Pinedo (2012). As the objective or constraints gets more complicated, the heuristic methods require more effort for obtaining a feasible solution. For instance, in order to solve the problem of minimizing total completion time with precedence constraints problem on parallel machines, the Critical Path (CP) rule can be applied (Pinedo, 2012). Sivrikaya-Şerifoğlu and Ulusoy (1999) worked on minimizing earliness and tardiness penalties on parallel machines. They proposed to use Genetic Algorithm (GA) for solving the complex problem. They concluded that GA is an efficient algorithm for solving parallel machine scheduling problems, and for the problems with larger size, Multi-Component Uniform Order-Based Crossover Operator improves the solution of GA. Another work of scheduling on parallel machines is conducted by Fang and Lin (2013) with the objective of minimizing weighted total tardiness penalty and total power cost. They used EDD and Weighted Shortest Processing Time (WSPT) rule as construction methods and Particle Swarm Optimization (PSO) for improving the results of the construction methods. They suggested that PSO algorithm gives sufficient solutions in a reasonable solution time. Another approach on parallel machine scheduling is adopted by Xing and Zhang (2000). They suggested the idea of splitting jobs on parallel machines in order to minimize the maximum completion time and used Maximum Likelihood (ML) method with the estimations on maximum completion time. As a result of the study, when the setup times are included, they obtained an interval for the worst-case performance.

When the setup times are included in parallel machine scheduling problem, it becomes more complex and the need for more advanced solution approached arises. Kim et al. (2002) proposed Simulated Annealing (SA) method for solving unrelated parallel machine scheduling problem with setup times. The setup times are sequence dependent and the objective is minimizing total tardiness. Their suggested SA method produces better results than traditional SA and Neighborhood Search (NS) algorithms. Vallada and Ruiz (2011) also worked on parallel machine scheduling with sequence dependent setup times. Their objective was minimizing the makespan by using GA with fast local search. With the calibration of parameters for fast local search, they obtained outperforming results with GA. In the study of Kim et al. (2003), a problem with the same concept with the objective of minimizing total weighted tardiness is solved by four heuristic methods which are Earliest Weighted Due Date (EWDD), the

Shortest Weighted Processing Time (SWPT), Two-Level Batch Scheduling Heuristic and SA. Out of the four methods, SA gives the best results. EWDD and SWPT are less successful than other two methods and their outcomes are used as lower bounds. Two-Level Batch Scheduling is mentioned to be practical for the problems having less than hundred jobs per machine. Lee and Pinedo (1997) developed a heuristic method based on Apparent Tardiness Cost with Setups (ATCS) which is modified to be suitable for parallel machines concept. They combined the modified ATCS rule with SA by using the results from ATCS as the initial solution for SA. The proposed procedure is applied on factories and resulted in satisfactory schedules. Pfund et al. (2008) modified the ATCS rule by considering the ready times. Their developed method is called Apparent Tardiness Cost with Setups and Ready Times (ATCSR). They concluded that the method outperforms under given conditions with better results than the previous works on parallel machine scheduling with setup and ready times.

2.2 Scheduling with Outsourcing Decisions

Outsourcing is a useful decision for manufacturing operations. In general, subcontractors are preferred as outsource. For the scheduling point of view, outsourcing can be used for minimizing makespan or tardiness by providing shorter completion times of jobs where these objectives override the outsourcing costs. In the work of Lee and Sung (2008), a single machine scheduling problem with outsourcing decision is solved by using Branch and Bound (BB) algorithm. BB resulted in solutions with a good performance for the objective of minimizing total completion times and outsourcing costs. Mokhtari and Abadi (2013) studied on the outsourcing concept for minimizing total weighted completion time and outsourcing cost. The setting of the study includes parallel machines for manufacturing environment where the subcontractor processing can be performed on a single machine. Firstly, they formulated the case with integer programming and proposed Lagrangian heuristic for decomposing the problem and used Lagrangian Dual as the base for Dynamic Programming. This procedure is suggested to solve large sized problems with high performance. A different heuristic method, Ant Colony Optimization (ACO) is used by Neto and Godinho Filho (2011) for solving a flowshop scheduling problem with outsourcing option with minimizing makespan objective. It is stated that ACO gives effective result in a short computation time. The algorithm is also used in a different setting and with a different objective by Tavares Neto et al. (2015). They solved a parallel machine scheduling problem and in the concept, outsourcing is allowed. The objective is minimizing the outsourcing and delay costs and ACO algorithm works better than solving mathematical programming of the problem with more than 20 jobs in terms of computation time. Guo and

Lei (2014) proposed using Two-Phase Neighborhood Search (TPNS) algorithm for scheduling with outsourcing decision in a job-shop environment. TPNS outperformed on several instances for minimizing total tardiness and outsourcing cost.

2.3 Cooperative Scheduling

Cooperative scheduling can be defined as the cooperation of two or more agents on scheduling decisions or methods. In the literature, there are various approaches on cooperative scheduling with different concepts. For example, Yang et al. (2012) adopted cooperative scheduling for subway systems. The cooperation is considered to be between consecutive trains and the objective is maximizing the overlapping time by synchronization of acceleration and breaking times. They constructed the mathematical model and used GA for solving the problem. Wang et al. (2012) considered the cooperation of Body Area Networks as the agents for cooperative scheduling in order to reduce the inter-BAN interference. The schedule is obtained by applications of Horse Racing Scheduling and Mixed Strategy Nash Equilibrium (MSNE). Zheng et al. (2013) proposed a cooperative scheduling model for vehicular communication networks and solved it by Kuhn-Munkres algorithm. Cooperation is performed between vehicles and it is concluded that cooperation increases the efficiency of the vehicular networks. Another cooperation approach is adopted by Numao and Morishita (1991) for improving scheduling of steelmaking process. The scheduling engine, user and the scheduling rule are assumed to be the agents of the cooperation. Schedule is obtained by using Artifical Intelligence (AI) techniques and as a result, both daily scheduling and average waiting times decreased significantly. Gil et al. (2003) worked on cooperation of Uninhabited Autonomous Vehicles (UAV) by choosing their tasks using the proposed strategy and therefore generating the schedule for minimizing the travel time. In the work of Fan et al. (2018), cooperative scheduling concept is constituted for energy hubs which are assumed to be agents and share energy and cooperate for minimizing operation cost. The schedule is obtained by Game Theory with energy and cost payoffs of the hubs.

2.4 Game Theory Applications on Scheduling

Game Theory is used for solving scheduling problems when there are multiple decision makers on schedule or conflicting multiple objectives to be achieved. Ahmad et al. (2008) modeled the problem of task scheduling on multi-core processors as a cooperative game for minimizing energy consumption while also minimizing the makespan. Li et al. (2012) also solved an integrated process planing and scheduling problem with multiple objectives of minimizing maximum and total machine workload, and the maximum completion time by using game theory approach. In the work of Zhang et al. (2017), a game theory based dynamic model for flexible job shop scheduling is adopted. The machines are considered to request tasks and the model assigns the tasks optimally according to machine status. Zhou et al. (2009) presented a game theory based model for scheduling jobs on a networked manufacturing system. The jobs are assumed to be the players and the payoff is considered as the makespan to be minimized. Sun et al. (2014) worked on flexible job shop scheduling with machine breakdowns. The objectives are defined as increased robustness and stabilized performance. Nash Equilibrium (NE) is adopted for achieving both objectives. Skowron and Rzadca (2013) proposed using game theory for obtaining a fair schedule in a multiorganisational system. Fairness, the payoff, is determined by the Shapley value and it considers the contribution of the organisations who are the players of the game.

The related works are summarized in Table 1. This study brings a novel perspective on cooperative scheduling with outsourcing decision. The presented concept includes parallel machine scheduling with setup times which are relatively high. Parallel machine setting is constructed with the consideration of individual capacities of decision makers and a shared capacity environment that the outsourcing decision is proposed to be performed on. The objective is designed as processing jobs with minimized tardiness, minimized processing and outsourcing costs while maximizing the yield that is based on the time that a job is processed. The cooperation of decision makers is considered to achieve this objective by minimizing the setup requirements on shared capacity by communication. The optimal cooperative schedule is obtained by solving the mathematical model and the cooperation decisions of the players are analyzed by using Game Theory approach.

Table 1. Related Scheduling Problems and Solution Methods

Authors	Problem Type			Objective	Solution Method	
	Parallel Machine Scheduling with Setup Times	Scheduling with Outsourcing	Cooperative Scheduling		Mathematical Model	Heuristic
Kim et al. (2002)	X			Tardiness		SA
Vallada and Ruiz (2011)	X			Makespan	X	GA
Kim et al. (2003)	X			Weighted Tardiness		EWDD, SWPT, SA
Pfund et al. (2008)	X			Weighted Tardiness		ATCSR
Lee and Pinedo (1997)	X			Weighted Tardiness		ATCS
Mokhtari and Abadi (2013)		X		Weighted Completion Time and Outsourcing Cost	X	Lagrangian
Neto and Godinho Filho (2011)		X		Makespan		ACO
Tavares Neto et al. (2015)		X		Outsourcing and Delay Costs	X	ACO
Guo and Lei (2014)		X		Tardiness and Outsourcing Cost	X	TPNS
Lee and Sung (2008)		X		Weighted Total Completion Times and Outsourcing Cost		BB
Yang et al. (2012)			X	Overlapping Time	X	GA
Wang et al. (2012)			X	Inter-BAN Interference		MSNE
Numao and Morishita (1991)			X	Scheduling Time		AI
Zheng et al. (2013)			X	Efficiency		Kuhn Munkres
Fan et al. (2018)			X	Operation Cost	X	Game Theory
Ahmad et al. (2008)				Makespan and Energy Consumption	X	Game Theory
Li et al. (2012)				Makespan and Machine Workload	X	Game Theory
Zhang et al. (2017)				Machine Task Assignment	X	Game Theory
Zhou et al. (2009)				Makespan		Game Theory
Sun et al. (2014)				Robustness and Performance		Game Theory
Skowron and Rzadca (2013)				Fairness		Game Theory
This Study	X	X	X	Profit	X	Game Theory

CHAPTER 3: PROBLEM DEFINITION

This chapter covers the problem definition and the description of the illustrative example case concept that is presented in Chapter 5.

The increase in the product range may require additional capacity in a production or processing system. Instead of increasing internal capacity, outsourcing option can be preferable for companies since it may be less costly than making new investments on expanding the capacity. However, job scheduling may require more effort with outsourcing. In addition, with high product variety, the setup necessity is likely to increase and consequently the completion times are affected.

For a production system with high setup times, an efficient schedule is needed. The type of the product is also important for scheduling decisions. For example, scheduling the processes of perishable products or products that lose end product yield over time can be challenging. This study covers an outsourcing scheduling problem with cooperation opportunity for maximizing profit. As an illustrative example, job scheduling decisions are practiced for the leaf tobacco processing sector.

In Turkey, the variety of grown tobacco leaves increases in addition to the existing volumes and varieties. With a new regulation (Official Gazette, 2020), cigarette producers are required to purchase a specific volume of tobacco grown in Turkey. Therefore, new origins of tobacco started to be grown and processed by tobacco companies in order to meet the demand. The form of the leaves that are from recently introduced origins require different processing systems. Consequently, the need for outsourcing to process new varieties arises for tobacco companies. Moreover, leaf tobacco processing requires relatively high setup times between processing different kinds of jobs and it is significant for the companies to process tobacco with high yield. Processing yield partially depends on the processing starting and completion times. The earlier the tobacco is processed, the higher the processing yield is. Therefore, scheduling is a significant operation for leaf tobacco sector.

This study examines the cooperation of companies to schedule their processes with outsourcing to minimize the setup requirements. The constructed example covers the problem of job scheduling with cooperative outsourcing including high setup times and due dates. The objective is maximizing the profit which is calculated by subtracting both in-company and subcontractor processing cost and penalty cost of tardy jobs from the revenue from yielded volume. Hence, the objective includes the maximization of the processing yield and minimization of the tardiness implicitly.

With the concept of the discussed problem, two cases are examined and compared in this study. First, non-cooperative scheduling with outsourcing is solved by mathematical modelling for each company separately for obtaining decentralized schedules. In non-cooperative case, one of the companies pays a fixed cost for booking the subcontractor first to start processing at time zero and becomes the leader company. Booking subcontractor capacity first requires a higher fixed cost since processing yield would be significantly affected by the timing of processing. After the processing jobs of the leader company at the subcontractor is completed, the capacity becomes available for the follower company. Since setup would be necessary between each job of the leader company, the availability for the follower company would be considered as the time after the completion of the last job of the leader company at subcontractor and the setup for the jobs of the follower company. In this case, since there is not any communication or cooperation between companies, even if the last job of the leader company and the first job of the follower company processed at the subcontractor are similar and do not require setup, the availability for the follower company would be stated as starting after setup. In summary, the scheduling problems of leader and follower companies are solved separately and consecutively for the first case, and the processing start time for the follower company at the subcontractor is fixed. Since the problem is solved separately for each company, the objective is maximizing their own profit.

The second case is constructed as cooperative scheduling with outsourcing. In cooperative scheduling case, companies share full information and the objective is maximizing the profit of both companies. They also share the fixed cost for the subcontractor since both companies' jobs are possible to be scheduled first. The cooperation is designed to be performed on subcontractor capacity with identical jobs that do not require setup. Consequently with cooperative scheduling, companies may have such a schedule that requires minimum number of setups and process their jobs earlier with higher yield and lower tardiness, therefore increase their profit.

Table 2. Non-cooperative and Cooperative Scheduling with Outsourcing Summary Table

Scheduling with Outsourcing:	**Non-cooperative**	**Cooperative**
Solution procedure:	Sequential	Simultaneous
Solution locations:	Company and subcontractor	All companies' and subcontractor
Subcontractor availability starts:	At time zero for leader After completion of the leader for follower	At time zero for all companies
Information sharing:	Low level	High level
Number of setups:	Between all jobs	Minimized
Subcontractor fixed cost:	Paid by the leader	Shared by all companies

Cooperative scheduling is discussed with two perspectives. A centralized cooperation is solved using mathematical model with the objective of maximizing total profit of all companies. On the other hand, individual profits are used as the objective to be maximized for the decentralized cooperative scheduling.

The main features of the non-cooperative and cooperative scheduling with outsourcing cases are summarized in Table 2.

CHAPTER 4: METHODOLOGY

In this chapter, firstly the solution approach for solving the cooperative scheduling with outsourcing problem is presented. Then, the notation used in the mathematical model is defined, and the mathematical model is described for the solution of the cases. Lastly, the Game Theory applications for both non-cooperative and cooperative cases are explained.

The solution procedure starts with solving the mathematical models of the constructed example problem optimally for non-cooperative and cooperative scheduling with outsourcing cases. There are two solution approaches to the given case. Firstly, the non-cooperative scheduling with outsourcing is solved with a decentralized non-cooperative solution process. In this process, the mathematical model is solved firstly for the leader company, then follower companies solve the same model according to the leader companies solution. This non-cooperative scheduling solution can be also found with the Stackelberg Game by constructing the non-cooperative scheduling payoff matrix. Second approach is solving a centralized cooperative scheduling with outsourcing which is the case with one decision maker and the solution is obtained by solving the mathematical model for all companies commonly with a single objective of maximizing total profit. This solution is expected to result in minimum setup requirements. However, the objective of maximizing total profit may not be preferred by all companies since there is a possibility that a company may obtain higher profit with another feasible cooperative schedule. Thus, the companies would make different decisions on the cooperative schedule. This situation is modeled as Nash Game, and the equilibrium search is performed with the cooperative scheduling payoff matrix.

4.1 Notation

Sets:

$\mathscr{J}$: jobs $j,k \in \mathscr{J}$

$\mathscr{I}$: time intervals $i \in \mathscr{I}$

$\mathscr{L}$: locations $l \in \mathscr{L}$ ($\mathscr{L}^c \bigcup \mathscr{L}^s = \mathscr{L}$)

$\mathscr{L}^c$: in-company processing locations $l^c \in \mathscr{L}^c$

$\mathscr{L}^s$: subcontractor processing location $l^s \in \mathscr{L}^s$

$\mathscr{A}$: processing order of jobs $a \in \mathscr{A}$

In the following cases of the study, jobs to be processed are denoted by j and k, where i represents the time intervals that the timeline is divided into. There are two types of processing locations l^c and l^s which is generalized as l. l^c represents the location of the companies that also means their capacities or processing lines, and the subcontractor location is denoted by l^s which is the shared capacity for the following example. Lastly, a represents the processing order of a job at a location.

Parameters:

p_j: processing time of job j

se_{jk}: setup time between jobs j and k

d_j: due date of job j

Yld_i: processing yield of time interval i

c: in-company processing cost per time unit

b: subcontractor processing cost per time unit

n: penalty cost for tardy jobs per time unit

m: sale price per time unit

Lo_i: lower bound of time interval i

Up_i: upper bound of time interval i

Decision variables:

$Comp_j$: completion time of processing job j

Str_{jl}: starting time of processing job j at location l

T_j: tardiness of job j

$$r_{jkl} = \begin{cases} 1, & \text{if job } k \text{ is processed immediately after job } j \text{ at location } l \\ 0, & \text{otherwise} \end{cases}$$

$$x_{jal} = \begin{cases} 1, & \text{if job } j \text{ is processed in order } a \text{ at location } l \\ 0, & \text{otherwise} \end{cases}$$

y_j: yield of job j

y_j^s: yield of job j at subcontractor location

Pr_{ji}: proportion of job j in interval i

$$I_{ji}^S = \begin{cases} 1, & \text{if starting time of job } j \text{ is in interval } i \\ 0, & \text{otherwise} \end{cases}$$

$$I_{ji}^C = \begin{cases} 1, & \text{if completion time of job } j \text{ is in interval } i \\ 0, & \text{otherwise} \end{cases}$$

$$I_{ji}^T = \begin{cases} 1, & \text{if processing of job } j \text{ is completely in interval } i \\ 0, & \text{otherwise} \end{cases}$$

$$I_{ji}^{T1} = \begin{cases} 1, & \text{if starting time of job } j \text{ is greater than the lower bound of interval } i \\ 0, & \text{otherwise} \end{cases}$$

$$I_{ji}^{T2} = \begin{cases} 1, & \text{if completion time of job } j \text{ is lower than the upper bound of interval } i \\ 0, & \text{otherwise} \end{cases}$$

4.2 Mathematical Model

In this section, the constructed mathematical model for non-cooperative and cooperative scheduling with outsourcing is presented.

For the non-cooperative scheduling, the model is solved iteratively with one l^c for each iteration and each company using Equation 4.1 as the objective function. For the follower company, the available starting time for shared capacity is updated according to the last processed job of the leader company at shared location .

$$\text{Maximize } Z = \sum_j \left(p_j * y_j * m\right) - \sum_j \sum_a \left(x_{jal^c} * p_j * c\right) - \sum_j \left(p_j * y_j^s * b\right) - \sum_j \left(T_j * n\right) \tag{4.1}$$

Subject to

$$Comp_j = \sum_l \left(Str_{jl} + \sum_a \left(p_j * x_{jal}\right)\right) \qquad \forall j \tag{4.2}$$

$$T_j \geq Comp_j - d_j \qquad \forall j \tag{4.3}$$

$$Str_{kl} \geq Str_{jl} + p_j + se_{jk} - M * \left(1 - r_{jkl}\right) \qquad \forall j,k, j \neq k, l \in \{l^c, l^s\} \tag{4.4}$$

$$Str_{jl} \leq M * \sum_a x_{jal} \qquad \forall j, l \in \{l^c, l^s\} \tag{4.5}$$

$$x_{jal} + x_{k(a+1)l} \leq r_{jkl} + 1 \qquad \forall j,k, j \neq k, a, l \in \{l^c, l^s\} \tag{4.6}$$

$$\sum_j x_{j(a+1)l} \leq \sum_j x_{jal} \qquad \forall a, l \in \{l^c, l^s\} \tag{4.7}$$

$$\sum_{a}\sum_{l} x_{jal} = 1 \qquad \forall j \tag{4.8}$$

$$\sum_{j} x_{jal} \leq 1 \qquad \forall a, l \in \{l^c, l^s\} \tag{4.9}$$

$$\sum_{i}\left(Lo_i * I_{ji}^S\right) \leq \sum_{l} Str_{jl} \leq \sum_{i}\left(Up_i * I_{ji}^S\right) \qquad \forall j \tag{4.10}$$

$$\sum_{i}\left(Lo_i * I_{ji}^C\right) \leq Comp_j \leq \sum_{i}\left(Up_i * I_{ji}^C\right) \qquad \forall j \tag{4.11}$$

$$Lo_i - \sum_{l} Str_{jl} + 1 \leq M * I_{ji}^{T1} \qquad \forall j, i \tag{4.12}$$

$$\sum_{l} Str_{jl} - Lo_i \leq M * \left(1 - I_{ji}^{T1}\right) \qquad \forall j, i \tag{4.13}$$

$$Comp_j - Up_i + 1 \leq M * I_{ji}^{T2} \qquad \forall j, i \tag{4.14}$$

$$Up_i - Comp_j \leq M * \left(1 - I_{ji}^{T2}\right) \qquad \forall j, i \tag{4.15}$$

$$\sum_{i} I_{ji}^S = 1 \qquad \forall j \tag{4.16}$$

$$\sum_{i} I_{ji}^C = 1 \qquad \forall j \tag{4.17}$$

$$I_{ji}^{T1} + I_{ji}^{T2} \leq I_{ji}^T + 1 \qquad \forall j, i \tag{4.18}$$

$$I_{ji}^{T1} + I_{ji}^{T2} \geq 2 * I_{ji}^T \qquad \forall j, i \tag{4.19}$$

$$Pr_{ji} \leq \left(\frac{Up_i - \sum_l Str_{jl}}{p_j}\right) + M * \left(1 - I_{ji}^S\right) \qquad \forall j, i \tag{4.20}$$

$$Pr_{ji} \leq \left(\frac{Up_i - Lo_i}{p_j}\right) + M * \left(1 - I_{ji}^T\right) \qquad \forall j, i \tag{4.21}$$

$$Pr_{ji} \leq \left(\frac{Comp_j - Lo_i}{p_j}\right) + M * \left(1 - I_{ji}^C\right) \qquad \forall j, i \tag{4.22}$$

$$Pr_{ji} \leq I_{ji}^{T} + I_{ji}^{S} + I_{ji}^{C} \qquad \forall j,i \tag{4.23}$$

$$\sum_{i} Pr_{ji} = 1 \qquad \forall j \tag{4.24}$$

$$y_j = \sum_{i} \left(Pr_{ji} * Yld_i \right) \qquad \forall j \tag{4.25}$$

$$y_j^s \geq y_j - M * \left(1 - \sum_{a} x_{jal^s} \right) \qquad \forall j \tag{4.26}$$

$$Comp_j, T_j \geq 0 \qquad \forall j \tag{4.27}$$

$$Str_{jl} \geq 0 \qquad \forall j, l \in \{l^c, l^s\} \tag{4.28}$$

$$r_{jkl} \in \{0,1\} \qquad \forall j,k,l \in \{l^c, l^s\} \tag{4.29}$$

$$x_{jal} \in \{0,1\} \qquad \forall j,a,l \in \{l^c, l^s\} \tag{4.30}$$

$$y_j, y_j^s \geq 0 \qquad \forall j \tag{4.31}$$

$$Pr_{ji} \geq 0 \qquad \forall j,i \tag{4.32}$$

$$I_{ji}^{S}, I_{ji}^{C}, I_{ji}^{T}, I_{ji}^{T1}, I_{ji}^{T2} \in \{0,1\} \qquad \forall j,i \tag{4.33}$$

The objective function (4.1) maximizes the total profit of a company by subtracting in-company processing cost, subcontractor processing cost and penalty cost for tardy jobs from the revenue. It is assumed that the volumes of the jobs are in proportion to the processing times. For a job with volume of n, the processing of the job is n time units and the revenue from this job is obtained by multiplying the sale price with the after processing volume which is yielded. Therefore, the processing time p_j is used as the volume in the model. The total revenue is calculated by the multiplication of yielded volume and the sale price per time unit. If a job is processed in-company, the processing cost basically depends on the volume which is equal to the multiplication of the processing time and in-company processing cost per time unit. In contrast, the subcontractor cost is in proportion to the processing end volume. Thus, it is obtained by multiplying the processing time with the yield of the job and subcontractor processing cost. Lastly, the penalty cost for tardy jobs is calculated by the tardiness with penalty cost per each time unit.

The constraints (4.2), (4.3), (4.4) are for the calculation of the completion time, tardiness and starting time of jobs, respectively. Constraint (4.5) guarantees that the starting time location is the same location as the job is assigned. Constraint (4.6) is for ensuring that jobs that are processed consecutively have successive orders, and constraint (4.7) provides that there is not any empty order between jobs. With equation (4.8), all jobs are assigned to exactly one location and one order, and equation (4.9) guarantees that for each location and order, at most one job is assigned. Constraints (4.10) and (4.11) are for finding the time intervals that a job's starting and completion times are in. If processing of a job is completely in one interval, meaning that the starting time is greater than the lower bound of the interval and the completion time is lower than the upper bound of the same interval, equations (4.12), (4.13), (4.14) and (4.15) are satisfied. Constraints (4.16) and (4.17) ensure that starting and completion times of jobs are assigned to the intervals. Equations (4.18) and (4.19) provide that if the starting time of a job is greater than the lower bound of an interval and the completion time is lower than the upper bound of the same interval for the same job, the job is processed completely in that interval. Constraints (4.20), (4.21) and (4.22) calculate the proportion of jobs that is processed on each time interval and constraint (4.24) ensures that the total of proportions is equal to one for each job. Lastly, constraint (4.25) calculates the processing yield of jobs, whereas constraint (4.26) calculates the processing yield of jobs that are processed by subcontractor.

The sets for the solution of the cooperative scheduling contain all jobs of companies to be processed and all in-company locations of companies. For the cooperative solution, the following constraints (4.35) and (4.36) are included in addition to all constraints of the model. These constraints ensure that a company can process only its own jobs in-company. The objective function used in the cooperative scheduling solution is the equation (4.34). For cooperative scheduling, the model is solved once for all companies commonly. And the objective function maximizes the total profit for all companies together.

Sets:

J_l: set of jobs that belong to in-company processing location l, $l \in \mathscr{L}^c$

$$\text{Maximize } Z = \sum_j (p_j * y_j * m) - \sum_j \sum_a \sum_{l \in \mathscr{L}^c} (x_{jal} * p_j * c) - \sum_j (p_j * y_j^s * b) - \sum_j (T_j * n) \tag{4.34}$$

Subject to

$$\sum_{l | j \notin J_l} Str_{jl} = 0 \qquad \forall j \tag{4.35}$$

$$\sum_a \sum_{l | j \notin J_l} x_{jal} = 0 \qquad \forall j \tag{4.36}$$

In order to calculate the individual profits of the companies for the cooperative scheduling, the following parameters and equations are included in the solution. For example if the profit of Company A to be calculated, the parameter $profitA$ is added to the model and found by the equation 4.37.

$profitA$: profit of Company A

$$profitA = \sum_{j \in J_l} (p_j * y_j * m) - \sum_{j \in J_l} \sum_a \sum_{l \in \mathscr{L}^c} (x_{jal} * p_j * c) - \sum_{j \in J_l} (p_j * y_j^s * b) - \sum_{j \in J_l} (T_j * n) \tag{4.37}$$

In Chapter 5, an example from leaf tobacco sector is discussed and solved as the non-cooperative and cooperative scheduling with outsourcing. For non-cooperative scheduling, the model is solved separately for each company. The company that pays the fixed cost has the right to book the shared capacity at time zero. Therefore, the non-cooperative schedule is firstly solved for the leader company. For the follower company, the same model is used with a small revision of the available starting time of processing at the shared capacity. Since it is assumed for the non-cooperative scheduling that there is not any communication between companies, even if the consecutively processed jobs would be similar and do not require setup at the shared capacity location, the starting time for the follower company at that location would be after setup from the last job of the leader company. For cooperative scheduling, the model is solved commonly for all companies including all companies' jobs.

The model used in this study is a special case of n jobs and m parallel machines problem with the objective of minimizing tardiness in case the yield yie_j is equal to 1 for all j and the sale price m is equal to both in-company and subcontractor processing costs c and b is identical to negative of the tardiness formula for equation (4.34). Since it is previously proved that minimizing total tardiness on parallel machines is NP-Hard, the given problem is also NP-Hard. The problem is solved optimally by mathematical modelling. Once the problem is solved optimally for non-cooperative and cooperative scheduling, the impact of the cooperation on the objective can be easily analyzed. In order to evaluate the improvement of the objective with cooperative scheduling further and companies' practical decisions on scheduling with outsourcing, the Game Theoretical approaches explained in Section 4.3 are adopted.

4.3 Analysis of Scheduling with Outsourcing Decisions with Game Theory

In this section, the non-cooperative and cooperative scheduling with outsourcing decisions are analyzed by two Game Theory approaches. Considering the presented problem, the players are the companies, the decisions or strategies are the jobs selected to be processed on and the payoff is the profit.

The analysis of decisions for non-cooperative scheduling with outsourcing is inspired from Stackelberg Game which is also called the leader and follower game. The leader selects the strategy first and imposes own optimal strategy, and the follower makes the decision according to the given strategy of the leader (Simaan and Cruz, 1973). For the cooperative outsourcing scheduling, the approach is based on Nash Equilibrium. It is a concept that players interact each other and a player cares other players actions during decision making process (Osborne and Rubinstein, 1994).

The cooperative solution with mathematical modelling is a centralized approach for cooperation by assuming there is one decision maker, since the objective function results in the maximum total profit of the companies. In fact, the cooperative scheduling concept for this study is scheduling selected jobs at shared location without privilege, meaning that there is allowance for all companies' jobs to be processed in any order by sharing the fixed cost. Therefore, although the optimal solution of the cooperative scheduling results in the highest total profit, the individual profits of the companies for that solution may not be the highest for all companies. Hence, the optimal solution of the model may not be preferred by companies who have the opportunity of obtaining higher profit by processing different jobs at shared location with cooperation. Consequently, companies are likely to have different decisions on the cooperative schedules which means that each company is actually a decision maker. In summary, both non-cooperative and cooperative problems can be modeled as decentralized or non-cooperative games considering the scheduling decisions. The non-cooperative outsourcing scheduling case has the leader and follower, and decisions are made sequentially. Therefore the concept fits to a decentralized Stackelberg Game. In cooperative outsourcing scheduling case, companies make their decisions simultaneously on the cooperative schedule with the objective of maximizing their own profits and determine the equilibrium. For this concept, the decentralized Nash Game is suitable for the cooperative scheduling with outsourcing decisions.

The payoff matrices for non-cooperative and cooperative schedules are constructed by calculating the individual profits of the companies for all possible decisions that are the combination of the jobs selected to be processed on shared capacity using the mathematical model. Then the Stackelberg Game is conducted on the non-cooperative schedules payoff matrix in order to evaluate the non-cooperative scheduling decision outcomes. Using the cooperative scheduling payoff matrix, the Nash Equilibrium search is performed. The objective function of the centralized cooperative scheduling is constructed to calculate the total profit of the companies. However, for the cooperative game payoff matrix, the profits of the companies calculated separately for each company.

CHAPTER 5: IMPLEMENTATION

In this chapter, firstly the illustrative example is described and solved optimally for non-cooperative and cooperative scheduling with outsourcing using the corresponding mathematical models presented in Chapter 4. The solutions are generated using GAMS version 23.9. For analyzing the impact of the cooperation on the objective, a computational experiment is conducted by solving the problem with different parameters optimally. Lastly, for the given case, Game Theoretical applications are performed on the constructed payoff matrices for non-cooperative and cooperative scheduling in order to evaluate the scheduling with outsourcing decisions of the companies.

5.1 Illustrative Example

In this section, the illustrative example from leaf tobacco sector is described. Leaf tobacco processing is a significant part of the agricultural and industrial sector in Turkey. In addition, the leaf tobacco sector plays an important role in international trade since tobacco is grown and processed locally and a big portion of processed tobacco is exported. Every year, around 80 million kgs of tobacco are grown and purchased by tobacco companies in Turkey (Tobacco Experts Association, 2020). Tobacco is considered as a perishable product before it is processed. The grown and purchased amount decreases during processing due to the changes in leaf humidity. In addition, leaves are sorted and cleaned from dust, twigs, stems, or non-tobacco materials. The proportion of the difference between purchased weight and after processing weight is an important indicator of the processing yield. Furthermore, the timing of processing is significant for the processing yield and quality results. The humidity kept in leaves before processing results in quality loss and the yield decreases. Therefore, the sooner tobacco is processed after purchasing, the higher yield is. In order to increase processing yield, companies tend to subcontract the processing of their tobacco. Subcontractors generally do not play a role in purchasing period, they are only used as an external capacity.

Tobacco processing is performed on a single line, which is continuously fed, and processed tobacco is packaged at the end of the line. It is only possible to process one variety on each line at a time. Moreover, during setup, processing lines are cleansed from tobacco remnants. The cleansing is a detailed process and duration is relatively high compared to other setup processes such as temperature and steam adjustments or vacuum processing before processing. This is why each variety is processed without preemption. Since it is only possible to process as many varieties as the number of available lines for a company, in order to process earlier with a higher yield, companies may prefer subcontracting opportunities.

The proposed cooperative scheduling with outsourcing method offers a shared capacity environment for two companies, Company A and Company B, in addition to their individual capacities. Company A is assumed to be the leader company that pays a fixed cost for starting the processing of jobs at subcontractor location at time zero in the non-cooperative scheduling, and Company B is the follower company that starts processing jobs after Company A's jobs are finished at the subcontractor location. The shared capacity environment is designed as subcontractor. It is assumed that there are 5 identical jobs representing 5 varieties to be processed without preemption. Each company has one available line that is available for processing only one job at a time and the subcontractor has the same available capacity. Companies either process their own jobs in-company or use the subcontractor capacity. It is assumed that they are not able to process other companies' jobs. Each job has given due dates and penalty cost for tardy jobs. All jobs are assumed to be ready at time zero and setup time is assumed to be fixed between different types of jobs. One of the main conditions to examine in this study is having zero setup time between similar jobs and therefore processing jobs earlier which is also the motivation for cooperation on scheduling. The processing time of a job depends on the volume of the tobacco to be processed. In addition, the processing line has a fixed speed described as the volume passing in a time unit which means that the processing times are proportional to the volume of the tobacco. In the examined case, time unit is assumed to be hour. Therefore, processing times, setup times, and due dates are given in hours. Penalty cost, processing costs for both in-company and subcontractor, and sale price are also given per hour. The timeline is divided into intervals and each time interval has a relative processing yield. The objective is to maximize the profit by processing jobs as early as possible with higher yield and lower tardiness.

The processing yield is a significant factor for the companies. The higher yield means the higher volume of the end product and the higher revenue. The reason of the yield loss is the moisture hidden in unprocessed leaves and non-tobacco material inside the leaf storage boxes previously counted in the leaf volume in kgs while purchasing. In time, the moisture decomposes the leaves and decomposed leaves are not used in final volume. Also, the moisture vaporizes if leaves are stored as unprocessed. Considering the timeline as intervals, for the first couple of time intervals, the yield difference would be higher, since the leaves lose moisture and some parts of the leaves are decomposed. After the moisture is almost completely lost, decomposition and

Table 3. Yield Percentages of Time Intervals

Interval	1	2	3	4	5	6
Time (h)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 200
Yield %	96	86	78	72	68	66

Table 4. Processing Times and Due Dates of Jobs of Company A and Company B

	Jobs	Processing Time (h)	Due Date (h)
Company A	1	9	25
	2	17	48
	3	13	56
	4	7	33
	5	11	29
Company B	6	12	44
	7	18	28
	8	9	36
	9	7	51
	10	11	27

volume loss would stop and the yield would depend on only the non-tobacco material volume. Therefore, the yield percentage decreases exponentially. The processing yield denoted by Yld_i for time interval i decreases according to processing time.

In the experiments, the timeline is set from 0 to 200 hours and divided into 6 time intervals. It is assumed that the yield decreases until the 6th time interval and then remains constant. First 5 time intervals consist of 10 hour units and the last interval consists of 150 hour units. The yield of each interval is given in Table 3 as percentages.

The jobs to be processed by each company are denoted by j and jobs 1 to 5 belong to Company A and jobs 6 to 10 belong to Company B. Jobs 1 and 6 are assumed to be similar and do not require setup if they are processed consecutively, which is also valid for the job pairs 2 and 7, 3 and 8, 4 and 9, 5 and 10. The processing times are given in hours according to the volume to be processed. The due date of each job is also given in hours. Table 4 shows the processing times and due dates of jobs.

There are 3 processing locations which are Company A, Company B and Subcontractor. Subcontractor processes both jobs of Company A and Company B where companies are assumed to be allowed to process only their own jobs.

The setup is basically for the cleaning of the line for processing different varieties that are jobs in this case. Therefore, the setup is only required if two non-identical jobs are being processed consecutively. The setup time is assumed to be fixed at 7 hours since the setup process is same for all jobs, and zero for the identical job pairs.

The objective is to maximize profit which is obtained by subtracting the in-company processing costs for each company, subcontractor processing cost and

Table 5. The Non-cooperative Scheduling with Outsourcing Solution Table for Company A

Company A	Processing Time (h)	Due Date (h)	Starting Time	Completion Time	Processing Yield	Lateness	Processing Location
Job 1	9	25	0	9	0.960	-	Subcontractor
Job 2	17	48	18	35	0.772	-	In-company
Job 3	13	56	42	55	0.672	-	In-company
Job 4	7	33	16	23	0.826	-	Subcontractor
Job 5	11	29	0	11	0.951	-	In-company
				Yielded Volume:	46.743		

penalty cost for tardy jobs from the revenue of the yielded final volume. For non-cooperative scheduling, the objective is formulated for each company separately and for centralized cooperative scheduling, the objective is maximizing total profit of the two companies.

The optimal solution for the non-cooperative scheduling with outsourcing results in the schedule that the jobs are ordered as job 5, job 2 and job 3 for the processing at Company A. The order of jobs for the subcontractor is job 1 and job 4. The model is then solved for Company B and the in-company processing order is job 7, job 10 and job 6 where the order for subcontractor is job 8 and job 9. When solving the problem for Company B, since the subcontractor schedule would start after the jobs of Company A, even if the similar job with the last job of Company A would be scheduled first for Company B at subcontractor, since there is not any communication in the non-cooperative scheduling, the setup time is counted. The profit of Company A is calculated as 1217.38 and the profit of Company B is 991.11 without any cooperation. Total profit is 2208.49.

The Gantt chart, solution table and profit calculation for non-cooperative scheduling with outsourcing are shown in Figure 1 and Tables 5, 6 and 7.

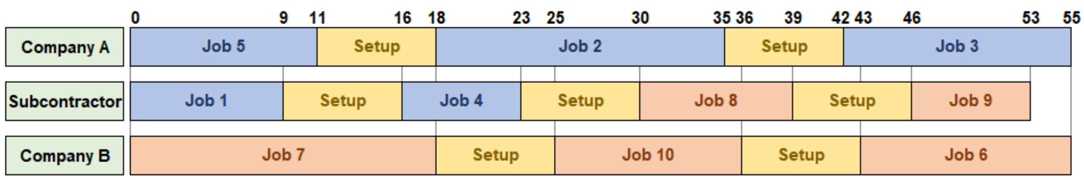


Figure 1. The Non-cooperative Scheduling with Outsourcing Solution Gantt Chart

Secondly, when the centralized cooperative scheduling with outsourcing problem is solved, the optimal schedule for Company A gives the order job 1, job 4, job 2 and job 3 where the optimal schedule for Company B is job 7, job 6 and job 9. The order for the subcontractor location is job 10, job 5 and job 8. With this cooperative schedule,

Table 6. The Non-cooperative Scheduling with Outsourcing Solution Table for Company B

Company B	Processing Time (h)	Due Date (h)	Starting Time	Completion Time	Processing Yield	Lateness	Processing Location
Job 6	12	44	43	55	0.672	11	In-company
Job 7	18	28	0	18	0.916	-	In-company
Job 8	9	36	30	39	0.720	3	Subcontractor
Job 9	7	51	46	53	0.671	2	Subcontractor
Job 10	11	27	25	36	0.747	9	In-company
				Yielded Volume:	43.946		

Table 7. The Non-cooperative Scheduling with Outsourcing Solution Profit Calculation Table

	Company A	Company B
In-company Processing Cost (15 per hour):	615	615
Subcontractor Processing Cost (35 per hour multiplied with the yield):	504.77	391.20
Penalty Cost (8 per hour):	0	200
Revenue (50 per hour unit):	2337.15	2197.30
Profit:	**1217.38**	**991.11**

the required setup is decreased to 6 which is 7 hours less than the non-cooperative schedule. The profit of Company A and Company B become 1109.93 and 1192.41 and the total profit is 2302.34.

The Gantt chart, solution table and profit calculation for the centralized cooperative scheduling with outsourcing are shown in Figure 2 and Tables 8, 9 and 10.

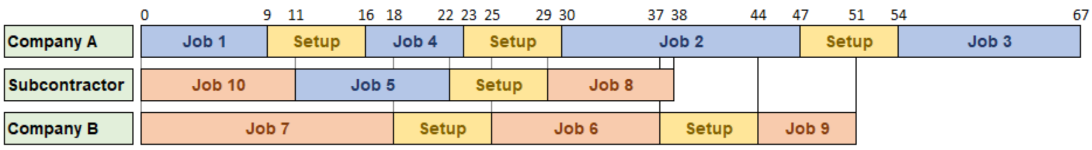


Figure 2. The Cooperative Scheduling with Outsourcing Solution Gantt Chart

Table 8. The Cooperative Scheduling with Outsourcing Solution Table for Company A

Company A	Processing Time (h)	Due Date (h)	Starting Time	Completion Time	Processing Yield	Lateness	Processing Location
Job 1	9	25	0	9	0.960	-	In-company
Job 2	17	48	30	47	0.704	-	In-company
Job 3	13	56	54	67	0.660	11	In-company
Job 4	7	33	16	23	0.826	-	In-company
Job 5	11	29	11	22	0.845	-	Subcontractor
				Yielded Volume:	44.265		

Table 9. The Cooperative Scheduling with Outsourcing Solution Table for Company B

Company B	Processing Time (h)	Due Date (h)	Starting Time	Completion Time	Processing Yield	Lateness	Processing Location
Job 6	12	44	25	37	0.745	-	In-company
Job 7	18	28	0	18	0.916	-	In-company
Job 8	9	36	29	38	0.727	2	Subcontractor
Job 9	7	51	44	51	0.677	-	In-company
Job 10	11	27	0	11	0.951	-	Subcontractor
				Yielded Volume:	47.171		

Table 10. The Cooperative Scheduling with Outsourcing Solution Profit Calculation Table

	Company A	Company B
In-company Processing Cost (15 per hour):	690	555
Subcontractor Processing Cost (35 per hour multiplied with the yield):	325.33	595.14
Penalty Cost (8 per hour):	88	16
Revenue (50 per hour unit):	2213.25	2358.55
Profit:	**1109.93**	**1192.41**

5.2 Computational Experiment

In this section, a computational experiment for comparing the results with different parameters that are considered to affect the companies' behaviors on cooperative scheduling with outsourcing is explained.

Following the solution of the given example, the same concept is solved with different values of the parameters that are setup time, subcontractor processing cost and penalty cost for tardy jobs. This experiment is conducted in order to analyze the companies' willingness to cooperate under different settings. The two approaches, non-cooperative and cooperative scheduling with outsourcing, are solved optimally with the combination of different parameter sets and the profits of the two companies are calculated.

The setup time has a significant impact on the cooperation since the identical jobs do not require setup, therefore the jobs are processed earlier, the tardiness is minimized and the yield becomes higher. For the comparison, in order to show the impact of the setup time on the cooperation, the setup time is chosen to be relatively low, medium and high according to the processing times. A low setup time is assumed to be 3 hours for the given case, where the initial setup time which is 7 hours considered to be medium and the highest setup time is 15 hours.

It is considered for the case in this study that subcontractor processing cost is another factor that has impact on companies' decision on cooperative scheduling. For a product that the processing yield is assumed to be important, the cost of subcontracting is likely to depend on the yield. Therefore, in this case, the subcontractor is paid according to the yield. For this study, it is calculated by multiplying the subcontractor processing cost with both the volume and the processing yield. Thus, when the subcontractor processing cost per hour is relatively higher than the in-company processing cost, the processing yield has more impact on the profit. In contrast, when the subcontractor processing cost is low, which is assumed to be slightly higher than the in-company processing cost, the yield has less effect on the profit. Different subcontractor processing costs may result in different choices of companies on the job set to be processed at the subcontractor. Therefore, in the experiment, two levels of subcontractor processing costs are chosen. The lower cost value is selected to be slightly higher than the in-company processing cost, and the higher value is slightly higher than the double of the in-company processing cost. The subcontractor processing cost is chosen as 35 per hour initially which is slightly higher than the double price of in-company processing. In order to examine the impact of the lower subcontractor processing cost and see if companies process more jobs at subcontractor with cooperation, it is also taken as 20 per hour which is slightly higher than in-company processing cost and a lot less than the initial subcontractor processing cost.

The penalty cost that is paid for tardy jobs per hour has effect on the decision of the job sets to be processed in-company and subcontractor. This effect can be described as the following. If penalty cost is low and the subcontractor cost is high, companies may tend to process higher number of jobs using their own capacity. In contrast, if the subcontractor cost is relatively low and the penalty cost is high, the optimal schedule may result in more jobs to be processed in subcontractor location. In order to analyze the impact of the penalty cost, three levels of penalty cost is used in the comparison experiment. The low cost is chosen as close to the half of the in-company processing cost, and the medium and high levels are chosen as the lowest penalty cost multiplied by two and three relatively. The penalty cost is initially taken as 8 per hour and that cost is assumed to be the lowest for the comparison. The medium penalty cost per hour is chosen as 16 and the highest is chosen as 24. This incremental increase examines if companies tend to cooperate in order to process jobs earlier with minimized setup and tardiness.

The non-cooperative and centralized cooperative scheduling problems are solved optimally with selected parameters and the results are shown in Table 11. With this experiment, the impact of the cooperation on profits can be analyzed with different parameter settings. When the results with negative profit of Company B are eliminated, the profit of Company A is decreases by 7 percent where the profit of Company B and total profit are increases by 69 percent and 16 percent relatively. Since the fixed cost

Table 11. Computational Experiment with Different Parameters

Cooperation	Setup Time	Subcontractor Processing Cost per hour	Penalty per hour	Profit of Company A	Profit of Company B	Total Profit
Non-cooperative	3	35	8	1,285	1,239	2,523
Cooperative				1,268	1,257	2,525
Non-cooperative	7	35	8	1,217	991	2,208
Cooperative				1,110	1,192	2,302
Non-cooperative	15	35	8	1,046	474	1,519
Cooperative				925	935	1,860
Non-cooperative	3	35	16	1,277	1,214	2,490
Cooperative				1,260	1,232	2,492
Non-cooperative	7	35	16	1,217	791	2,008
Cooperative				1,115	1,159	2,274
Non-cooperative	15	35	16	1,014	-632	381
Cooperative				753	741	1,494
Non-cooperative	3	35	24	1,273	1,201	2,474
Cooperative				1,255	1,233	2,488
Non-cooperative	7	35	24	1,217	595	1,812
Cooperative				1,098	1,160	2,258
Non-cooperative	15	35	24	990	-1,512	-523
Cooperative				609	525	1,134
Non-cooperative	3	20	8	1,526	1,216	2,742
Cooperative				1,415	1,503	2,918
Non-cooperative	7	20	8	1,490	864	2,354
Cooperative				1,371	1,447	2,818
Non-cooperative	15	20	8	1,378	377	1,755
Cooperative				1,202	1,172	2,374
Non-cooperative	3	20	16	1,526	1,064	2,590
Cooperative				1,415	1,503	2,918
Non-cooperative	7	20	16	1,490	424	1,914
Cooperative				1,405	1,396	2,802
Non-cooperative	15	20	16	1,354	-503	851
Cooperative				1,058	956	2,014
Non-cooperative	3	20	24	1,526	912	2,438
Cooperative				1,415	1,503	2,918
Non-cooperative	7	20	24	1,490	-16	1,474
Cooperative				1,405	1,380	2,786
Non-cooperative	15	20	24	1,330	-1,383	-53
Cooperative				879	775	1,654

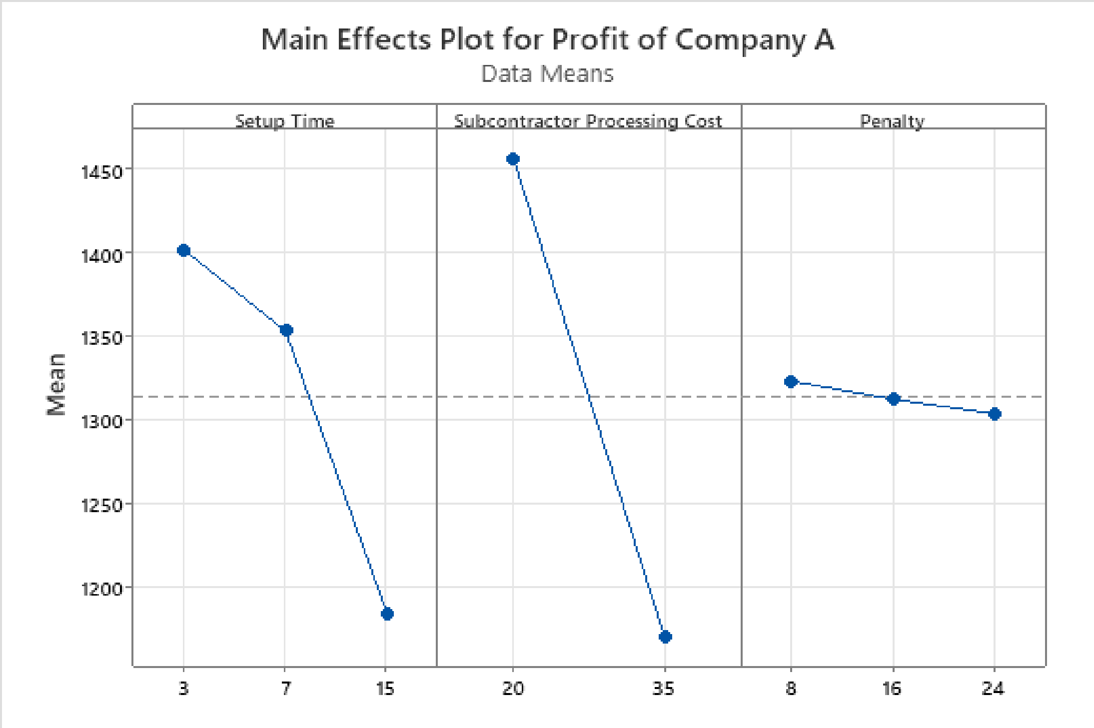
is not included in the calculations, the decrease in the profit of Company A is expected from the beginning of the calculations. For a specific range of fixed cost, it is possible for the profit of Company A to increase with cooperative scheduling where the rise in the profit of Company B gets smaller since the fixed cost would be shared among companies.

The results of the computational experiment are analyzed by graphical representations in Figures 3a-11 using the main effects and interaction plots for profits of Company A, Company B and total profit for both non-cooperative and cooperative scheduling with outsourcing results. The graphics are generated with Minitab Statistical Software version 21. Same plots are also used for the percentage changes in profits from non-cooperative to cooperative scheduling.

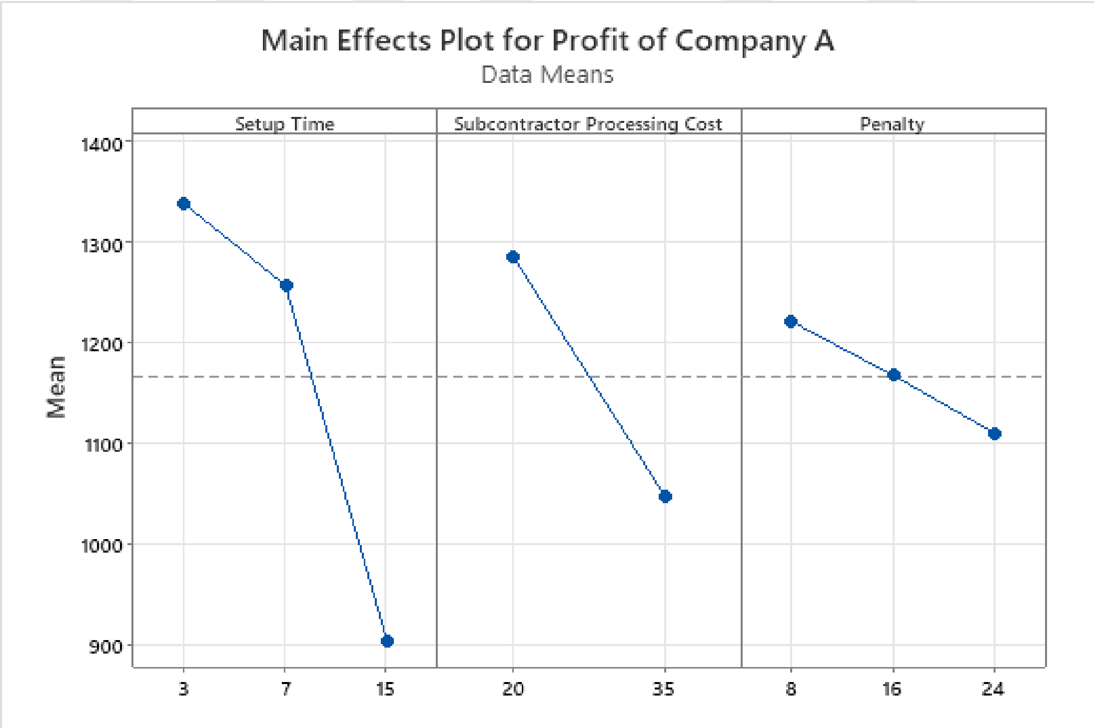
It can be seen from Figures 3a, 3b, 4a, 4b, 5a and 5b that as the setup time increases, the profits decline with the same ratio for both companies. However, as the setup time increases from 3 to 7, the decrease is more dramatic in non-cooperative scheduling where the dramatic decrease occurs as the setup time increases for 7 to 15 for cooperative scheduling. Therefore, it can be inferred that a smaller increase affects the profit more in the non-cooperative scheduling. With the increase in subcontractor processing cost, the profit of Company A in the non-cooperative scheduling decreases sharply since the jobs of that company is processed with higher yield at subcontractor and the cost is multiplied with the yield. In contrast, the profit of Company B in non-cooperative scheduling increases slightly. This increase can be explained with the decrease in the number of jobs of Company B processed at subcontractor location. For cooperative scheduling, the rise in the subcontractor cost has the same impact on the profits of both companies since the same number of jobs are processed at subcontractor with similar yields. This situation has the same impact when the penalty cost for tardy jobs increases. Since the similar number of jobs would be tardy for both companies, the decline in the profits are likewise for cooperative scheduling. However, for non-cooperative scheduling, Company A is less affected by the chance in penalty cost since the tardiness is low.

Figures 6, 7 and 8 show that when the setup time is high, the profit is more affected by the changes in penalty cost. With higher setup times, the tardiness increases and the profit declines especially with high penalty cost. In addition, for the difference in profits between the non-cooperative scheduling and the cooperative scheduling, Figure 9b shows that for Company A, there is a slight interaction between the penalty cost and the subcontractor processing cost. Moreover, the profit is more improved with the higher setup time even the subcontractor processing cost and penalty cost are high. According to Figures 10b and 11b, for the improvement in the profit of Company B and total profit, the setup is more affected by the changes in the subcontractor processing cost and penalty cost. This interaction is also valid between the penalty cost and the subcontractor processing cost.

The differences in percentage between non-cooperative and cooperative scheduling are also analyzed with main effects plot in Figures 9a, 10a and 11a. The decrease in the profit of Company A scales up with higher setup time due to the reduction in the processing yield, which is also the reason for only slight change with subcontractor processing cost. On the other hand, the increment in the profit of Company B is the highest with the middle level of setup since the effect of the cooperation is less with small setup time and the high setup time results in decrease of the profit. Also, the higher subcontractor processing cost results in less profit for Company B and total profit since the yield increases with cooperation. In addition, with minimized setup, the tardiness becomes lower and penalty cost has less impact on the profit.

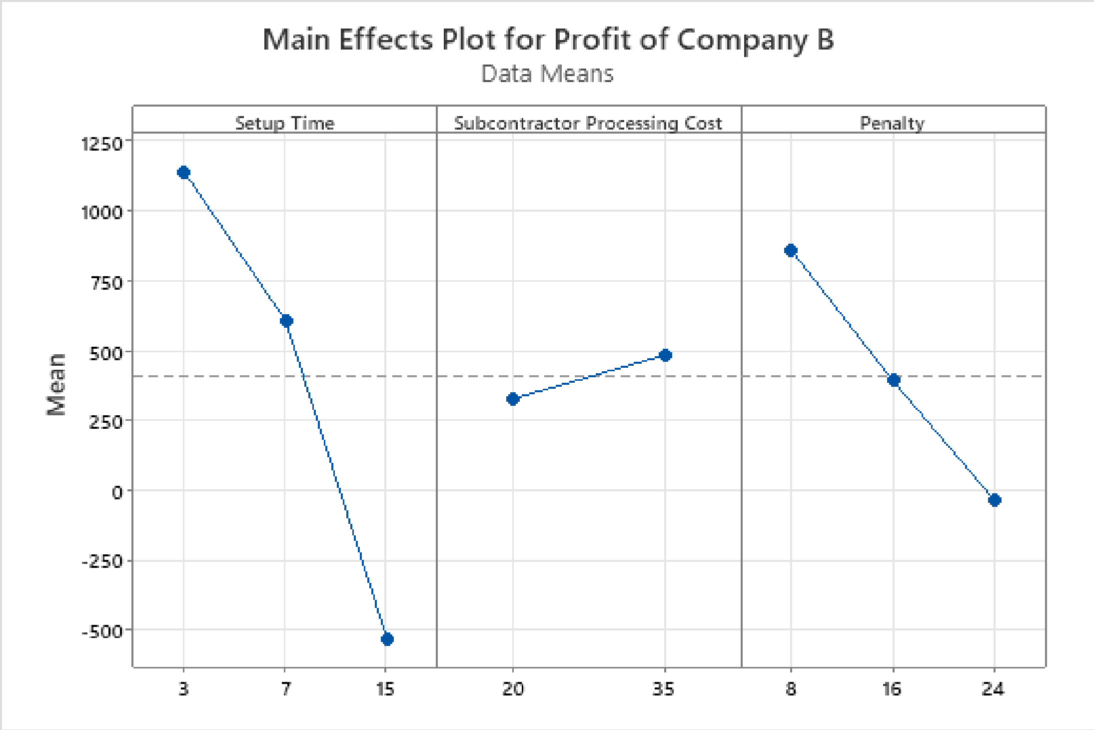


(a) Main Effects Plot for the Profit of Company A (Non-cooperative Scheduling with Outsourcing)

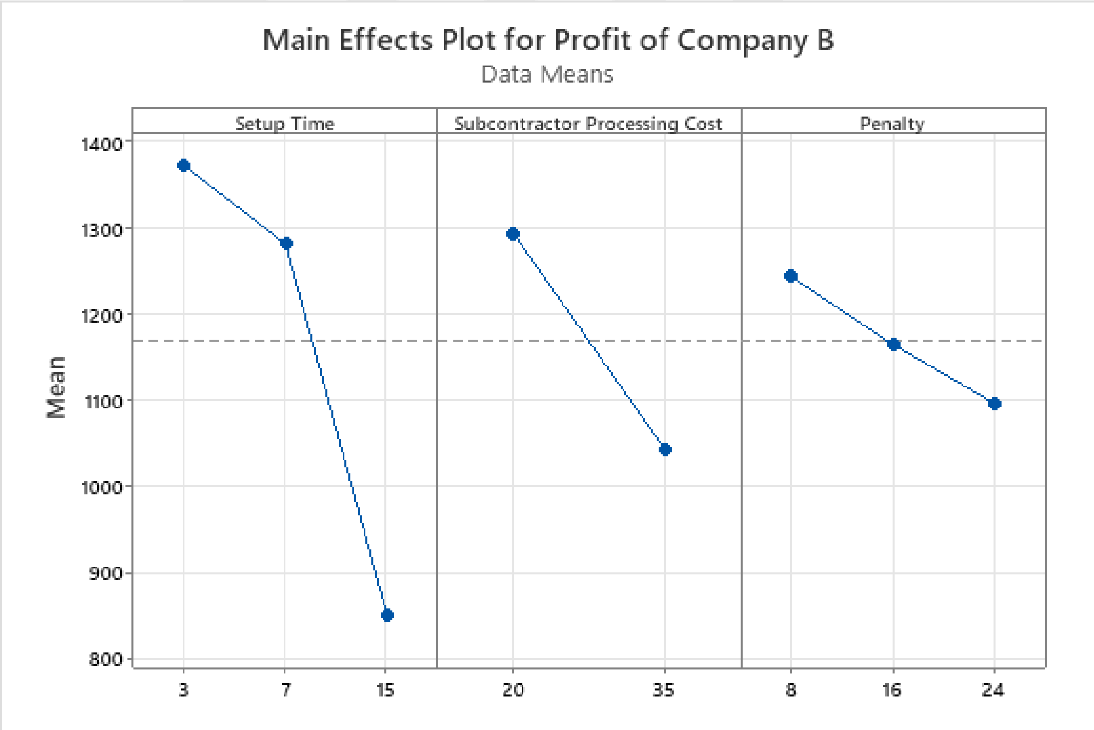


(b) Main Effects Plot for the Profit of Company A (Cooperative Scheduling with Outsourcing)

Figure 3. Main Effects Plots for the Profit of Company A

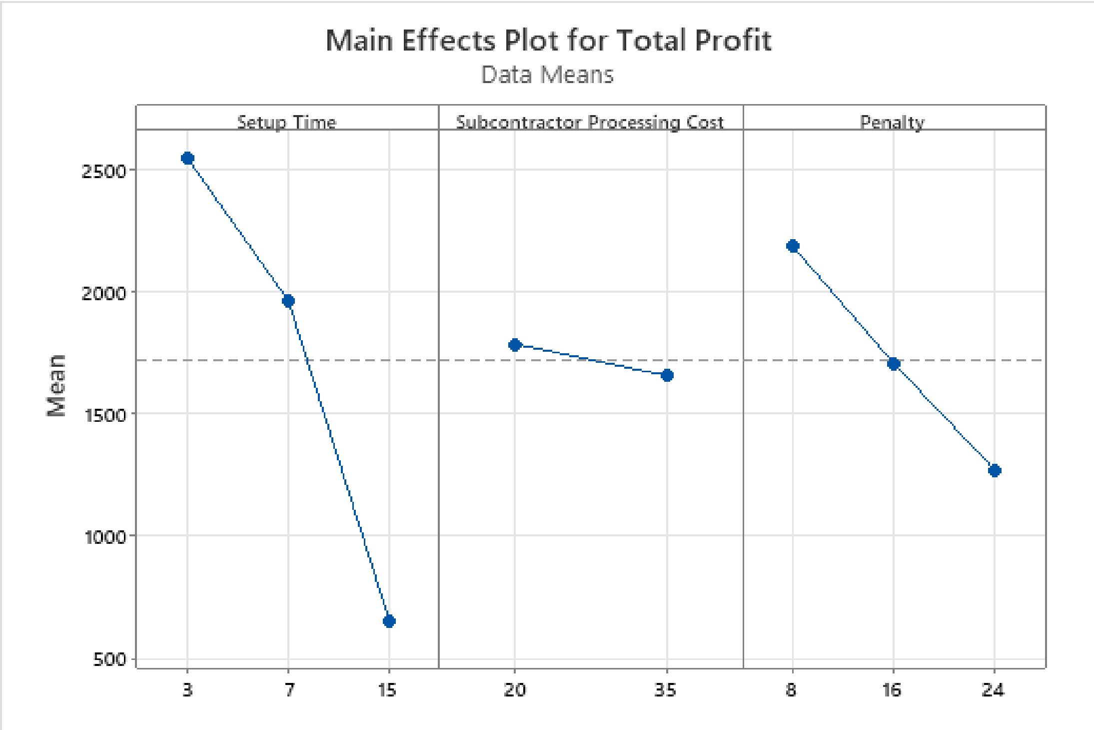


(a) Main Effects Plot for the Profit of Company B (Non-cooperative Scheduling with Outsourcing)

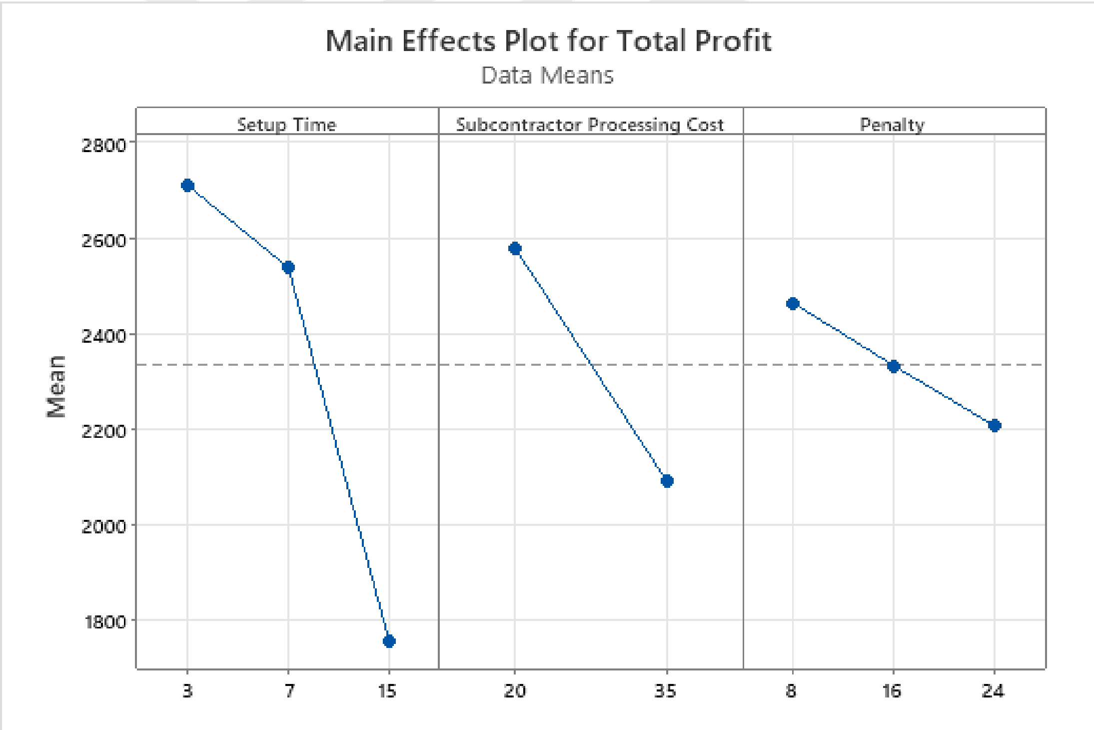


(b) Main Effects Plot for the Profit of Company B (Cooperative Scheduling with Outsourcing)

Figure 4. Main Effects Plots for the Profit of Company B

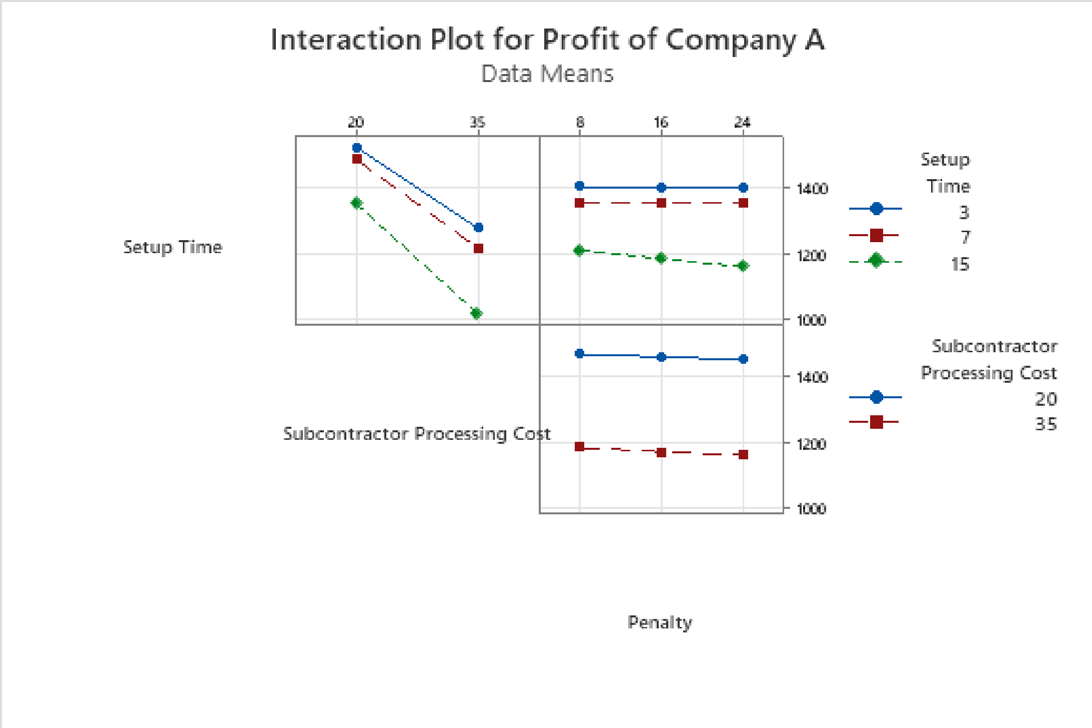


(a) Main Effects Plot for the Total Profit (Non-cooperative Scheduling with Outsourcing)

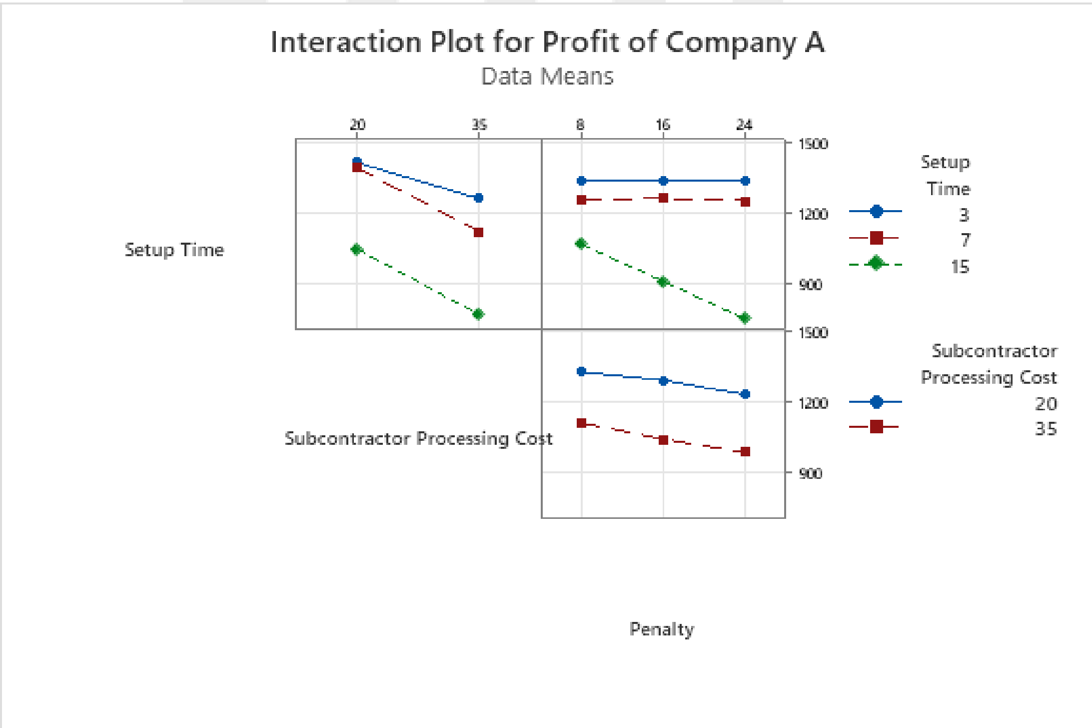


(b) Main Effects Plot for the Total Profit (Cooperative Scheduling with Outsourcing)

Figure 5. Main Effects Plots for the Total Profit

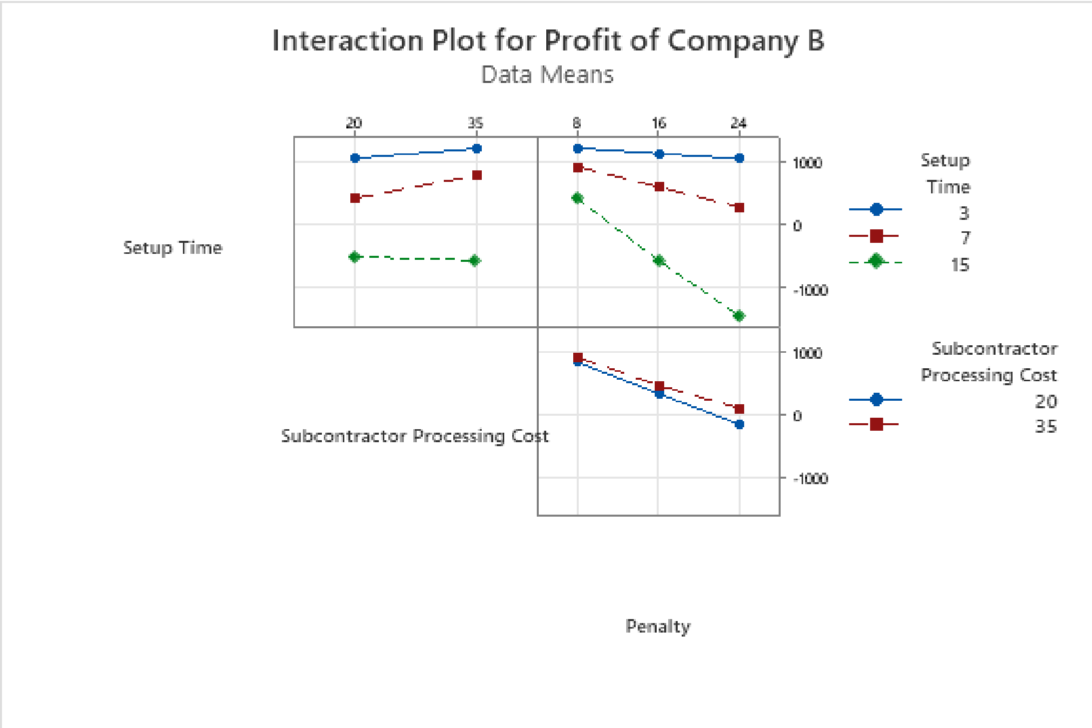


(a) Interaction Plot for the Profit of Company A (Non-cooperative Scheduling with Outsourcing)

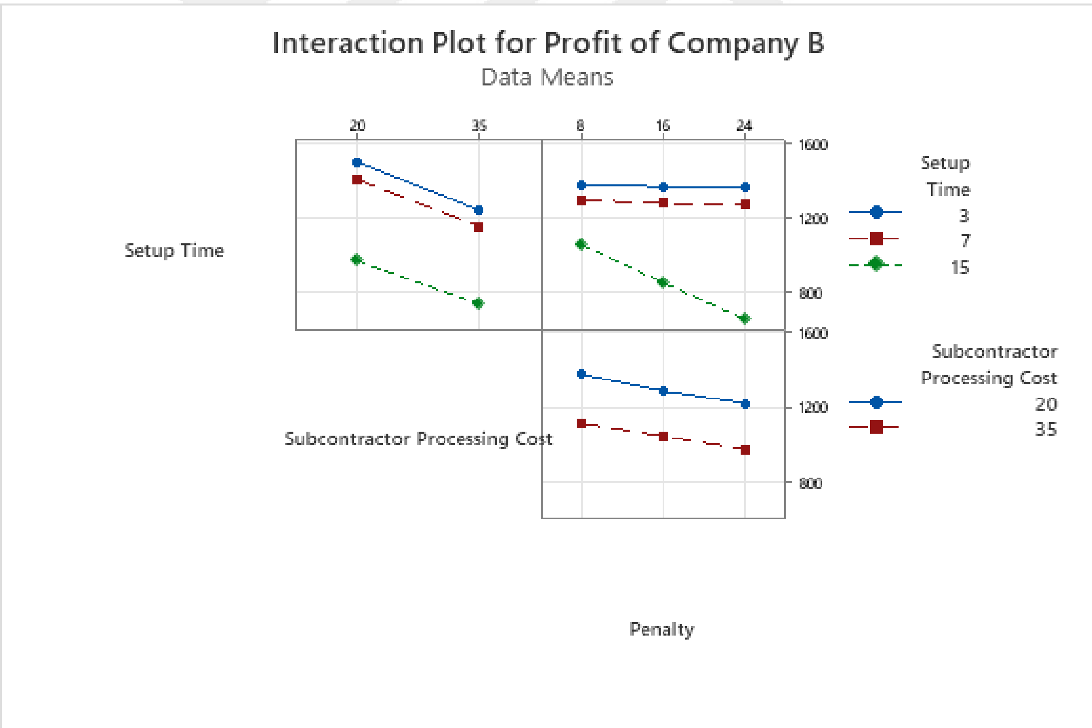


(b) Interaction Plot for the Profit of Company A (Cooperative Scheduling with Outsourcing)

Figure 6. Interaction Plots for the Profit of Company A

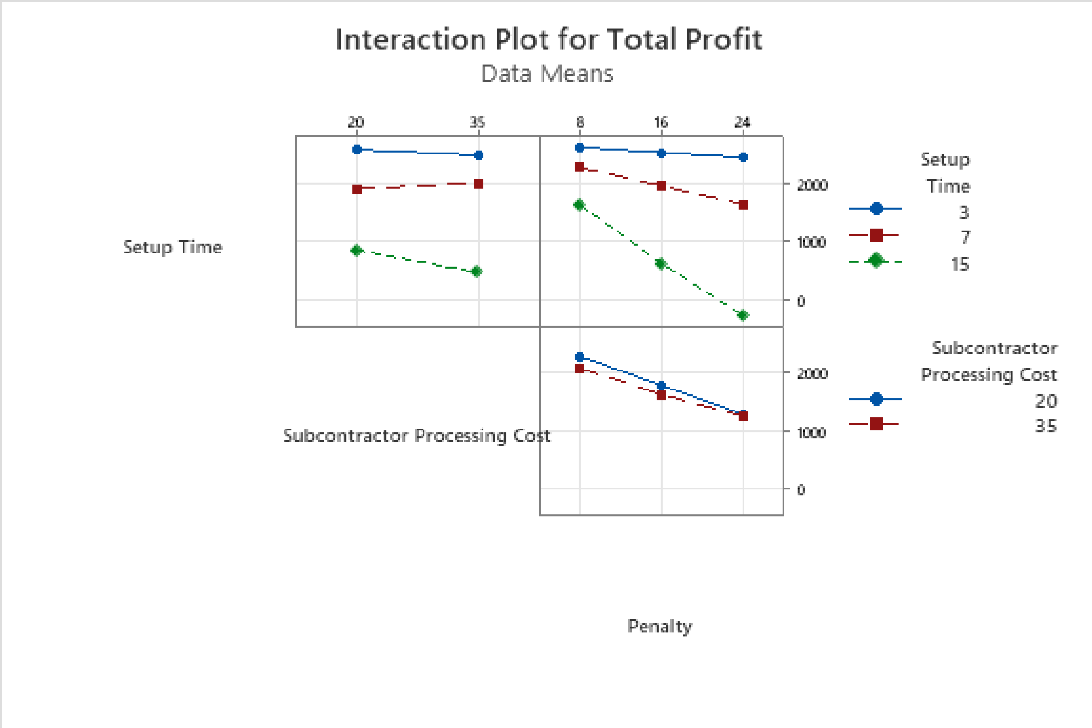


(a) Interaction Plot for the Profit of Company B (Non-cooperative Scheduling with Outsourcing)

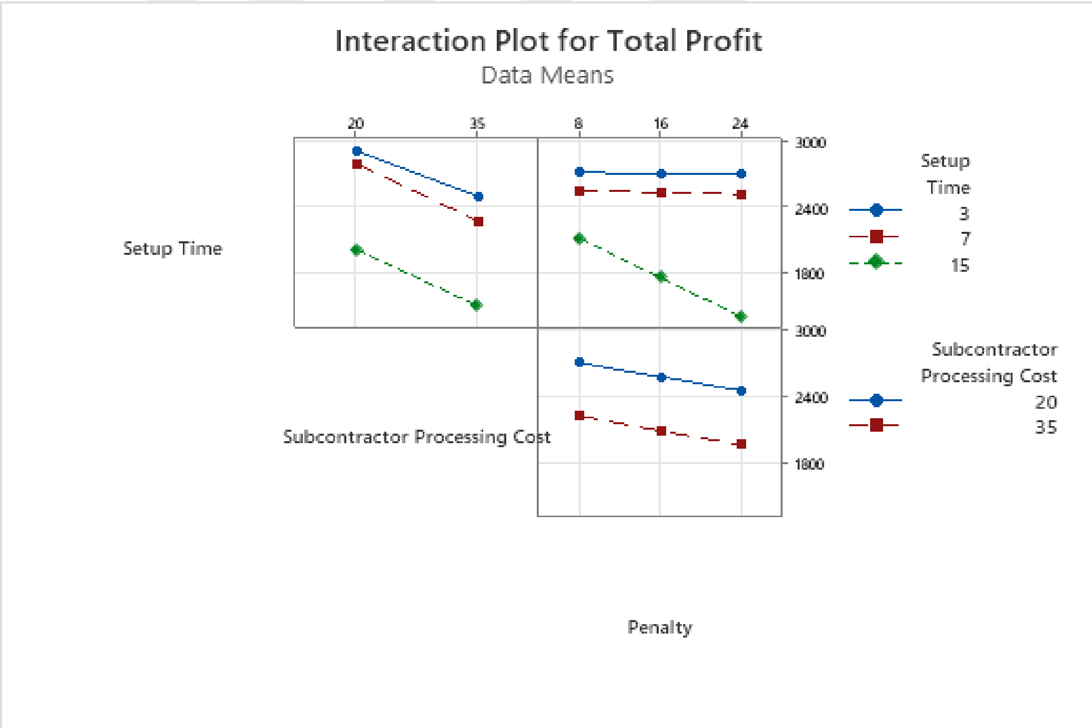


(b) Interaction Plot for the Profit of Company B (Cooperative Scheduling with Outsourcing)

Figure 7. Interaction Plots for the Profit of Company B

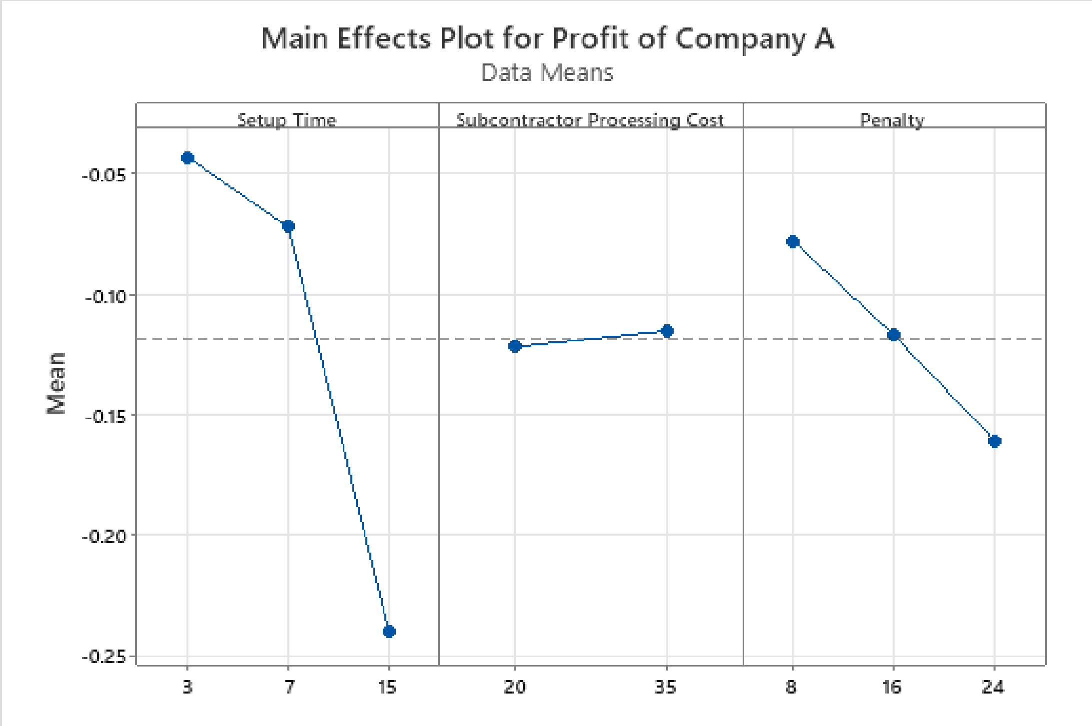


(a) Interaction Plot for the Total Profit (Non-cooperative Scheduling with Outsourcing)

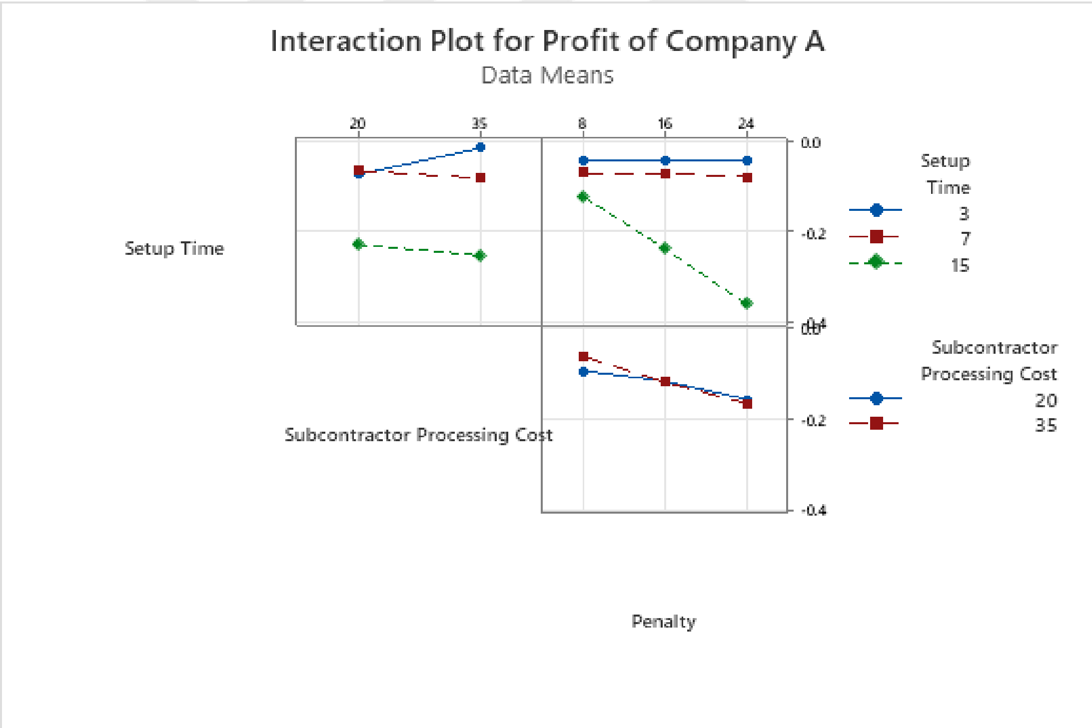


(b) Interaction Plot for the Total Profit (Cooperative Scheduling with Outsourcing)

Figure 8. Interaction Plots for the Total Profit

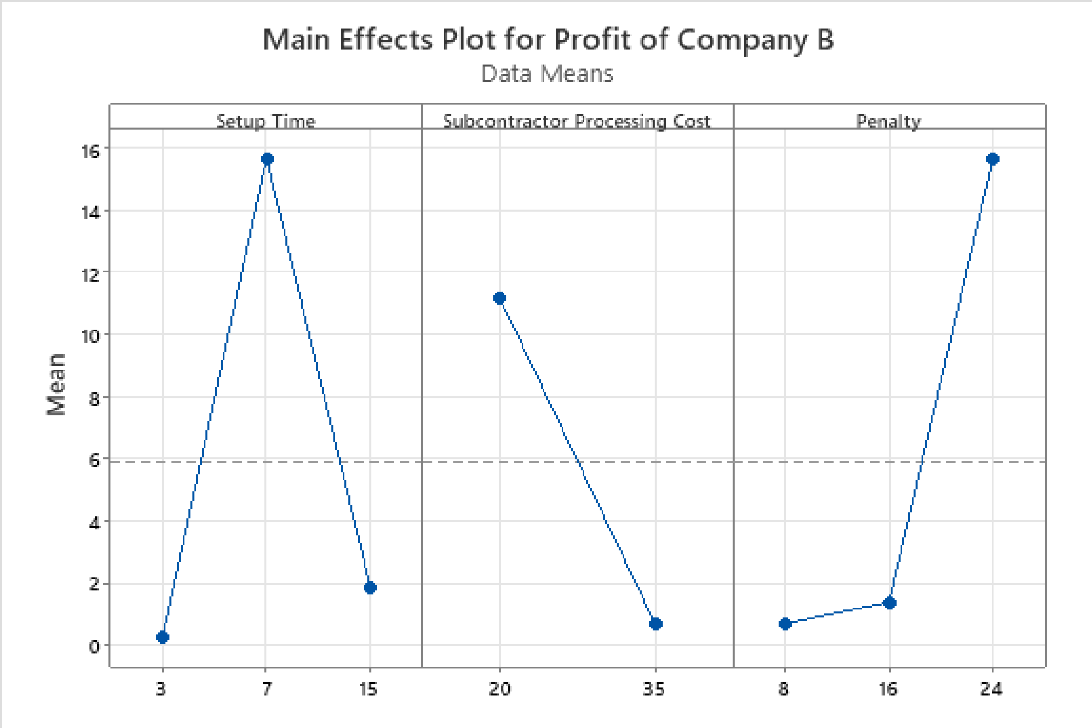


(a) Main Effects Plot for the Difference in Profit of Company A

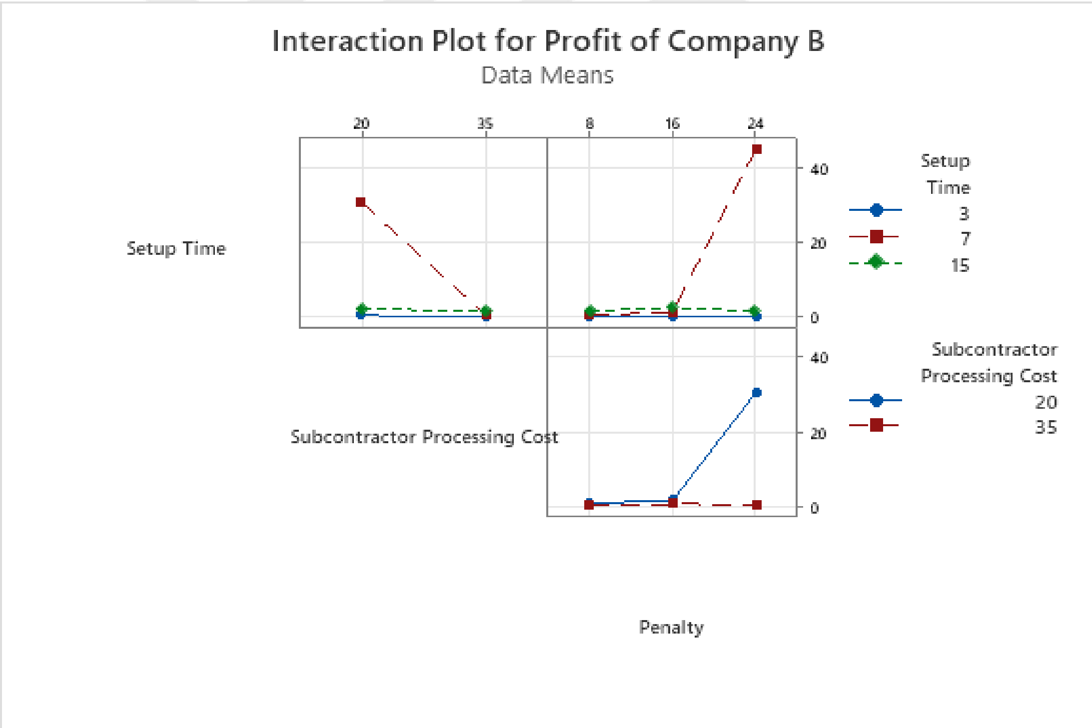


(b) Interaction Plot for the Difference in Profit of Company A

Figure 9. Difference Percentages for the Profit of Company A Main Effects and Interaction Plots

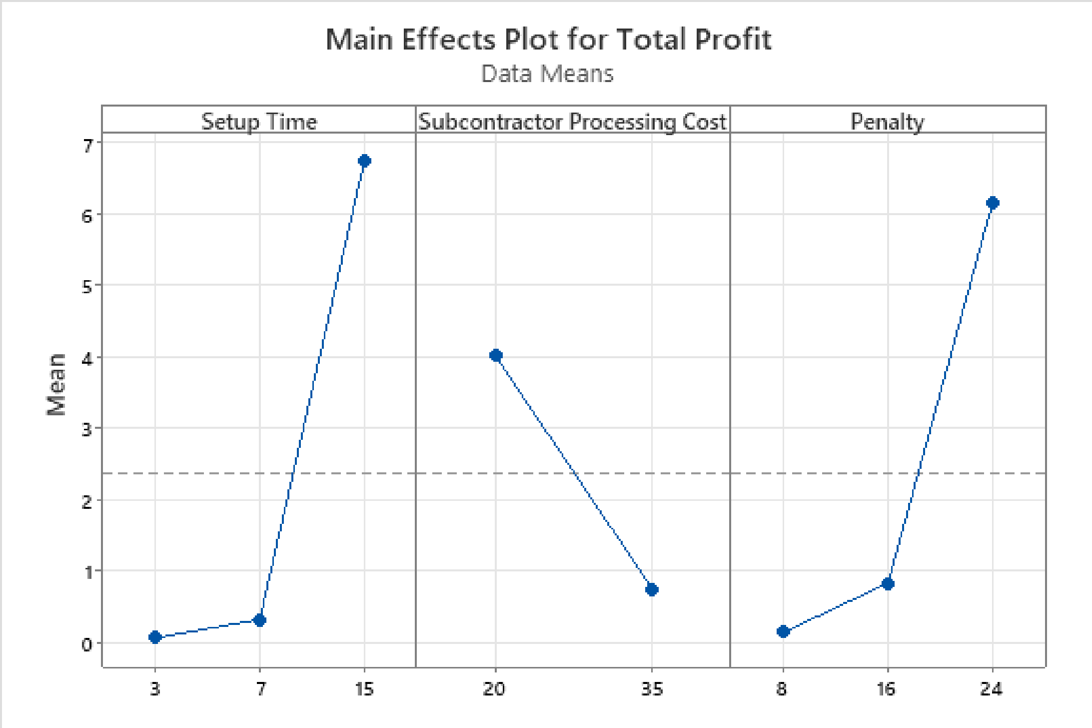


(a) Main Effects Plot for the Difference in Profit of Company B

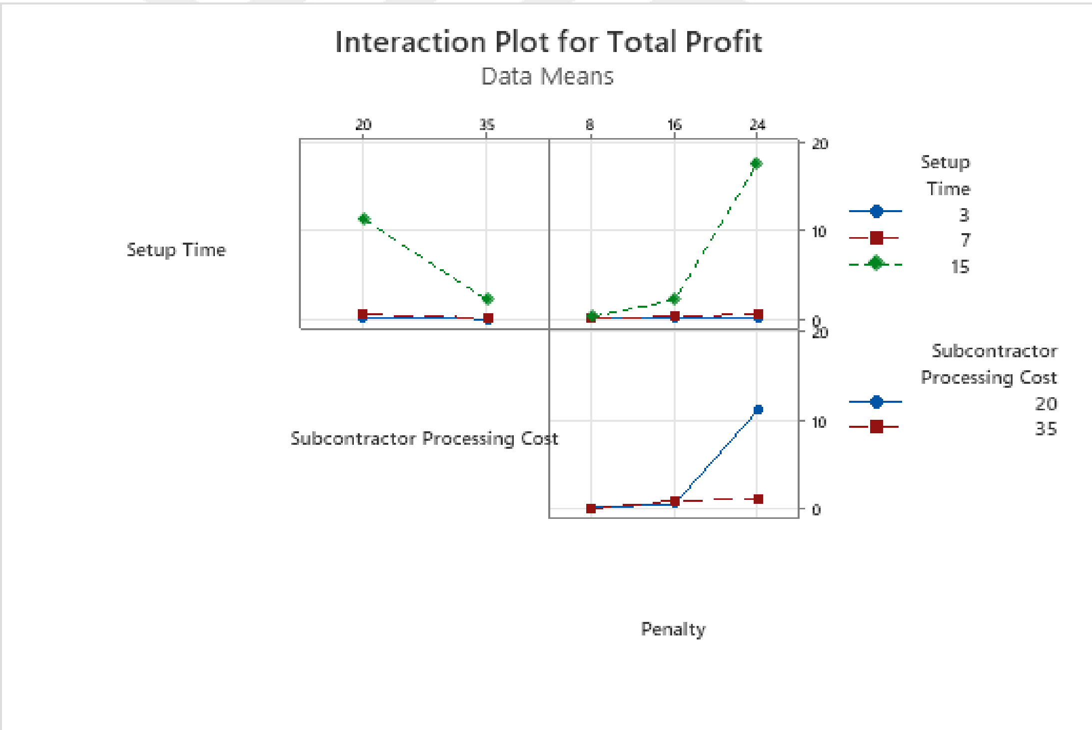


(b) Interaction Plot for the Difference in Profit of Company B

Figure 10. Difference Percentages for the Profit of Company B Main Effects and Interaction Plots



(a) Main Effects Plot for the Difference in Total Profit



(b) Interaction Plot for the Difference in Total Profit

Figure 11. Difference Percentages for Total Profit Main Effects and Interaction Plots

5.3 Evaluation of Decisions using Game Theoretical Approach

In this section, the evaluation of the decisions using Stackelberg Game and Nash Equilibrium is conducted.

As it is explained in Section 4.3, the non-cooperative outsourcing scheduling decisions are analyzed using Stackelberg Game, and cooperative outsourcing scheduling decisions are examined using Nash Equilibrium considering both scheduling decisions as non-cooperative games.

Firstly, the payoff matrices for both non-cooperative and cooperative scheduling with outsourcing are constructed. Since there are five jobs for each company, the number of subsets which are the jobs that companies would outsource the processing is found as $2^5 = 32$. For each job set, the optimal schedules are found and the profits are calculated. The profits are the payoffs for companies according to their decisions of the job sets processed by subcontractor. The non-cooperative and cooperative scheduling with outsourcing payoff matrices are given in Appendices.

For the Stackeberg Game, the non-cooperative outsourcing scheduling payoff matrix is used. The leader company, Company A, makes the decision first and selects the job pairs 1 and 4 to be processed at subcontractor location. According to the decision of Company A, Company B, the follower, makes the decision and selects jobs 8 and 9. According to these decisions, the schedule is the same as the obtained schedule by the non-cooperative solution of the mathematical model. The profit of Company A is 1217 and the profit of Company B is 991. The Stackelberg Game solution is shown in Figure 12.

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{ }	864	1022	864	1191	864	1142	864	1133	864	1050	864	1079	864	1218	864	1152
{1}	1125	989	1125	1135	1125	937	1125	1041	1125	877	1125	1049	1125	1126	1125	1120
{2}	1088	975	1088	992	1088	837	1088	922	1088	734	1088	1037	1088	985	1088	1043
{3}	1013	980	1013	1062	1013	875	1013	992	1013	804	1013	1042	1013	1054	1013	1079
{4}	1102	992	1102	1154	1102	973	1102	1062	1102	897	1102	1051	1102	1146	1102	1123
{5}	1127	985	1127	1099	1127	900	1127	1021	1127	841	1127	1046	1127	1090	1127	1100
{1,2}	1128	783	1128	722	1128	591	1128	704	1128	463	1128	824	1128	730	1128	788
{1,3}	1152	843	1152	788	1152	657	1152	738	1152	530	1152	891	1152	796	1152	855
{1,4}	1217	927	1217	889	1217	758	1217	817	1217	630	**1217**	**991**	1217	896	1217	955
{1,5}	1204	876	1204	822	1204	691	1204	756	1204	563	1204	924	1204	829	1204	888
{2,3}	1038	751	1038	657	1038	526	1038	671	1038	398	1038	775	1038	665	1038	755
{2,4}	1151	810	1151	755	1151	624	1151	721	1151	497	1151	857	1151	763	1151	822
{2,5}	1102	767	1102	689	1102	559	1102	688	1102	431	1102	792	1102	697	1102	771
{3,4}	1152	876	1152	822	1152	691	1152	756	1152	563	1152	924	1152	829	1152	888
{3,5}	1135	810	1135	755	1135	624	1135	721	1135	497	1135	857	1135	763	1135	822
{4,5}	1197	910	1197	855	1197	724	1197	784	1197	596	1197	958	1197	863	1197	921

Figure 12. The Stackelberg Game Solution

Using the cooperative scheduling with outsourcing payoff matrix, the Nash Equilibrium search is conducted. However, there is not a pure equilibrium found in the payoff matrix which means that companies do not agree on a cooperative schedule for outsourcing for the given example. The decisions are continuously change among the following job sets. If Company B selects the jobs 6 and 10, Company A selects 4 and 5. Company B responses with the jobs 7 and 9, then Company A selects the jobs 1 and 5 which changes the decision of Company B to 6 and 10. Thus, it can be concluded that there is not a pure equilibrium. The Nash Equilibrium search is shown in Figure 13.

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{ }	864	1022	864	1191	864	1142	864	1133	864	1050	864	1079	864	1218	864	1152
{1}	1125	1002	1087	1180	1125	937	1125	1041	1125	877	1125	1049	1125	1126	1125	1120
{2}	1088	975	980	1191	1007	1086	1051	1125	989	1050	1088	1037	1007	1218	1051	1139
{3}	1013	980	964	1191	966	1142	963	1133	961	1050	1013	1053	973	1218	984	1141
{4}	1102	1002	1085	1175	1081	1037	1081	1129	1081	925	1096	1069	1085	1167	1081	1152
{5}	1127	985	1127	1160	1037	1003	1037	1125	1127	928	1127	1046	1110	1192	1127	1127
{1,2}	1109	929	938	1180	1079	798	1079	927	1079	733	1077	966	943	1126	1079	1002
{1,3}	1126	1002	1063	1180	1067	937	1027	1041	995	877	1140	1036	1125	1126	1084	1120
{1,4}	1214	976	1003	1180	1217	758	1082	1008	1142	757	1217	991	1142	1008	1142	1116
{1,5}	1190	956	1075	1124	1204	691	1204	756	1120	782	1204	924	1120	1033	1120	1107
{2,3}	909	975	769	1191	920	1012	880	1125	786	1050	1022	974	842	1218	880	1139
{2,4}	1036	1002	929	1175	1107	834	966	1129	1091	772	1057	1069	953	1167	1024	1152
{2,5}	913	985	938	1160	1058	751	1040	891	929	945	1058	928	963	1175	1068	1065
{3,4}	1120	1002	1036	1175	1056	1037	1073	1129	984	925	1126	1063	1103	1167	1100	1152
{3,5}	1046	985	1070	1160	948	1003	908	1125	1022	928	1124	1017	1114	1175	1094	1144
{4,5}	1193	920	1148	1081	1197	724	922	1129	1165	805	1124	1042	1165	1056	1165	1124

Figure 13. The Nash Equilibirum Search Solution

CHAPTER 6: CONCLUSION

In the case of increasing product varieties, the producers may need additional capacity for supplying the demand with higher profitability. In addition, for the perishable products or the goods that the processing yield affects the final volume, outsourcing would bring the opportunity of earlier processing since higher capacity may be available in former phases of production. Although the outsourcing option provides an external capacity with acceptable processing costs, the job scheduling may require more effort since the number of resources with different processing costs are increased. Moreover, with high sequence dependent setup times, the complexity of the scheduling increases. In this study, cooperative scheduling with outsourcing option is proposed in order to increase profitability. The cooperation is designed to be performed by companies that would outsource a part of their processes. The purpose of the cooperation is minimizing the setup requirements by scheduling identical jobs consecutively on the shared capacity. With the cooperative scheduling, the profit is aimed to be maximized by increased processing yield and decreased tardiness. An illustrative example from the leaf tobacco sector is constructed for demonstrating the impact of cooperative scheduling with outsourcing. The example case includes two companies with similar jobs and subcontractor as a shared capacity that can be considered as a parallel machine. The setup times are relatively high according to the processing times and each job has a due date. The non-cooperative and cooperative scheduling with outsourcing approaches differs in the solution procedure. In the non-cooperative scheduling with outsourcing, the leader company pays a fixed cost for booking the subcontractor capacity at time zero and for the follower company, the availability of the subcontractor capacity starts after the leader company's processing is finished. However, for the cooperative scheduling with outsourcing, companies share the fixed cost and schedule their jobs commonly at the subcontractor capacity.

In order to solve the problem with and without cooperative scheduling, a mathematical model is constructed and solved optimally for both scheduling approaches. Also, in order to analyze the impact of cooperation on scheduling with outsourcing, the model is solved with different parameters and the profits are compared for non-cooperative and cooperative schedules. Results show that the overall profitability increases with cooperative scheduling.

Considering the companies' individual profits, the preferences may differ from the optimal solutions. Thus, in addition to the solution of the mathematical model for both scheduling approaches, companies' decisions are modeled as non-cooperative games and the payoff matrices for non-cooperative and cooperative scheduling with outsourcing are constructed. For the non-cooperative scheduling with outsourcing decisions, the Stackelberg Game is used for finding the leader and follower game solution. On the other hand, the Nash Equilibrium search is conducted using the cooperative scheduling with outsourcing decisions. Stackelberg Game results in the same schedule obtained by solving the mathematical model for non-cooperative scheduling with outsourcing where for the cooperative scheduling, a pure equilibrium can not be found with the Nash Game according to the given illustrative example.

As a future work of this study, a dispatching rule can be proposed for finding the job sets that companies would prefer to outsource with cooperation. In addition, the analysis of the fixed cost for subcontractor can be performed for obtaining the interval of the cooperative scheduling feasibility. Lastly, cooperative scheduling decisions can be analyzed under the job eligibility of the subcontractor.

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APPENDICES

Appendix A-Non-cooperative Outsourcing Schedule Payoff Matrix

A \ B	{ }		{6}		{7}		{8}		{9}		{10}		{6,7}		{6,8}	
{ }	864	728	864	953	864	1107	864	1007	864	887	864	1080	864	1108	864	1137
{1}	1125	728	1125	928	1125	1024	1125	987	1125	873	1125	1057	1125	940	1125	1098
{2}	1088	728	1088	918	1088	947	1088	980	1088	867	1088	983	1088	797	1088	1021
{3}	1012	728	1013	922	1013	983	1013	982	1013	868	1013	1019	1013	867	1013	1059
{4}	1102	728	1102	931	1102	1044	1102	990	1102	875	1102	1059	1102	978	1102	1102
{5}	1127	728	1127	926	1127	1004	1127	985	1127	870	1127	1038	1127	904	1127	1079
{1,2}	1128	728	1128	841	1128	801	1128	865	1128	857	1128	843	1128	571	1128	776
{1,3}	1152	728	1152	876	1152	837	1152	899	1152	860	1152	878	1152	638	1152	842
{1,4}	1217	728	1217	911	1217	890	1217	950	1217	862	1217	929	1217	722	1217	942
{1,5}	1204	728	1204	893	1204	855	1204	916	1204	861	1204	895	1204	671	1204	875
{2,3}	1038	728	1038	808	1038	768	1038	832	1038	857	1038	810	1038	506	1038	711
{2,4}	1151	728	1151	859	1151	819	1151	882	1151	859	1151	860	1151	605	1151	809
{2,5}	1102	728	1102	824	1102	785	1102	849	1102	857	1102	826	1102	538	1102	743
{3,4}	1152	728	1152	893	1152	855	1152	916	1152	861	1152	895	1152	671	1152	875
{3,5}	1135	728	1135	859	1135	819	1135	882	1135	859	1135	860	1135	605	1135	809
{4,5}	1197	728	1197	910	1197	872	1197	934	1197	862	1197	912	1197	704	1197	909

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{ }	864	1022	864	1191	864	1142	864	1133	864	1050	864	1079	864	1218	864	1152
{1}	1125	989	1125	1135	1125	937	1125	1041	1125	877	1125	1049	1125	1126	1125	1120
{2}	1088	975	1088	992	1088	837	1088	922	1088	734	1088	1037	1088	985	1088	1043
{3}	1013	980	1013	1062	1013	875	1013	992	1013	804	1013	1042	1013	1054	1013	1079
{4}	1102	992	1102	1154	1102	973	1102	1062	1102	897	1102	1051	1102	1146	1102	1123
{5}	1127	985	1127	1099	1127	900	1127	1021	1127	841	1127	1046	1127	1090	1127	1100
{1,2}	1128	783	1128	722	1128	591	1128	704	1128	463	1128	824	1128	730	1128	788
{1,3}	1152	843	1152	788	1152	657	1152	738	1152	530	1152	891	1152	796	1152	855
{1,4}	1217	927	1217	889	1217	758	1217	817	1217	630	1217	991	1217	896	1217	955
{1,5}	1204	876	1204	822	1204	691	1204	756	1204	563	1204	924	1204	829	1204	888
{2,3}	1038	751	1038	657	1038	526	1038	671	1038	398	1038	775	1038	665	1038	755
{2,4}	1151	810	1151	755	1151	624	1151	721	1151	497	1151	857	1151	763	1151	822
{2,5}	1102	767	1102	689	1102	559	1102	688	1102	431	1102	792	1102	697	1102	771
{3,4}	1152	876	1152	822	1152	691	1152	756	1152	563	1152	924	1152	829	1152	888
{3,5}	1135	810	1135	755	1135	624	1135	721	1135	497	1135	857	1135	763	1135	822
{4,5}	1197	910	1197	855	1197	724	1197	784	1197	596	1197	958	1197	863	1197	921

A \ B	{6,7,8}		{6,7,9}		{6,7,10}		{6,8,9}		{6,8,10}		{6,9,10}		{7,8,9}		{7,8,10}	
{ }	864	932	864	1034	864	814	864	1079	864	1091	864	1131	864	1074	864	822
{1}	1125	622	1125	752	1125	559	1125	979	1125	864	1125	999	1125	763	1125	589
{2}	1088	481	1088	591	1088	353	1088	845	1088	659	1088	816	1088	654	1088	384
{3}	1013	550	1013	671	1013	455	1013	907	1013	760	1013	895	1013	691	1013	485
{4}	1102	649	1102	790	1102	594	1102	999	1102	900	1102	1034	1102	816	1102	625
{5}	1127	586	1127	716	1127	507	1127	943	1127	812	1127	947	1127	718	1127	538
{1,2}	1128	107	1128	270	1128	-46	1128	504	1128	276	1128	434	1128	313	1128	1
{1,3}	1152	205	1152	362	1152	53	1152	602	1152	374	1152	533	1152	412	1152	99
{1,4}	1217	353	1217	494	1217	201	1217	751	1217	522	1217	681	1217	560	1217	248
{1,5}	1204	254	1204	412	1204	102	1204	652	1204	423	1204	582	1204	461	1204	149
{2,3}	1038	10	1038	206	1038	-143	1038	423	1038	179	1038	369	1038	232	1038	-96
{2,4}	1151	156	1151	313	1151	3	1151	553	1151	325	1151	484	1151	363	1151	50
{2,5}	1102	59	1102	238	1102	-95	1102	456	1102	227	1102	401	1102	265	1102	-47
{3,4}	1152	254	1152	412	1152	102	1152	652	1152	423	1152	582	1152	461	1152	149
{3,5}	1135	156	1135	313	1135	3	1135	553	1135	325	1135	484	1135	363	1135	50
{4,5}	1197	304	1197	461	1197	151	1197	701	1197	473	1197	631	1197	511	1197	198

A \ B	{7,9,10}		{8,9,10}		{6,7,8,9}		{6,7,8,10}		{6,7,9,10}		{6,8,9,10}		{7,8,9,10}		{6,7,8,9,10}	
{ }	864	978	864	1161	864	713	864	440	864	588	864	926	864	607	864	94
{1}	1125	689	1125	1015	1125	347	1125	85	1125	260	1125	611	1125	326	1125	-349
{2}	1088	548	1088	810	1088	149	1088	-184	1088	14	1088	342	1088	57	1088	-682
{3}	1013	617	1013	911	1013	244	1013	-51	1013	124	1013	475	1013	190	1013	-517
{4}	1102	720	1102	1051	1102	383	1102	137	1102	312	1102	663	1102	379	1102	-281
{5}	1127	653	1127	964	1127	295	1127	17	1127	192	1127	543	1127	259	1127	-432
{1,2}	1128	165	1128	459	1128	-320	1128	-695	1128	-497	1128	-137	1128	-422	1128	-1289
{1,3}	1152	264	1152	557	1152	-190	1152	-565	1152	-366	1152	-7	1152	-291	1152	-1127
{1,4}	1217	412	1217	706	1217	7	1217	-369	1217	-170	1217	189	1217	-95	1217	-882
{1,5}	1204	313	1204	607	1204	-124	1204	-500	1204	-301	1204	58	1204	-226	1204	-1045
{2,3}	1038	100	1038	378	1038	-433	1038	-824	1038	-594	1038	-250	1038	-535	1038	-1434
{2,4}	1151	215	1151	508	1151	-255	1151	-630	1151	-431	1151	-72	1151	-357	1151	-1208
{2,5}	1102	133	1102	411	1102	-384	1102	-760	1102	-546	1102	-202	1102	-486	1102	-1369
{3,4}	1152	313	1152	607	1152	-124	1152	-500	1152	-301	1152	58	1152	-226	1152	-1045
{3,5}	1135	215	1135	508	1135	-255	1135	-630	1135	-431	1135	-72	1135	-357	1135	-1208
{4,5}	1197	363	1197	656	1197	-59	1197	-434	1197	-236	1197	124	1197	-160	1197	-964

A \ B	{ }		{6}		{7}		{8}		{9}		{10}		{6,7}		{6,8}	
{1,2,3}	996	728	996	678	996	638	996	702	996	727	996	680	996	248	996	453
{1,2,4}	1071	728	1071	726	1071	686	1071	750	1071	775	1071	728	1071	344	1071	549
{1,2,5}	998	728	998	694	998	654	998	718	998	743	998	696	998	280	998	485
{1,3,4}	1114	728	1114	758	1114	718	1114	782	1114	807	1114	760	1114	408	1114	613
{1,3,5}	1090	728	1090	726	1090	686	1090	750	1090	775	1090	728	1090	344	1090	549
{1,4,5}	1091	728	1091	774	1091	735	1091	799	1091	824	1091	777	1091	441	1091	646
{2,3,4}	1026	728	1026	694	1026	654	1026	718	1026	743	1026	696	1026	280	1026	485
{2,3,5}	966	728	966	662	966	622	966	686	966	711	966	664	966	216	966	421
{2,4,5}	1036	728	1036	710	1036	670	1036	734	1036	759	1036	712	1036	312	1036	517
{3,4,5}	1091	728	1091	742	1091	702	1091	766	1091	791	1091	744	1091	376	1091	581
{1,2,3,4}	794	728	794	566	794	526	794	590	794	615	794	568	794	24	794	229
{1,2,3,5}	656	728	656	534	656	494	656	558	656	583	656	536	656	-40	656	165
{1,2,4,5}	721	728	721	582	721	542	721	606	721	631	721	584	721	56	721	261
{1,3,4,5}	889	728	889	614	889	574	889	638	889	663	889	616	889	120	889	325
{2,3,4,5}	733	728	733	550	733	510	733	574	733	599	733	552	733	-8	733	197
{1,2,3,4,5}	45	728	236	422	236	382	236	446	236	471	236	424	236	-264	236	-59

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{1,2,3}	996	493	996	399	996	268	996	414	996	140	996	517	996	407	996	497
{1,2,4}	1071	589	1071	495	1071	364	1071	510	1071	236	1071	613	1071	503	1071	593
{1,2,5}	998	525	998	431	998	300	998	446	998	172	998	549	998	439	998	529
{1,3,4}	1114	653	1114	559	1114	428	1114	574	1114	300	1114	677	1114	567	1114	657
{1,3,5}	1090	589	1090	495	1090	364	1090	510	1090	236	1090	613	1090	503	1090	593
{1,4,5}	1091	686	1091	591	1091	461	1091	606	1091	333	1091	710	1091	600	1091	690
{2,3,4}	1026	525	1026	431	1026	300	1026	446	1026	172	1026	549	1026	439	1026	529
{2,3,5}	966	461	966	367	966	236	966	382	966	108	966	485	966	375	966	465
{2,4,5}	1036	557	1036	463	1036	332	1036	478	1036	204	1036	581	1036	471	1036	561
{3,4,5}	1091	621	1091	527	1091	396	1091	542	1091	268	1091	645	1091	535	1091	625
{1,2,3,4}	794	269	794	175	794	44	794	190	794	-84	794	293	794	183	794	273
{1,2,3,5}	656	205	656	111	656	-20	656	126	656	-148	656	229	656	119	656	209
{1,2,4,5}	721	301	721	207	721	76	721	222	721	-52	721	325	721	215	721	305
{1,3,4,5}	889	365	889	271	889	140	889	286	889	12	889	389	889	279	889	369
{2,3,4,5}	733	237	733	143	733	12	733	158	733	-116	733	261	733	151	733	241
{1,2,3,4,5}	236	-19	236	-113	236	-244	236	-99	236	-372	236	5	236	-105	236	-15

A \ B	{6,7,8}		{6,7,9}		{6,7,10}		{6,8,9}		{6,8,10}		{6,9,10}		{7,8,9}		{7,8,10}	
{1,2,3}	996	-376	996	-180	996	-529	996	37	996	-207	996	-17	996	-153	996	-482
{1,2,4}	1071	-232	1071	-36	1071	-385	1071	181	1071	-63	1071	127	1071	-9	1071	-338
{1,2,5}	998	-328	998	-132	998	-481	998	85	998	-159	998	31	998	-105	998	-434
{1,3,4}	1114	-136	1114	60	1114	-289	1114	277	1114	33	1114	223	1114	87	1114	-241
{1,3,5}	1090	-232	1090	-36	1090	-385	1090	181	1090	-63	1090	127	1090	-9	1090	-338
{1,4,5}	1091	-87	1091	109	1091	-241	1091	326	1091	81	1091	272	1091	135	1091	-193
{2,3,4}	1026	-328	1026	-132	1026	-481	1026	85	1026	-159	1026	31	1026	-105	1026	-434
{2,3,5}	966	-424	966	-228	966	-577	966	-11	966	-255	966	-65	966	-201	966	-530
{2,4,5}	1036	-280	1036	-84	1036	-433	1036	133	1036	-111	1036	79	1036	-57	1036	-386
{3,4,5}	1091	-184	1091	12	1091	-337	1091	229	1091	-15	1091	175	1091	39	1091	-290
{1,2,3,4}	794	-712	794	-516	794	-865	794	-299	794	-543	794	-353	794	-489	794	-818
{1,2,3,5}	656	-808	656	-612	656	-961	656	-395	656	-639	656	-449	656	-585	656	-914
{1,2,4,5}	721	-664	721	-468	721	-817	721	-251	721	-495	721	-305	721	-441	721	-770
{1,3,4,5}	889	-568	889	-372	889	-721	889	-155	889	-399	889	-209	889	-345	889	-674
{2,3,4,5}	733	-760	733	-564	733	-913	733	-347	733	-591	733	-401	733	-537	733	-866
{1,2,3,4,5}	236	-1144	236	-948	236	-1297	236	-731	236	-975	236	-785	236	-921	236	-1250

A \ B	{7,9,10}		{8,9,10}		{6,7,8,9}		{6,7,8,10}		{6,7,9,10}		{6,8,9,10}		{7,8,9,10}		{6,7,8,9,10}	
{1,2,3}	996	-286	996	-8	996	-947	996	-1338	996	-1108	996	-764	996	-1049	996	-2076
{1,2,4}	1071	-142	1071	136	1071	-755	1071	-1146	1071	-916	1071	-572	1071	-857	1071	-1836
{1,2,5}	998	-238	998	40	998	-883	998	-1274	998	-1044	998	-700	998	-985	998	-1996
{1,3,4}	1114	-46	1114	232	1114	-627	1114	-1018	1114	-788	1114	-444	1114	-729	1114	-1676
{1,3,5}	1090	-142	1090	136	1090	-755	1090	-1146	1090	-916	1090	-572	1090	-857	1090	-1836
{1,4,5}	1091	3	1091	281	1091	-562	1091	-953	1091	-723	1091	-379	1091	-664	1091	-1595
{2,3,4}	1026	-238	1026	40	1026	-883	1026	-1274	1026	-1044	1026	-700	1026	-985	1026	-1996
{2,3,5}	966	-334	966	-56	966	-1011	966	-1402	966	-1172	966	-828	966	-1113	966	-2156
{2,4,5}	1036	-190	1036	88	1036	-819	1036	-1210	1036	-980	1036	-636	1036	-921	1036	-1916
{3,4,5}	1091	-94	1091	184	1091	-691	1091	-1082	1091	-852	1091	-508	1091	-793	1091	-1756
{1,2,3,4}	794	-622	794	-344	794	-1395	794	-1786	794	-1556	794	-1212	794	-1497	794	-2636
{1,2,3,5}	656	-718	656	-440	656	-1523	656	-1914	656	-1684	656	-1340	656	-1625	656	-2796
{1,2,4,5}	721	-574	721	-296	721	-1331	721	-1722	721	-1492	721	-1148	721	-1433	721	-2556
{1,3,4,5}	889	-478	889	-200	889	-1203	889	-1594	889	-1364	889	-1020	889	-1305	889	-2396
{2,3,4,5}	733	-670	733	-392	733	-1459	733	-1850	733	-1620	733	-1276	733	-1561	733	-2716
{1,2,3,4,5}	236	-1054	236	-776	236	-1971	236	-2362	236	-2132	236	-1788	236	-2073	236	-3356

Appendix B-Cooperative Outsourcing Schedule Payoff Matrix

A \ B	{ }		{6}		{7}		{8}		{9}		{10}		{6,7}		{6,8}	
{ }	864	728	864	953	864	1107	864	1007	864	887	864	1080	864	1108	864	1137
{1}	1125	728	1125	938	1125	1024	1125	987	1125	873	1125	1057	1025	1084	1110	1122
{2}	1088	728	1088	918	1051	1107	1088	980	1088	867	1051	1080	1051	1015	998	1137
{3}	1012	728	1013	922	975	1107	1013	991	1013	868	984	1080	953	1108	1013	1105
{4}	1102	728	1084	953	1081	1107	1088	1007	1102	881	1085	1080	1081	1041	1088	1119
{5}	1127	728	1127	926	1037	1107	1127	985	1127	870	1127	1063	1037	1007	1127	1079
{1,2}	1128	728	1094	953	1079	1024	1097	987	1128	857	1079	1057	976	931	993	1107
{1,3}	1152	728	1122	953	1126	1024	1140	987	1152	860	1131	1057	943	1084	1114	1122
{1,4}	1217	728	1214	919	1217	890	1217	950	1217	868	1142	1057	1214	788	1214	968
{1,5}	1204	728	1190	915	1204	855	1204	916	1204	861	1120	1057	1190	751	1190	932
{2,3}	1038	728	1023	918	986	1107	1022	991	1038	857	986	1080	840	1015	874	1105
{2,4}	1151	728	1081	953	1107	1044	1102	1007	1151	862	1091	1080	1107	838	947	1119
{2,5}	1102	728	1031	926	1040	1004	1058	985	1102	857	1068	1080	1040	765	897	1079
{3,4}	1152	728	1107	953	1100	1107	1135	990	1136	881	1109	1080	976	1041	1101	1079
{3,5}	1135	728	1115	926	1022	1107	1124	985	1135	859	1121	1063	868	1007	1030	1079
{4,5}	1197	728	1197	910	981	1107	1197	934	1193	870	1165	1063	1197	704	1197	909

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{ }	864	1022	864	1191	864	1142	864	1133	864	1050	864	1079	864	1218	864	1152
{1}	1125	1002	1087	1180	1125	937	1125	1041	1125	877	1125	1049	1125	1126	1125	1120
{2}	1088	975	980	1191	1007	1086	1051	1125	989	1050	1088	1037	1007	1218	1051	1139
{3}	1013	980	964	1191	966	1142	963	1133	961	1050	1013	1053	973	1218	984	1141
{4}	1102	1002	1085	1175	1081	1037	1081	1129	1081	925	1096	1069	1085	1167	1081	1152
{5}	1127	985	1127	1160	1037	1003	1037	1125	1127	928	1127	1046	1110	1192	1127	1127
{1,2}	1109	929	938	1180	1079	798	1079	927	1079	733	1077	966	943	1126	1079	1002
{1,3}	1126	1002	1063	1180	1067	937	1027	1041	995	877	1140	1036	1125	1126	1084	1120
{1,4}	1214	976	1003	1180	1217	758	1082	1008	1142	757	1217	991	1142	1008	1142	1116
{1,5}	1190	956	1075	1124	1204	691	1204	756	1120	782	1204	924	1120	1033	1120	1107
{2,3}	909	975	769	1191	920	1012	880	1125	786	1050	1022	974	842	1218	880	1139
{2,4}	1036	1002	929	1175	1107	834	966	1129	1091	772	1057	1069	953	1167	1024	1152
{2,5}	913	985	938	1160	1058	751	1040	891	929	945	1058	928	963	1175	1068	1065
{3,4}	1120	1002	1036	1175	1056	1037	1073	1129	984	925	1126	1063	1103	1167	1100	1152
{3,5}	1046	985	1070	1160	948	1003	908	1125	1022	928	1124	1017	1114	1175	1094	1144
{4,5}	1193	920	1148	1081	1197	724	922	1129	1165	805	1124	1042	1165	1056	1165	1124

A \ B	{6,7,8}		{6,7,9}		{6,7,10}		{6,8,9}		{6,8,10}		{6,9,10}		{7,8,9}		{7,8,10}	
{ }	864	932	864	1034	864	814	864	1079	864	1091	864	1131	864	1074	864	822
{1}	1110	703	1025	937	1087	662	1110	1060	1087	944	1087	1102	1125	763	1125	589
{2}	1007	738	1051	827	989	676	879	1079	846	1091	862	1131	1007	905	1007	673
{3}	966	826	839	1034	807	814	1013	1010	879	1091	895	1131	966	992	887	822
{4}	1081	712	1081	913	1085	615	1102	1012	1085	921	1085	1062	1081	915	1085	646
{5}	1127	586	1037	819	1127	625	1127	943	1110	971	1110	1083	1037	822	1110	697
{1,2}	978	565	707	941	938	524	864	1060	809	944	825	1102	1079	504	943	453
{1,3}	1114	597	831	937	1063	500	1129	939	989	944	949	1102	1140	613	1125	485
{1,4}	1214	436	1192	678	1003	548	1192	871	1003	830	1003	1045	1158	670	1142	359
{1,5}	1190	367	1190	547	1092	498	1190	740	1075	798	1075	956	1204	461	1120	408
{2,3}	920	560	728	827	634	676	761	1010	652	1029	571	1131	667	992	842	569
{2,4}	1107	374	812	913	929	479	899	1018	801	921	868	1066	836	915	953	510
{2,5}	897	450	1040	450	920	506	785	943	809	954	825	1065	938	575	945	559
{3,4}	1056	608	920	913	1036	455	1126	903	1103	816	980	1062	1000	915	1103	542
{3,5}	1087	421	756	819	1053	482	1124	769	1097	866	940	1083	1124	578	1097	591
{4,5}	1198	305	1193	527	1165	416	1124	842	1165	722	1148	964	1124	651	1148	464

A \ B	{7,9,10}		{8,9,10}		{6,7,8,9}		{6,7,8,10}		{6,7,9,10}		{6,8,9,10}		{7,8,9,10}		{6,7,8,9,10}	
{ }	864	978	864	1161	864	713	864	440	864	588	864	926	864	607	864	94
{1}	1125	689	1125	1015	1110	484	1087	221	1087	419	1087	723	1125	326	476	94
{2}	989	848	1007	1043	1007	408	846	304	989	322	734	926	887	464	590	94
{3}	847	978	973	1132	966	527	936	275	695	588	973	752	973	467	567	94
{4}	1081	799	1012	1157	1081	498	1085	158	1085	395	1012	825	1012	540	798	38
{5}	1110	791	1127	1105	1127	295	1110	233	1110	384	1127	741	1110	474	487	94
{1,2}	960	553	831	1015	864	348	809	85	825	283	697	723	831	190	697	-317
{1,3}	1084	529	1125	911	1114	274	989	117	949	259	877	723	1125	118	878	-285
{1,4}	1142	629	1142	891	1177	254	874	109	946	363	946	667	1142	146	946	-293
{1,5}	1120	589	1120	850	1190	20	939	87	1075	185	1092	472	1120	74	1075	-503
{2,3}	674	848	729	1132	920	125	652	139	522	322	577	752	729	331	577	-320
{2,4}	1024	657	823	1157	905	317	801	22	873	259	671	825	823	404	671	-199
{2,5}	946	638	849	1105	785	159	792	97	832	231	697	741	832	338	697	-315
{3,4}	1104	629	1056	1059	1000	394	964	30	976	239	1061	618	1061	334	1061	-374
{3,5}	1112	576	1114	1000	1124	17	1114	6	940	224	1114	532	1097	265	1098	-475
{4,5}	1165	678	1165	939	1124	138	1014	7	1148	209	1165	519	1165	235	1165	-457

A \ B	{ }		{6}		{7}		{8}		{9}		{10}		{6,7}		{6,8}	
{1,2,3}	996	728	888	953	825	1024	866	987	905	857	825	1057	611	945	758	1107
{1,2,4}	1071	728	971	919	924	890	941	950	1013	873	849	1057	971	649	841	968
{1,2,5}	998	728	880	915	998	710	861	916	998	743	819	1057	880	614	751	932
{1,3,4}	1114	728	1103	919	1008	890	1108	950	1105	873	989	1057	1103	625	1037	968
{1,3,5}	1090	728	1046	915	955	855	1085	916	1043	861	984	1057	1046	590	972	932
{1,4,5}	1091	728	976	915	1091	735	1091	799	1091	857	915	1057	976	638	976	818
{2,3,4}	1026	728	838	953	873	1044	912	1007	1007	862	856	1080	721	838	712	1079
{2,3,5}	966	728	749	926	766	1004	822	985	859	857	868	1063	614	765	632	1017
{2,4,5}	1036	728	875	910	1036	727	899	934	970	870	891	1080	875	568	747	909
{3,4,5}	1091	728	1018	910	754	1107	1085	934	1082	870	1035	1080	1018	544	1085	804
{1,2,3,4}	794	728	623	919	504	890	651	950	680	873	460	1057	623	489	484	968
{1,2,3,5}	656	728	474	915	512	710	537	916	456	861	421	1057	474	454	329	932
{1,2,4,5}	721	728	509	915	721	598	592	799	664	857	456	1057	509	502	381	818
{1,3,4,5}	889	728	678	915	889	574	817	799	833	857	625	1057	678	478	489	968
{2,3,4,5}	733	728	451	910	589	727	598	934	615	866	532	1080	451	408	446	804
{1,2,3,4,5}	45	728	84	614	236	438	91	799	123	857	24	835	-72	342	-333	969

A \ B	{6,9}		{6,10}		{7,8}		{7,9}		{7,10}		{8,9}		{8,10}		{9,10}	
{1,2,3}	776	944	620	1180	753	798	713	927	681	733	753	1036	707	1126	625	1120
{1,2,4}	876	1016	614	1180	941	620	1017	714	849	621	825	1045	720	1008	792	1116
{1,2,5}	767	956	676	1124	861	555	878	619	819	645	749	924	689	1033	705	1107
{1,3,4}	1016	1013	754	1180	1108	652	817	1008	989	597	1081	959	931	1008	932	1116
{1,3,5}	932	956	841	1124	1085	587	1043	595	984	621	1085	820	911	1033	871	1107
{1,4,5}	919	956	789	1106	1091	461	1091	696	915	669	1091	800	915	920	826	1142
{2,3,4}	746	1002	574	1175	619	1037	651	1129	712	772	833	1063	721	1167	738	1148
{2,3,5}	551	985	582	1178	822	647	654	891	584	928	709	1017	724	1175	681	1127
{2,4,5}	819	916	737	1081	899	588	974	696	909	667	769	1042	761	1073	851	1124
{3,4,5}	958	920	881	1081	1085	620	639	1129	1035	660	1043	940	978	1056	977	1142
{1,2,3,4}	480	1013	129	1180	651	516	536	719	460	461	569	959	316	1008	348	1116
{1,2,3,5}	249	956	199	1106	537	451	456	459	421	485	425	820	275	1033	195	1107
{1,2,4,5}	396	956	235	1106	592	325	664	560	456	533	536	800	328	920	329	1124
{1,3,4,5}	565	956	403	1106	817	357	833	536	625	509	761	800	416	1073	480	1142
{2,3,4,5}	339	916	243	1063	598	484	471	696	550	507	496	944	403	1056	418	1142
{1,2,3,4,5}	-241	956	-452	1124	91	221	123	400	-117	373	-21	800	-398	1074	-300	1124

A \ B	{6,7,8}		{6,7,9}		{6,7,10}		{6,8,9}		{6,8,10}		{6,9,10}		{7,8,9}		{7,8,10}	
{1,2,3}	744	461	499	662	620	364	632	954	475	944	395	1102	753	477	707	349
{1,2,4}	841	300	876	556	614	412	748	886	486	830	500	1102	825	534	720	223
{1,2,5}	751	231	767	411	693	362	639	740	565	780	581	938	749	325	689	272
{1,3,4}	1037	332	1016	532	754	388	959	871	1037	492	698	1045	1076	485	916	255
{1,3,5}	972	263	932	387	841	356	860	740	769	798	729	956	1085	253	911	304
{1,4,5}	976	142	976	434	789	386	741	871	789	668	826	892	1091	281	915	183
{2,3,4}	712	378	603	662	574	319	681	903	569	816	462	1062	506	915	721	406
{2,3,5}	632	285	502	450	582	346	557	769	554	866	375	1065	709	442	724	438
{2,4,5}	747	168	819	391	755	280	617	842	627	722	699	946	769	515	761	328
{3,4,5}	1085	96	962	363	881	273	1038	640	964	635	842	946	1038	449	978	343
{1,2,3,4}	484	196	480	396	129	252	335	886	340	492	17	1045	564	349	316	119
{1,2,3,5}	329	127	249	251	199	202	105	740	38	798	87	778	425	117	275	168
{1,2,4,5}	381	6	396	355	217	268	76	886	89	686	233	874	536	145	328	47
{1,3,4,5}	489	188	565	331	403	227	340	886	314	686	401	874	761	177	416	232
{2,3,4,5}	302	96	335	231	226	137	344	640	234	635	114	964	500	309	386	224
{1,2,3,4,5}	-333	52	-241	195	-452	108	-538	886	-596	686	-510	892	-21	41	-380	79

A \ B	{7,9,10}		{8,9,10}		{6,7,8,9}		{6,7,8,10}		{6,7,9,10}		{6,8,9,10}		{7,8,9,10}		{6,7,8,9,10}	
{1,2,3}	625	393	595	911	632	138	331	117	403	123	251	723	451	118	251	-421
{1,2,4}	792	493	664	891	748	118	212	109	558	170	430	610	664	10	373	-429
{1,2,5}	719	453	577	850	495	20	268	87	581	32	436	489	433	75	453	-656
{1,3,4}	932	469	860	891	944	150	562	5	641	203	944	325	875	42	944	-739
{1,3,5}	871	429	799	850	625	130	480	1	729	25	675	472	605	137	657	-607
{1,4,5}	826	567	698	957	726	110	428	31	844	47	844	351	716	107	844	-697
{2,3,4}	594	653	619	1059	675	109	286	30	462	99	471	618	619	202	471	-510
{2,3,5}	664	458	594	1017	413	17	428	6	358	88	442	549	594	129	460	-628
{2,4,5}	833	559	705	957	617	2	332	7	681	73	571	519	705	116	571	-593
{3,4,5}	994	518	922	939	756	2	446	23	842	31	922	415	905	148	922	-665
{1,2,3,4}	348	333	204	891	335	14	340	-582	17	10	191	324	204	-94	191	-875
{1,2,3,5}	195	293	51	850	105	-220	55	-271	-42	-111	-169	471	51	-166	-186	-743
{1,2,4,5}	311	431	201	811	75	-26	107	-391	216	-71	107	351	73	-29	87	-815
{1,3,4,5}	480	407	304	957	361	6	314	-341	384	-95	312	369	304	20	312	-783
{2,3,4,5}	435	382	291	939	348	-309	234	-456	117	-87	139	415	291	-5	122	-783
{1,2,3,4,5}	-300	253	-566	957	-538	-130	-596	-477	-492	-249	-654	369	-548	-150	-654	-919