



**UNRELATED PARALLEL MACHINE SCHEDULING  
WITH SEQUENCE-DEPENDENT SETUP TIMES  
AND MACHINE ELIGIBILITY:  
AN APPLICATION AT EURO GIDA**

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Master's Thesis

Graduate School  
Izmir University of Economics  
İzmir  
2022

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# ABSTRACT

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Master Program in Industrial Engineering

Advisor: Prof. Dr. Selin Özpeynirci

June, 2022

Unrelated parallel machine scheduling problem considers assigning a group of jobs to one of the parallel machines and sequencing with the aim of optimizing an objective. Although unrelated parallel machine problems are the most realistic scenario for the modern manufacturing industry, they have not been studied as much as the other parallel machine cases. In this study, a real-life unrelated parallel machine scheduling problem with sequence-dependent setup times and machine eligibility restrictions is studied with an application at Euro Gıda, which is one of the largest brands of food industry in Turkey. A mathematical model and a heuristic method are proposed. The results of the study indicate that the methods developed are successful to find solutions to most instances.

Keywords: Parallel machine scheduling, Unrelated parallel machines, Sequence-dependent setup time, machine eligibility.

# ÖZET

## SIRAYA BAĞLI KURULUM SÜRELERİ VE MAKİNE UYGUNLUĞU İLE BAĞLANTISIZ MAKİNE ÇİZELGELEMESİ: EURO GIDA'DA BİR UYGULAMA

Ersan, Tuğçe

Endüstri Mühendisliği Yüksek Lisans Programı

Tez Danışmanı: Prof. Dr. Selin Özpeynirci

Haziran, 2022

Bağlantısız paralel makine çizelgeleme problemi, bir amacı en iyilemek için bir grup işi paralel makinelere atama ve sıralamalarını belirleme ile ilgilenir. Bağlantısız paralel makine çizelgeleme problemleri modern imalat endüstrisi için en gerçekçi senaryo olmasına rağmen, diğer paralel makine çizelgeleme problemleri kadar çalışılmamıştır. Bu çalışmada, sıraya bağlı kurulum süreleri ve makine uygunluk kısıtları içeren bir gerçek hayat bağlantısız makine çizelgeleme problemi ele alınmıştır. Çalışma Türkiye'de gıda sektörünün en büyük markalarından biri olan Euro Gıda firmasında yürütülmüştür. Bir matematiksel model, bir de sezgisel yöntem sunulmuştur. Çalışmanın sonuçları, geliştirilen yöntemlerin çoğu örneğe çözüm bulmakta başarılı olduğunu göstermektedir.

Anahtar Kelimeler: Paralel makine çizelgelemesi, Bağlantısız paralel makineler, Sıraya bağlı kurulum süresi, Makine uygunluğu.

To those who have encouraged me to become the person who I am ...



## ACKNOWLEDGEMENTS

First and foremost, I appreciate to my advisor Prof. Dr. Selin Özpeynirci for her support, advice and patience during the study. I hope I have a chance to be as successful as she is both in my career and personal life.

I would like to express my gratitude to Euro Gıda and Izmir University of Economics for their support during my studies.

I want to express my deepest gratitude to Prof. Dr. K. Nazan Turhan for being more than an academician in my life. I always feel her support in my life.

I also thank my current supervisor Özgür Tuğgan for his patience and support during my study.

Finally, I would like to thank my family Sevgi Ersan, Nadir Ersan and Tolga Ersan, they always support and encourage me to do what I want in my life. You are golden chance for me and I am so lucky to have you all.

# TABLE OF CONTENTS

ABSTRACT .....	iii
ÖZET.....	iv
ACKNOWLEDGEMENTS .....	vi
LIST OF TABLES .....	ix
LIST OF FIGURES .....	xi
CHAPTER 1: INTRODUCTION .....	1
1.1 <i>Unrelated Parallel Machine Scheduling</i> .....	1
1.2 <i>Euro Gıda</i> .....	2
1.2.1 <i>Production of Semi-Finished Products</i> .....	2
1.2.2 <i>Production of Finished Products</i> .....	3
1.2.3 <i>Configuration of Labelling Lines</i> .....	3
1.3 <i>Purpose of the Study</i> .....	5
1.4 <i>Structure of the Thesis</i> .....	5
CHAPTER 2: LITERATURE REVIEW .....	6
2.1 <i>Analytical Hierarchy Process (AHP)</i> .....	6
2.2 <i>Parallel Machine Scheduling Problems</i> .....	7
CHAPTER 3: PROBLEM DEFINITION .....	10
3.1 <i>Problem Statement</i> .....	10
3.2 <i>Problem Environment</i> .....	10
CHAPTER 4: METHODOLOGY .....	13
4.1 <i>Analytical Hierarchy Process (AHP)</i> .....	13
4.2 <i>Mathematical Model</i> .....	17
4.3 <i>Heuristic Method</i> .....	20
4.3.1 <i>Phase 1 - Pre-processing</i> .....	20
4.3.2 <i>Phase 2 - Dispatching</i> .....	22
4.3.3 <i>Phase 3 - Post-processing</i> .....	23
CHAPTER 5: EXPERIMENTAL RESULTS .....	24
5.1 <i>Experimental Results of Mathematical Model</i> .....	24
5.2 <i>Experimental Results of Heuristic Method</i> .....	29
5.3 <i>Comparison of Mathematical Model and Heuristic Method</i> .....	39

CHAPTER 6: CONCLUSION..... 42  
REFERENCES..... 43





## LIST OF TABLES

Table 1. The scores for the importance levels of the variables.....	14
Table 2. Saaty's pair-wise comparison matrix (W).....	14
Table 3. The summation of the relative importance values. ....	15
Table 4. The weights of each criterion.....	15
Table 5. The sum/weight ratios.....	16
Table 6. The Value of Random Consistency Index. ....	16
Table 7. Objective function values and solution times of problems with $n=10$ .....	24
Table 8. Available time of lines. ....	25
Table 9. Due date data.....	25
Table 10. Machine eligibility data. ....	26
Table 11. Processing time data.....	26
Table 12. Setup times (in minutes). ....	27
Table 13. Solution for Line 1.....	27
Table 14. Solution for Line 2. ....	28
Table 15. Objective function values and solution times of problems with $n=15$ .....	29
Table 16. Objective function values of the heuristic method with $n=10$ . ....	30
Table 17. Indices at $t=0$ for line 1. ....	32
Table 18. Indices at $t=280$ for line 1. ....	32
Table 19. Indices at $t=675$ for line 1. ....	33
Table 20. Indices at $t=1,950$ for line 1. ....	33
Table 21. Indices at $t=3,159$ for line 1. ....	33
Table 22. Indices at $t=4,061$ for line 1. ....	34
Table 23. Indices at $t=4,121$ for line 1. ....	34
Table 24. Indices at $t=0$ for line 2. ....	34
Table 25. Indices at $t=2,100$ for line 2. ....	35
Table 26. Indices at $t=7,145$ for line 2. ....	35
Table 27. Solution for Line 1 in step 2.....	36
Table 28. Solution for Line 2 in step 2.....	37
Table 29. Solution for Line 1 in step 3.....	38
Table 30. Solution for Line 2 in step 3.....	39

Table 31. Comparison between the mathematical model and the heuristic method for  
10-jobs-sized examples. .... 40

Table 32. Comparison between the mathematical model and the heuristic method for  
15-jobs-sized examples. .... 41



## LIST OF FIGURES

Figure 1. Layout representation of the automated labelling lines.....	4
Figure 2. Hierarchy of determining significance level.....	13
Figure 3. The schedule of the small-sized problem that is obtained by solving the developed MILP model.....	28



## **CHAPTER 1: INTRODUCTION**

In this chapter, firstly unrelated parallel machine scheduling problems and their usage areas are explained. In the next section, the company, where the study is performed, Euro Gıda is introduced, presenting its products, processes and role in the market. Then, in the following section the problem which the thesis focuses on is defined in detail. In the final section, the general structure of the thesis is presented.

### ***1.1 Unrelated Parallel Machine Scheduling***

Scheduling problems are one of the most commonly studied problems, both by academicians and practitioners. They are vital for the operation management in terms of production planning and control. There are plenty of types depending on the constraints, problem environment, resources and number of machines. Single machine scheduling, parallel machine scheduling, flow shop scheduling, job-shop scheduling problems are some of the examples (Panneerselvam, 2012).

Parallel machine scheduling problems are common in different industries varying from food industry to automotive industry. Therefore, they have been extensively studied in the past few decades. The parallel machine problems can involve uniform or unrelated parallel machines. In the uniform parallel machine cases, the machines have different speeds so, the processing time of a job can vary depending on the machine it is processed. In the unrelated parallel machine cases, the processing times of the jobs on the machines are arbitrary for all jobs. This may be due to technological differences or different features of the machines and the jobs. Although the parallel machine scheduling problems are widely studied, unrelated parallel machine problems have not been studied as much as the other parallel machine cases (Vallada and Ruiz, 2011).

In general, the unrelated parallel machine scheduling problems consist of a set of jobs which must be processed on absolutely one machine among a set of parallel machines. The machines are defined as unrelated since each job has a specific processing time depending on the machine that is assigned to.

Furthermore, setup time is another concept that is taken into consideration in this type of problems (Vallada and Ruiz, 2011). Although the setup time is a crucial part of real-life problems, the concept of setup time has not been widely studied in the literature. In general, the setup times between consecutive jobs are assumed as zero. For instance, Bank and Werner (2001) studied an unrelated parallel machine scheduling case with the objective of minimizing total weighted tardiness and earliness. They assumed that there is zero setup time between succeeding jobs. The main reason to assume zero setup time is to lower the complexity of the problem and the mathematical model to solve the problem, since involving setup times, especially sequence-dependent setup times, increases complexity of the decision process.

This study considers an unrelated parallel machine scheduling problem where there are 2 unrelated parallel lines and a set of jobs to be processed on one of these lines. The setup times are sequence dependent; i.e. change depending on the job order. There is also machine eligibility that means each job can be processed on a subset of lines. The objective of the study is minimizing total tardiness, total earliness, total idle time of the lines and total setup time. We focus on the application of the problem at Euro Gıda. In the next section, a detailed explanation of production activities is given.

## ***1.2 Euro Gıda***

Euro Gıda is one of the largest brands of food industry in Turkey and has a wide range of product portfolio from pickles to canned food and gourmet sauces. It has the largest factory of the sector in Turkey and it ranks in the first 5 in Europe. The company appeals consumers with its own brands while undertaking contract manufacturing for many Turkish and European brands on B2B basis. 70% of its production is exported. The products produced by the company can be classified as semi-finished and finished.

### ***1.2.1 Production of Semi-Finished Products***

Fresh raw materials (vegetables and fruits) are supplied in their harvesting seasons (approximately from May to October). Some part of these fresh raw materials is

directly used in the production of semi-finished products when they are fresh. On the other hand, the rest is directly treated with brine in tanks to be processed in the winter season. In other words, after all required raw and auxiliary materials (vegetables, fruits, jars, caps, aromas, etc.) are supplied, semi-finished products are produced (basically: preparing raw materials, filling jars, adding brine, capping and pasteurization). There are 11 parallel production lines each having roughly from 1,500 to 10,000 jars/hour production capacity depending on the product that is produced. The semi-finish product production is done through May to December in 2 shifts (9 available working hours/shift), in high volumes and stored both in internal and external warehouses.

### ***1.2.2 Production of Finished Products***

Finished products are obtained by making the semi-finished products ready to be distributed. The stored semi-finished products are labelled, packed, wrapped, palletized and transferred/shipped to the determined locations. There are 2 parallel automated and 1 manual production lines each having nearly from 1,500 to 14,000 jars/hour production capacity depending on the product that is produced (The manual production line will not be under consideration in this work). The finished products are produced through the year generally in 3 shifts (7.5 available working hours/shift).

This study focuses only on the production planning of the finished products.

### ***1.2.3 Configuration of Labelling Lines***

A simple representation of the labelling lines is indicated in Figure 1, and an explanation related with each process is provided.

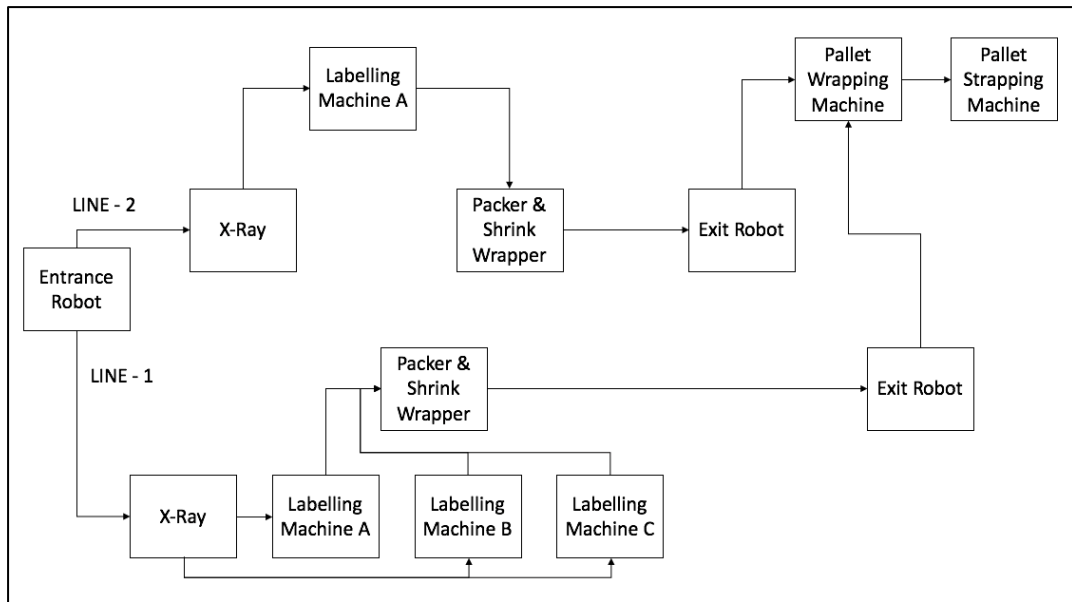


Figure 1. Layout representation of the automated labelling lines.

- Entrance Robot: There is an entrance robot that feeds both lines. It is a depalletiser.
- X-Rays: Critical control points to detect any foreign materials in jars or any non-vacuumed jars.
- Labelling Machines: There are 4 labelling machines in total in 2 lines. Both lines have the labelling machine A (does only labelling of one-labelled products). Line – 1 has also the labelling machine B (does labelling of two-labelled products) and labelling machine C (does labelling of products having transparent labels).
- Packer & Shrink Wrappers: In these machines, 3, 6, 8 or 10 jars are gathered, and wrapped with shrink. Depending on the customer, some products are only gathered in cartoons, and not wrapped with shrink, whereas some products are gathered in cartoons, and cartoon top covers are used instead of shrink.
- Exit Robots: The main purpose of the exit robots is to palletise the products.
- Pallet Wrapping Machine: The pallets leaving the exit robots are wrapped with stretch film in this work station.
- Pallet Strapping Machine: After the pallets are wrapped with stretch film, they are strapped in this machine.

### ***1.3 Purpose of the Study***

The aim of this study is to offer a solution to the company, to facilitate their scheduling process by minimizing the manual work, to improve their process by finding the optimal solution that minimizes the weighted sum of total tardiness, the total setup time, the total idle time and the total earliness. In other words, the goal of the study is to solve a real-life problem.

By the scope of the study, a mathematical model and a heuristic algorithm are proposed to be able to solve this real-life problem successfully. The proposed heuristic algorithm consists of 3 different phases; pre-processing, dispatching and post-processing. It is a modified version of a heuristic method that is found in the literature. In this study, the pre-processing phase is slightly changed, the dispatching phase is directly used as it was in the literature. However, the post-processing phase is completely changed and arranged according to the study.

### ***1.4 Structure of the Thesis***

In the next chapters of the thesis, the following topics are covered. In Chapter 2, the literature review which consists of the basics, the solution methods of the unrelated parallel machine problems with sequence dependent setup times and the basics of the heuristic that is proposed are given. In Chapter 3, the problem environment is clearly defined and all assumptions are explained. In Chapter 4, the steps of the Analytical Hierarchy Process (AHP), that is used to determine the weights of the objectives, are explained. The mathematical model is presented. Also, the heuristic method that is specifically developed for the real-life problem of this thesis is explained. In Chapter 5, all experimental results of the proposed heuristic method are reported and compared with the results of the mathematical model. Finally, in Chapter 6, the overall summary of the study and the future work is stated.



## **CHAPTER 2: LITERATURE REVIEW**

In this chapter, the information which is gathered by examining the previous studies which have been performed in the same or the similar areas is explained. The section is divided into two separate parts to provide a clear background for the methodology part: Firstly, brief information about Analytical Hierarchy Process (AHP) is given. Next, the studies that have been done about unrelated parallel machine scheduling problems with sequence dependent setup times are examined and summarized. Furthermore, the heuristic methods which have been proposed for solving unrelated parallel machine scheduling problems with sequence dependent setup times are presented.

### ***2.1 Analytical Hierarchy Process (AHP)***

The Analytical Hierarchy Process (AHP) is a widely used, decision-aiding method in which a complex multi-factor problem is decomposed into a hierarchy, each level having specific elements. It was developed by Saaty in 1980 as a vital tool to manage qualitative and quantitative multi-criteria elements including decision-making behaviour. It is totally based on a hierarchical structure (Taherdoost, 2017).

The main objective of the decision is located at the top of the hierarchy and the criteria, the sub-criteria and the other decision alternatives are located on each descending level of the hierarchy. When the hierarchical model is defined, the next step is providing pair-wise comparisons for each level so as to obtain the weight factor of the related level with respect to the one element in the next higher level. The obtained weight factor gives a measure of the relative significance of the element to make decision (Partovi et al., 1989).

The Eigenvector value problem is solved to compute the priorities of each element in each matrix. Next, the new vector is weighted with the weight factors of the upper level element that was used as the criterion in making the pair-wise comparison. The steps are repeated by moving downwards along the hierarchy, at every level the weights of each element are computed and these are used to identify composite

weight for lower levels. The optimal solution is the alternative with the greatest cumulative weight (Saaty, 1982).

In the literature, there are different operation management areas in which AHP is applied such as planning and scheduling, pricing decisions and international expansion decisions. For instance, Goedert and Sekpe (2013) used AHP technique to schedule multiple projects with competing priorities. Furthermore, Kaka et al. (2008) applied AHP technique for the selection of appropriate pricing system for a project. Moreover, Gunhan and Arditi (2005) used AHP to decide whether international expansion should be done or not (Darko et al., 2018).

## ***2.2 Parallel Machine Scheduling Problems***

Parallel scheduling problems have always been one of the challenging topics in manufacturing, especially in multi-criteria and multi-machine environment. A lot of studies have been conducted related to parallel machine scheduling and a huge number of papers have been published in the last decades (Kayvanfar et al., 2014).

Parallel machine scheduling problems can be divided into three main categories regarding the machine characteristics: identical machines, uniform machines and unrelated machines. In identical machine scheduling cases, all machines are identical so, the processing time of a job and the setup time between two succeeding jobs are the same for all machines. In the uniform machine scheduling problems, machines have dissimilar pace and the processing time of a job varies depending on the machine on which it is processed. Lastly, in the unrelated parallel machine scheduling problems, the processing times of the jobs have no special characteristics (Ekici et al., 2019).

There are several studies in the literature about each of the parallel machine scheduling cases. For instance, Alvarez-Valdes et al. (2015) studied an identical parallel machine scheduling problem with the objective of minimization of the total weighted tardiness and earliness of the jobs. They proposed a hybrid heuristic method which is depended on priority rules while assigning the jobs to the machines. Moreover, an identical parallel machine problem with sequence-dependent setup

times between consecutive jobs studied by Anderson et al. (2013). Their objective was to minimize total tardiness and earliness. A network-based solution was developed by them.

Armento and Filho (2007) performed a study about the uniform parallel machine case with sequence-dependent setup times. The objective of the study was to minimize total tardiness. They proposed a greedy random search adaptive procedure (GRASP). Furthermore, Toksari and Guner (2010) studied the same problem with the total weighed tardiness and earliness minimization objective. In their case, there was a presence of a learning effect and deteriorating jobs.

Although unrelated parallel machine problems are the most realistic scenario for the modern manufacturing industry, they have not been studied as much as the other parallel machine cases. In addition, the cases involving sequence-dependent setup times between consecutive jobs have not been considered until recent years (Vallada and Ruiz, 2011). For example, Fanjul-Peyro and Ruiz (2010) studied unrelated parallel machine scheduling problem with zero setup times to minimize make-span. Bank and Werner (2001) examined an unrelated parallel machine scheduling case in which jobs have a common due date and there is zero setup time between succeeding jobs. The objective of the study was to minimize total weighted tardiness and earliness by applying constructive and iterative algorithms. Cheng and Huang (2017) also studied an unrelated parallel machine scheduling problem without considering setup times. They conducted their study in the absence of setup times between jobs. The objective was to minimize total tardiness and earliness. Although these studies are all related to unrelated parallel machine scheduling problem, the setup times between consecutive jobs are not considered.

However, in real-life manufacturing processes, a company's responsibilities are keeping the stock levels at the desired level and fulfilling the customer orders on time obeying due dates since tardiness can result in penalty costs. While considering these, resources should also be used efficiently. For example, idle time of equipment, operators and machines should be minimized. Furthermore, make-to-order strategy of companies does not allow earliness of the customer orders. In other words, the orders should be completed just before the delivery date in order to not occupy

warehouse space bringing about holding cost (Ekici et al., 2019). These restrictions and constraints make it difficult to model these kinds of cases. In other words, in real-life unrelated parallel machine scheduling problems involve sequence-dependent setup times and machine eligibility restrictions while aiming several objectives simultaneously, such as minimizing total tardiness, total earliness, total setup time and total idle time.

Afzalirad and Rezaeian (2016) studied resource-constrained type of the unrelated parallel machine scheduling problem containing sequence dependent setup times, precedence constraints, machine eligibility restrictions and unequal release times. A pure integer model was developed by them to solve the problem but the problem was vigorously NP-hard. Hence, they proposed two meta-heuristic algorithms to solve large problems. For a similar problem, Kim et al. (2002) proposed a simulated annealing method with the aim of minimizing total tardiness. The same problem was also studied by Chen and Wu (2006) where resource constraints were also considered. Their objective was to minimize make-span, maximum tardiness and total tardiness, respectively. Rabadi et al. (2006) developed a heuristic for the same problem with the objective of minimizing make-span. Lee et al. (1997) proposed a three-phase heuristic for a single machine scheduling problem with sequence-dependent setup times. Their objective was to minimize total weighted tardiness.

In this paper, we deal with a real-life unrelated parallel machine scheduling problem with sequence-dependent setup times between consecutive jobs and machine eligibility restrictions. The objective is to minimize total tardiness, total earliness, total idle time of lines and total setup time. Some of the above methods which are available in the literature are evaluated and compared. A mathematical model, which is successful and rapid for solving small-sized instances, is developed. Since the developed mathematical model is slow for large-sized instances, a heuristic method which is a modification of Lee and Pinedo's (1997) three-phased heuristic is also proposed. The proposed heuristic indicates a satisfactory performance for both small and large-sized instances.

## **CHAPTER 3: PROBLEM DEFINITION**

In this section, a detailed explanation of the problem is stated. The problem environment is clearly defined and all assumptions are explained.

### ***3.1 Problem Statement***

In Euro Gıda, as it was stated above, there are 2 unrelated parallel lines to produce the finished goods. These lines are not identically the same. Therefore, some products can be labelled only on a specific line or on the both lines depending on the specifications of the product and the capability of the line. In other words, there are machine eligibility restrictions. Furthermore, the setup times between jobs are sequence dependent. Yearly, monthly, weekly and daily production plans for these labelling lines are prepared considering related constraints and available resources. The main goal is to minimize setup time and not to be tardy. However, the scheduling is done manually in MS Excel without using any optimization tools. This situation can be useful for the case “to not to be tardy” but can fail “minimizing setup times”. Moreover, the objective of the scheduling does not cover any earliness issue unless there is space in the warehouse of the facility. On the other side, if there is not sufficient space in the warehouse, a loose scheduling is done without considering minimizing idle time of the labelling lines. The problem environment is explained below, in details.

### ***3.2 Problem Environment***

After an order comes via e-mail or from order portals of customers, it is transferred to the ERP system of the company. While transferring the order to the ERP system, its due date is defined as Monday of the week in which the shipment will be done regardless the exact day of the shipment. For instance, an order that must be shipped on Thursday of the calendar week 6<sup>th</sup>, its due date is defined as 08.02.2021 (Monday of the calendar week 6). As a result of this, in the system, all jobs that are going to be shipped in the same week but in different days of the week, have the same due date. The scheduling of the jobs, which will be shipped in the same week, requires due date arrangement by the person who does planning depending on his/ her know how.

Cut-off date can be defined as the last date that a container can be returned to the port terminal to be loaded to the scheduled ship. It is generally two days before the expected departure date but can vary based on the carrier and the port. While doing monthly plans, for the orders that will be shipped by seaway, cut-off dates are unknown. Therefore, the dates are estimated by the person who does the planning depending on his/her know how. For example, the cut-off date of a ship arriving at Hamburg port of Germany is the next Monday of the relevant loading week, while the cut-off date of a ship arriving in Rotterdam, Netherlands is on the Thursday of the relevant loading week. The cut-off dates for the orders to be loaded in the relevant week begin to be exact as of the Monday of the week. For this reason, there can be sudden changes in the plan.

While most of the export shipments are shipped by sea, some export shipments and all inland shipments are shipped by road. For the highway dispatches, the logistics department prepares a loading plan considering truck supply and operator availability for loading. Changes in the loading plan leads to the changes in the production plan.

There are also other types of problems which result in changes in the production plan. For instance, there may be supply problems for auxiliary materials such as carton and label. The problems in supply of these or any delay in their arrival result in changes in the production plan. Moreover, low overall equipment efficiency and breakdowns in machines are other examples. However, in this thesis, main problem environment aspects which are taken into consideration are, having same due date in the ERP system, inexact and changeable cut-off dates and the loading plan.

The assumptions done in the study are listed below:

- Labelling plan can vary depending on stock of the semi-finished products. In other words, if a semi-finished product is not in stock, the labelling plan is done according to the production time of it. In the study, it is assumed that all products that will be labelled are in stock.
- When semi-finished products are produced, it is waited for seven days for incubation. Before the incubation period is done, they cannot be involved in the labelling plan. After incubation period, some quality tests are performed and if the test results are acceptable, they can be labelled. However, in this study, all

products that will be labelled are finished their incubation process and passed all quality tests.

- Processing times of some jobs slightly differ depending on the line that they are processed. However, for easiness, the processing time of the jobs are assumed as equal in both lines.
- Due dates for shipments, cut-off dates, are defined by a pre-process and assumed stable.
- For the mathematical model, it is assumed that all lines are available at the beginning of the day. If a line is busy due to maintenance or processing of a job a constraint can be added to the model to set a lower bound on the starting time on that line.

In this thesis, the focus is on the scheduling of the 2 unrelated parallel lines by considering the sequence dependent setup times and the machine eligibility restrictions with the objective of minimizing total tardiness, total setup time, total idle time and total earliness.

## CHAPTER 4: METHODOLOGY

In this chapter, firstly the steps of the Analytical Hierarchy Process (AHP) that is performed to determine the weights of different criteria on the objective function, is explained. Then, the mathematical model developed to solve the 2-unrelated parallel machines problem with sequence-dependent setup times and machine eligibility, is presented. Finally, a heuristic method, which is developed by modifying the heuristic that was proposed by Lee and Pinedo (1997), is explained.

### 4.1 Analytical Hierarchy Process (AHP)

In this study, Analytical Hierarchy Process (AHP) is applied to identify the weights of different criteria on the objective function, which are total tardiness, total earliness, total idle time and total setup time. The stepwise procedure is explained below.

**Step 1.** Firstly, the criteria that are used to evaluate the performance of a production plan should be identified. The objective of this study is to minimize total tardiness, total setup time, total idle time and total earliness. In this real-life problem, each criterion has a different importance level. Therefore, the effect of each criterion on the objective function should be determined quantitatively.

**Step 2.** The hierarchical model of these criteria is developed and illustrated in Figure 2.

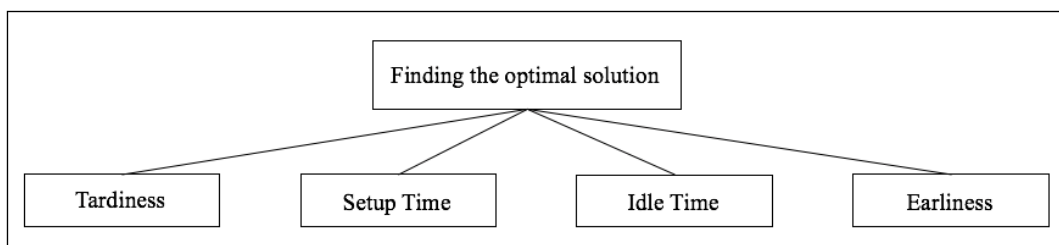


Figure 2. Hierarchy of determining significance level.

**Step 3.** To specify each criterion's contribution to the overall objective, the relative significance of each is determined by asking questions such as "What is the



importance of an increase in total idle time of lines relative to any tardiness of jobs to achieve the overall target?”. By making similar pair-wise comparisons for all criteria, Saaty’s pair-wise comparison matrix is generated (Table 2). To obtain this matrix, a table for the scores for the significance levels of the variables was prepared (Table 1). By using this score table, a questionnaire was performed and all questions were asked to the production planning responsible of these labelling lines who has been working at the same position for 3 years. The change in preferences can result in changes in weights. The change in the weights can cause the change in the objective function value.

Table 1. The scores for the importance levels of the variables  
(Source: Taherdoost, 2017).

<b>Importance Scale</b>	<b>Definition</b>
1	Equally important preferred
2	Equally to moderately important preferred
3	Moderately important preferred
4	Moderately to strongly important preferred
5	Strongly important preferred
6	Strongly to very strongly important to preferred
7	Very strongly important preferred
8	Very strongly o extremely important preferred
9	Extremely important preferred

Table 2. Saaty’s pair-wise comparison matrix (W).

	<b>Tardiness</b>	<b>Setup Time</b>	<b>Idle Time</b>	<b>Earliness</b>
<b>Tardiness</b>	1	4	7	9
<b>Setup Time</b>	1/4	1	2	2
<b>Idle Time</b>	1/7	1/2	1	1
<b>Earliness</b>	1/9	1/2	1	1

**Step 4.** After this evaluation, matrix W and the consistency ratio (CR) are calculated as explained below. The consistency ratio is the ratio of the inconsistencies of the decision maker and the inconsistencies due to randomly generated preferences. It should be known that if the consistency ratio is lower than 0.10, it is verified that the results are acceptable (Taherdoost, 2017).

- The relative importance values in each column are summed (Table 3).

Table 3. The summation of the relative importance values.

	<b>Tardiness</b>	<b>Setup Time</b>	<b>Idle Time</b>	<b>Earliness</b>
<b>Tardiness</b>	1	4	7	9
<b>Setup Time</b>	1/4	1	2	2
<b>Idle Time</b>	1/7	1/2	1	1
<b>Earliness</b>	1/9	1/2	1	1
<b>SUM</b>	<b>1.50</b>	<b>6.00</b>	<b>11.00</b>	<b>13.00</b>

- Each relative importance value is divided by the sum of the related column and the weights are calculated by taking the average of the values in each row (Table 4).

Table 4. The weights of each criterion.

	<b>Tardiness</b>	<b>Setup Time</b>	<b>Idle Time</b>	<b>Earliness</b>	<b>Weight</b>	<b>Weight (%)</b>
<b>Tardiness</b>	0.66	0.66	0.63	0.69	0.66	66%
<b>Setup Time</b>	0.16	0.16	0.18	0.15	0.17	17%
<b>Idle Time</b>	0.09	0.08	0.09	0.07	0.09	9%
<b>Earliness</b>	0.07	0.08	0.09	0.07	0.08	8%

- The summation of the values in each row is calculated and the sum/weight ratio for each row is calculated (Table 5).

Table 5. The sum/weight ratios.

	<b>Tardiness</b>	<b>Setup Time</b>	<b>Idle Time</b>	<b>Earliness</b>	<b>Sum</b>	<b>Sum / Weight</b>
<b>Tardiness</b>	0.66	0.66	0.63	0.69	2.66	4.03
<b>Setup Time</b>	0.16	0.16	0.18	0.15	0.67	3.94
<b>Idle Time</b>	0.09	0.08	0.09	0.07	0.35	3.89
<b>Earliness</b>	0.07	0.08	0.09	0.07	0.33	4.13

- Finally, for the validation of the results the AHP, the consistency ratio (CR) is calculated using the formula;

$$CR = CI/RI$$

where;

$$CI = \frac{\text{maximum } \frac{\text{sum}}{\text{weight}} \text{ value} - \text{number of criteria}}{(\text{number of criteria} - 1)}$$

The value of RI is found depending on the dimension of the matrix W. To obtain the value of it Table 6 is used.

Table 6. The Value of Random Consistency Index. (Source: Taherdoost, 2017)

<b>Dimension</b>	<b>RI value</b>
1	0
2	0
3	0.5799
4	0.8921
5	1.1159

So, in this study;

$$CI = \frac{4.13 - 4}{(4 - 1)} = 0.043$$

Since it is a 4 x 4 matrix, RI = 0.8921 (Table 6).

Finally,

$$CR = \frac{0.043}{0.8921} \cong 0.05$$

It can be concluded as the results of the comparison are acceptable since the CR values is lower than 0.10.

#### **4.2 Mathematical Model**

In this section, the mathematical model developed to solve this real-life problem is explained. In this mathematical model, each job can be processed on a line at a time depending on machine eligibility restrictions.

##### **Sets:**

$N$ : set of jobs to be processed,  $N = \{1, 2, \dots, n\}$

$M$ : set of lines,  $M = \{1, 2\}$

##### **Indices:**

$j, k$ : job indices, where  $j \neq k$  and  $j, k \in N$

$l$ : line index, where  $l \in M$

$i$ : order index, where  $i \in N$

##### **Parameters:**

$p_{jl}$  = processing time of job  $j$  on line  $l$

$s_{kj}$  = setup time of job  $j$  if it is processed immediately after job  $k$

$d_j$  = due date of job  $j$

$BM$ : very large positive number

$e_{jl} = \begin{cases} 1, & \text{if job } j \text{ can be processed on line } l \\ 0, & \text{otherwise} \end{cases}$

$A_l$  = available time of line  $l$

$w_I$  = weight of idle time

$w_T$  = weight of tardiness

$w_E$  = weight of earliness

$w_S$  = weight of setup time

**Decision variables:**

$C_j$  = completion time of job  $j$

$B_{jl}$  = beginning time of job  $j$  on line  $l$

$X_{jli} = \begin{cases} 1, & \text{if job } j \text{ is processed on line } l \text{ in the } i\text{th order} \\ 0, & \text{otherwise} \end{cases}$

$Y_{kjl} = \begin{cases} 1, & \text{if job } j \text{ immediately follows job } k \text{ on line } l \\ 0, & \text{otherwise} \end{cases}$

$T_j$  = tardiness of job  $j$

$EA_j$  = earliness of job  $j$

$idletime_l$  = idle time of line  $l$

$$Min z = w_I \sum_l idletime_l + w_T \sum_j T_j + w_E \sum_j EA_j + w_S \sum_k \sum_{j \neq k} \sum_l s_{kj} Y_{kjl} \quad (4.1)$$

Subject to

$$C_j = \sum_l \sum_i (B_{jl} + p_{jl} X_{jli}) \quad \forall j \quad (4.2)$$

$$\sum_l \sum_i X_{jli} = 1 \quad \forall j \quad (4.3)$$

$$\sum_i X_{jli} \leq e_{jl} \quad \forall j, l \quad (4.4)$$

$$B_{jl} \leq BM \sum_i X_{jli} \quad \forall j, l \quad (4.5)$$

$$B_{jl} \geq B_{kl} + p_{kl} + s_{kj} - BM(1 - Y_{kjl}) \quad \forall k, j \neq k, l \quad (4.6)$$

$$T_j - EA_j = C_j - d_j \quad \forall j \quad (4.7)$$

$$idletime_l = A_l - \sum_i \sum_j p_{jl} X_{jli} - \sum_k \sum_{j \neq k} s_{kj} Y_{kjl} \quad \forall l \quad (4.8)$$

$$X_{jl(i+1)} + X_{kli} \geq 2Y_{kjl} \quad \forall k, j \neq k, l, i \quad (4.9)$$

$$X_{jl(i+1)} + X_{kli} \leq Y_{kjl} + 1 \quad \forall k, j \neq k, l, i \quad (4.10)$$

$$\sum_j X_{jl(i+1)} \leq \sum_j X_{jli} \quad \forall l, i \quad (4.11)$$

$$C_j \geq 0 \quad \forall j \quad (4.12)$$

$$T_j \geq 0 \quad \forall j \quad (4.13)$$

$$EA_j \geq 0 \quad \forall j \quad (4.14)$$

$$B_{jl} \geq 0 \quad \forall j, l \quad (4.15)$$

$$idletime_l \geq 0 \quad \forall l \quad (4.16)$$

$$X_{jli} \in \{0,1\} \quad \forall j, l, i \quad (4.17)$$

$$Y_{kjl} \in \{0,1\} \quad \forall k, j \neq k, l \quad (4.18)$$

The objective function (4.1) minimizes total idle time, total tardiness, total earliness and total setup time. Constraint (4.2) calculates the completion time of the job depending on the beginning time of it on the assigned line, in the assigned order and its processing time. Restriction (4.3) guarantees that each job is assigned to an appropriate line to be processed and each job is assigned to exactly one line. Constraint (4.4) is the machine eligibility constraint. It helps the model to assign jobs to the lines according to the eligibility data. Constraint (4.5) ensures that a job can begin on the line that it is assigned to. Constraint (4.6) guarantees that the beginning time of a job is greater than or equal to the completion time of the previous job. Constraint (4.7) defines the tardiness and earliness of jobs. Constraint (4.8) calculates the idle time of a related line. Constraints (4.9) and (4.10) are for the relation between binary variables. The jobs that are assigned to the same line are ordered with these constraints. Constraint (4.11) is an order constraint to define the

processing order of a job on an assigned line. Constraints (4.12), (4.13), (4.14), (4.15), (4.16), (4.17) and (4.18) are sign restrictions.

Although the mathematical model is rapid to solve small-sized problems, it is slow to solve large-sized problems (explained in detail in Chapter 5). Since the real-life problem is a large-sized problem, a heuristic method is also developed to solve.

### ***4.3 Heuristic Method***

In this section, the heuristic method developed to solve the problem is explained. This heuristic method is a derivation of Lee and Pinedo's (1997) three-phased heuristic proposed for a parallel machine scheduling problem to minimize the total weighted tardiness in the existence of sequence-dependent setup times. In this study, this heuristic method is modified for an unrelated parallel machine scheduling problem with sequence-dependent setup times and machine eligibility restrictions. The objective is not only minimizing the total tardiness, it is minimizing total tardiness, total earliness, total idle time and total setup time.

In the developed heuristic method, the first two phases are applied with slight changes whereas the third phase is completely different from the Lee and Pinedo's heuristic. The details are explained below.

#### ***4.3.1 Phase 1 – Pre-processing***

In the first phase, all factors and statistics which are required to determine the scaling parameters that will be used in the phase 2 are calculated as defined below (Lee and Pinedo, 1997).

$m$  = number of machines

$n$  = number of jobs

$\bar{p}$  = average processing time

$\bar{s}$  = average setup time

$\bar{d}$  = average due date

$d_{max}$  = maximum due date

$d_{min}$  = minimum due date

$c_{max} = \max\{c_1|c_2\} = \text{maximum makespan}$

$$\mu = \frac{n}{m} = \text{job - machine factor}$$

$$\eta = \frac{\bar{s}}{\bar{p}} = \text{setup time severity factor}$$

$\beta = a$  coefficient which takes into account the effect of setup times on makespan =  $0.4 + 10/\mu^2 - n/7$

(4.19)

$$c_1 = \sum p_{j1} + \frac{(\sum p_{j(1,2)})}{2} + \bar{s} * \beta = \text{make span of line 1}$$

(4.20)

$$c_2 = \sum p_{j2} + \frac{(\sum p_{j(1,2)})}{2} + \bar{s} * \beta = \text{make span of line 2}$$

(4.21)

$$\tau = 1 - \frac{\bar{d}}{c_{max}} = \text{due date tightness}$$

(4.22)

$$R = \frac{d_{max} - d_{min}}{c_{max}} = \text{due date range factor}$$

(4.23)

In this study, equations (4.19), (4.22) and (4.23) are directly taken from the study of Lee and Pinedo (1997). However, the calculation of  $c_{max}$  is performed different than Lee and Pinedo's calculations.

In the study of Lee and Pinedo (1997), make spans of lines are not calculated separately. They state only one formula to calculate  $c_{max}$ .

The formula stated by them:

$$c_{max} = (\beta \bar{s} + \bar{p}) \mu$$

However, in this study  $c_{max}$  is found by using this formula:

$$c_{max} = \max\{c_1, c_2\}$$



Since there are machine eligibility restrictions in this case, the number of jobs that can be processed on lines differ. Hence, the make-spans of the lines are calculated separately by equations (4.20) and (4.21) and maximum of them is accepted as  $c_{max}$ .

#### 4.3.2 Phase 2 – Dispatching

In the second phase, a scheduling of jobs is constructed depending on their index values. Indices of unassigned jobs are calculated by the formula below. It is assumed that all jobs are available at time zero. At time  $t$ , a job having the largest index value is assigned to the related line at the first order. This process is repeated until there are no unassigned jobs (Lee and Pinedo, 1997).

$$I_{j(t,l)} = \left(\frac{1}{p_j}\right) \exp\left(-\frac{\max(d_j - p_j - t, 0)}{k_1 \bar{p}}\right) * \exp\left(-\frac{s_{lj}}{k_2 \bar{s}}\right) \quad (4.24)$$

where  $I_{j(t,l)}$  is the index for job  $j$  at time  $t$  given that job  $l$  is the last job which is completed on the related machine.  $p_j$  is the processing time of job  $j$ .  $d_j$  is the due date of job  $j$ .  $s_{lj}$  is the setup time between job  $l$  and job  $j$ .  $\bar{p}$  is the average processing time of jobs.  $\bar{s}$  is average setup time.  $k_1$  and  $k_2$  are parameters which can be characterized by the factors  $R$ ,  $\mu$ ,  $\tau$  and  $\eta$  (Lee and Pinedo, 1997).

$$k_1 = 1.2 \ln(\mu) - R \quad (4.25)$$

$$k_2 = \frac{\tau}{A_2 \sqrt{\eta}} \quad (4.26)$$

where  $A_2 = 1.8$  if  $\tau < 0.8$ , and  $A_2 = 2.0$  if  $\tau \geq 0.8$ .

Also, the following modifications must be done (Lee and Pinedo, 1997):

Subtract 0.5 from  $k_1$  if  $\tau < 0.5$ ,

Subtract 0.5 from  $k_1$  if  $\eta < 0.5$  and  $\mu > 5$ .

In this study, equations (4.24), (4.25) and (4.26) are directly taken from the study of Lee and Pinedo. All values depend on their experimental results.

### ***4.3.3 Phase 3 – Post-processing***

In the third phase, after all jobs are scheduled according to their indices calculated in phase 2, a post-processing is done to minimize earliness and improve the objective function value without deteriorating other objectives. The beginning and completion times of the scheduled jobs are manually changed to minimize earliness without causing any tardiness. Assumptions that are made during this phase are listed below.

- If the final job that is assigned to a line is early, the beginning and completion times of all jobs are shifted without causing any tardiness to minimize the total earliness.
- If there are jobs having higher due dates than the job assigned to the last order at a line, assigning the job which has the higher due date value to the last order is done without causing any tardiness and increase in total setup time to minimize the total earliness.
- For the jobs which can be processed at both lines, assign the job to the line at which it has the greatest index value.

This phase of the study is completely different from the study of Lee and Pinedo. Since their objective is to minimize the total weighted tardiness, they do not include any earliness minimizing activity in their study. In their phase 3, they try to improve the sequence obtained in the second phase. They propose a simulated annealing algorithm that uses the obtained schedule as a seed solution. However, they do not aim earliness minimization.

## CHAPTER 5: EXPERIMENTAL RESULTS

In this chapter, all computational experiments performed to evaluate both the mathematical model and the heuristic method are explained in detail. Also, the comparison between the success of the mathematical model and the heuristic method is stated.

All data used in the computational experiments is the real data of the company. After data collection, the data was pre-processed to be used in the experiments. For instance, due date data exists as in date form but it is converted into minutes to be used in the model as a due date data. In addition, setup time data is converted into a matrix version. Moreover, processing time data is found by multiplying the order size (pieces of jars) and the capacity (minutes per pieces of jars).

### *5.1 Experimental Results of Mathematical Model*

Firstly, 10 problem instances with 10 jobs are solved by using the mathematical model via GAMS. The objective function values and solution times in seconds are illustrated in Table 7.

Table 7. Objective function values and solution times of problems with  $n=10$ .

<b>Instance</b>	<b>Objective Function Value</b>	<b>Solution Time (s)</b>
1	938.32	8
2	678.30	7
3	865.99	4
4	1,060.47	59
5	803.94	8
6	753.10	14
7	705.72	41
8	549.84	56
9	870.50	15
10	648.56	7

To provide better insight, Problem 8 is solved as an illustrative example. The illustrative example involves 10 jobs to be processed and 2 available lines. All data used in the problem are taken from the company's real data and the parameters are given in Tables 8-12. The problem is solved with the developed MILP model. The value of the objective function is 549.84. The detailed solutions for the lines are provided in Tables 13 and 14. The obtained schedule of the problem is also illustrated in Figure 3.

Table 8. Available time of lines.

<b>Line</b>	<b>Available Time (min)</b>
1	8,100
2	8,100

Table 9. Due date data.

<b>Job</b>	<b>Due Date (min)</b>
1	1,000
2	500
3	2,000
4	8,000
5	7,800
6	8,100
7	6,600
8	5,200
9	4,800
10	3,600

Table 10. Machine eligibility data.

<b>Job</b>	<b>Line 1</b>	<b>Line 2</b>
1	1	0
2	1	0
3	1	0
4	1	0
5	0	1
6	0	1
7	1	0
8	1	0
9	1	0
10	0	1

Table 11. Processing time data.

<b>Job</b>	<b>Processing Time on Line 1 (min)</b>	<b>Processing Time on Line 2 (min)</b>
1	320	-
2	280	-
3	1,200	-
4	432	-
5	-	5,000
6	-	620
7	1,960	-
8	1,134	-
9	842	-
10	-	2,100

Table 12. Setup times (in minutes).

Job	1	2	3	4	5	6	7	8	9	10
1	0	75	75	45	45	45	45	45	45	30
2	75	0	120	120	60	60	20	75	120	75
3	75	120	0	75	75	300	75	75	120	75
4	45	120	75	0	20	60	30	20	60	45
5	45	60	75	20	0	60	30	20	60	45
6	45	60	300	60	60	0	60	60	20	45
7	45	20	75	30	30	60	0	30	60	45
8	45	75	75	20	20	60	30	0	60	45
9	45	120	120	60	60	20	60	60	0	45
10	30	75	75	45	45	45	45	45	45	0

Table 13. Solution for Line 1.

	JOB ORDER							Total Earliness (min)	Total Tardiness (min)
	2	1	3	8	9	7	4		
<b>Beginning Time (min)</b>	50	405	800	2,544	3,738	4,640	7,568	-	-
<b>Completion Time (min)</b>	330	725	2,000	3,678	4,580	6,600	8,000	-	-
<b>Due Date (min)</b>	500	1,000	2,000	5,200	4,800	6,600	8,000	-	-
<b>Earliness (min)</b>	170	275	0	1,522	220	0	0	2,187	-
<b>Tardiness (min)</b>	0	0	0	0	0	0	0	-	0
<b>Idle Time (min)</b>	1,557								
<b>Total Setup Time (min)</b>	375								

Table 14. Solution for Line 2.

	JOB ORDER			Total Earliness (min)	Total Tardiness (min)
	10	5	6		
Beginning Time (min)	275	2,420	7,480	-	-
Completion Time (min)	2,375	7,420	8,100	-	-
Due Date (min)	3,600	7,800	8,100	-	-
Earliness (min)	1,225	380	0	1,605	-
Tardiness (min)	0	0	0	-	0
Idle Time (min)	275				
Total Setup Time (min)	105				

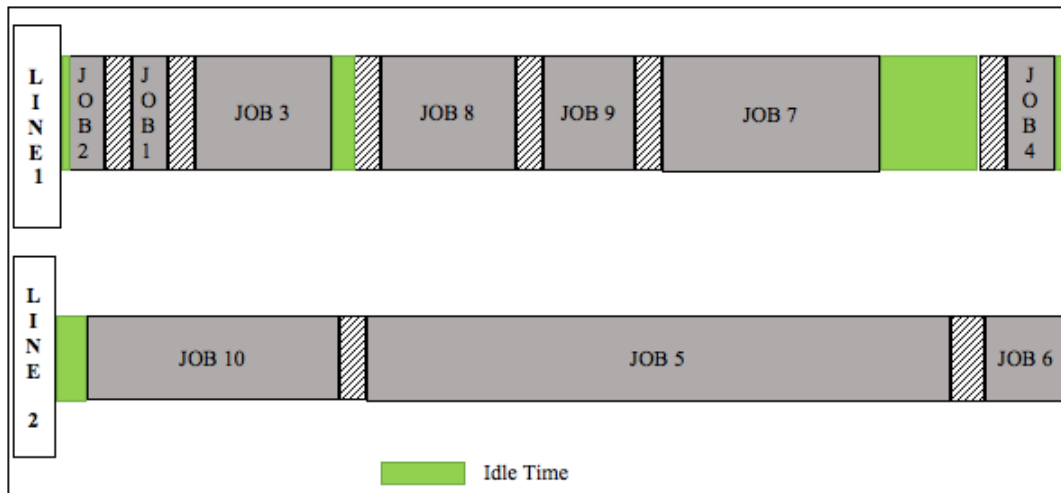


Figure 3. The schedule of the small-sized problem that is obtained by solving the developed MILP model.

Then, 5 problem instances with 15 jobs are solved by using the same model via GAMS. The objective function values and solution times in seconds are illustrated in Table 15.

Table 15. Objective function values and solution times of problems with  $n=15$ .

<b>Instance</b>	<b>Objective Function Value</b>	<b>Solution Time (s)</b>
1	702.75	3,823
2	625.66	3,395
3	474.45	7,764
4	753.70	9,799
5	480.45	605

The mathematical model is rapid while solving 10-jobs-sized problems. However, it is slow for solving 15-jobs-sized problems. When the number of jobs to be scheduled increases from 10 to 15, the average solving time rises nearly from 21.90 seconds to 5,077.20 seconds (84.62 minutes). When a real-life case is considered, it includes nearly 40 jobs to be scheduled in a weekly plan. Since it is slow only for 15-jobs-sized problems, other problems including more than 15 jobs are not tried to be solved by the mathematical model.

### ***5.2 Experimental Results of Heuristic Method***

The same 10-jobs-sized problems are solved by using the developed heuristic method and the objective function values are shown in Table 16.



Table 16. Objective function values of the heuristic method with  $n=10$ .

Instance	Objective Function Value of the Heuristic Method
1	938.32
2	681.70
3	882.99
4	1,569.03
5	811.59
6	892.87
7	705.72
8	549.84
9	1,456.20
10	648.56

For a better explanation, the same problem, Problem 8, is solved step-by-step as an illustrative example by using the developed heuristic method below. The parameters of the problem are given in Tables 8-12. The value of the objective function is found as 549.84 which is the same result with the mathematical model solution.

**Phase 1.** Pre-processing - firstly, all required parameters are calculated as shown below.

$$m = \text{number of machines} = 2$$

$$n = \text{number of jobs} = 10$$

$$\mu = \frac{n}{m} = \text{job - machine factor} = 5$$

$$\bar{p} = \text{average processing time} = 1,388.80 \text{ min}$$

$$\bar{s} = \text{average setup time} = 56.30 \text{ min}$$

$$\eta = \frac{\bar{s}}{\bar{p}} = \text{setup time severity factor} = 0.04$$

$\bar{d}$  = average due date = 4,760 min

$d_{max}$  = maximum due date

$d_{min}$  = minimum due date

$\beta$  = a coefficient which takes into account the effect of

setup times on makespan =  $0.4 + \frac{10}{\mu^2} - \frac{n}{7} = 0.79$

$$c_1 = \sum p_{j1} + \frac{(\sum p_{j(1,2)})}{2} + \bar{s} * \beta = \text{makespan of line 1} = 6,212.71 \text{ min}$$

$$c_2 = \sum p_{j2} + \frac{(\sum p_{j(1,2)})}{2} + \bar{s} * \beta = \text{makespan of line 2} = 7,764.71 \text{ min}$$

$$c_{max} = \max\{c_1|c_2\} = \text{maximum makespan} = 7,764.71$$

$$\tau = 1 - \frac{\bar{d}}{c_{max}} = \text{due date tightness} = 0.39$$

$$R = \frac{d_{max} - d_{min}}{c_{max}} = \text{due date range factor} = 0.98$$

**Phase 2.** Dispatching- in this second step, dispatching rule is applied by calculating the indices of jobs.

### Line 1

At  $t=0$ , all jobs which can be processed on line 1 are available and the calculated indices are shown in Table 17.

Table 17. Indices at  $t=0$  for line 1.

<b>Job</b>	<b>Index</b>
1	0.001869
2	0.003024
3	0.000455
4	0.000008
7	0.000015
8	0.000041
9	0.000060

Since job 2 has the maximum index value, it is assigned to the line 1 at the first order.

At  $t=280$ , the indices of unassigned jobs are calculated and illustrated in Table 18.

Table 18. Indices at  $t=280$  for line 1.

<b>Job</b>	<b>Index</b>
1	0.000663
3	0.000076
4	0.000001
7	0.000014
8	0.000014
9	0.000010

Since job 1 has the maximum index value, it is assigned to the line 1 at the second order.

At  $t=675$ , the indices of unassigned jobs are calculated and illustrated in Table 19.

Table 19. Indices at  $t=675$  for line 1.

<b>Job</b>	<b>Index</b>
3	0.00021774
4	0.00000598
7	0.00001205
8	0.00003214
9	0.00004697

Since job 3 has the maximum index value, it is assigned to line 1 at the third order.

At  $t=1,950$ , the indices of unassigned jobs are calculated and illustrated in Table 20.

Table 20. Indices at  $t=1,950$  for line 1.

<b>Job</b>	<b>Index</b>
4	0.00000951
7	0.00001918
8	0.00005115
9	0.00003536

Since job 8 has the maximum index value, it is assigned to the line 1 at the fourth order.

At  $t=3,159$ , the indices of unassigned jobs are calculated and illustrated in Table 21.

Table 21. Indices at  $t=3,159$  for line 1.

<b>Job</b>	<b>Index</b>
4	0.00005924
7	0.00010111
9	0.00023928

Since job 9 has the maximum index value, it is assigned to the line 1 at the fifth order.

At  $t=4,061$ , the indices of unassigned jobs are calculated and illustrated in Table 22.

Table 22. Indices at  $t=4,061$  for line 1.

<b>Job</b>	<b>Index</b>
4	0.00006022
7	0.00012139

Since job 7 has the maximum index value, it is assigned to the line 1 at the sixth order.

Finally, at  $t=4,121$ , the only unassigned job which is job 4 is assigned to the line 1 at the seventh order. Its index is calculated and illustrated in Table 23.

Table 23. Indices at  $t=4,121$  for line 1.

<b>Job</b>	<b>Index</b>
4	0.00010379

Job order for line 1: 2 – 1 – 3 – 8 – 9 – 7 – 4.

## **Line 2**

At  $t=0$ , all jobs that can be processed on line 2 are available and the calculated indices are shown in Table 24.

Table 24. Indices at  $t=0$  for line 2.

<b>Job</b>	<b>Index</b>
5	0.000024
6	0.000006
10	0.000153

Since job 10 has the maximum index value, it is assigned to the line 2 at the first order.

At  $t=2,100$ , the indices of unassigned jobs are calculated and illustrated in Table 25.

Table 25. Indices at  $t=2,100$  for line 2.

<b>Job</b>	<b>Index</b>
5	0.00005573
6	0.00001307

Since job 5 has the maximum index value, it is assigned to the line 2 at the second order.

Finally, at  $t=7,145$ , the only unassigned job which is job 6 is assigned to the line 2 at the third order. Its index is calculated and illustrated in Table 26.

Table 26. Indices at  $t=7,145$  for line 2.

<b>Job</b>	<b>Index</b>
6	0.00046149

Job order for line 2: 10 – 5 – 6.

After finishing assigning jobs according to their indices, the solutions which are shown in Tables 27 and 28 are obtained.

Table 27. Solution for Line 1 in step 2.

	JOB ORDER							Total Earliness (min)	Total Tardiness (min)
	2	1	3	8	9	7	4		
<b>Beginning Time (min)</b>	0	355	750	2,025	3,219	4,121	6,111	-	-
<b>Completion Time (min)</b>	280	675	1,950	3,159	4,061	6,081	6,543	-	-
<b>Due Date (min)</b>	500	1,000	2,000	5,200	4,800	6,600	8,000	-	-
<b>Earliness (min)</b>	200	325	50	2,041	739	519	1,457	5,351	-
<b>Tardiness (min)</b>	0	0	0	0	0	0	0	-	0
<b>Idle Time (min)</b>	1,557								
<b>Total Setup Time (min)</b>	375								

Table 28. Solution for Line 2 in step 2.

	<b>JOB ORDER</b>				
	<b>10</b>	<b>5</b>	<b>6</b>	<b>Total Earliness (min)</b>	<b>Total Tardiness (min)</b>
<b>Beginning Time (min)</b>	0	2,145	7,205	-	-
<b>Completion Time (min)</b>	2,100	7,145	7,825	-	-
<b>Due Date (min)</b>	3,600	7,800	8,100	-	-
<b>Earliness (min)</b>	1,500	655	275	2,430	-
<b>Tardiness (min)</b>	0	0	0	-	0
<b>Idle Time (min)</b>				275	
<b>Total Setup Time (min)</b>				105	

**Phase 3.** Post-processing - after scheduling jobs according to their indices in step 2, finally, a post-processing is done to minimize earliness. The beginning and completion times of the scheduled jobs are manually changed to minimize earliness without causing any tardiness. After performing this post-processing, the solutions which are shown in Tables 29 and 30 are obtained.



Table 29. Solution for Line 1 in step 3.

	JOB ORDER								
	2	1	3	8	9	7	4	Total Earliness (min)	Total Tardiness (min)
<b>Beginning Time (min)</b>	50	405	800	2,544	3,738	4,640	7,568	-	-
<b>Completion Time (min)</b>	330	725	2,000	3,678	4,580	6,600	8,000	-	-
<b>Due Date (min)</b>	500	1,000	2,000	5,200	4,800	6,600	8,000	-	-
<b>Earliness (min)</b>	170	275	0	1,522	220	0	0	2,187	-
<b>Tardiness (min)</b>	0	0	0	0	0	0	0	-	0
<b>Idle Time (min)</b>	1,557								
<b>Total Setup Time (min)</b>	375								

Table 30. Solution for Line 2 in step 3.

	JOB ORDER			Total Earliness (min)	Total Tardiness (min)
	10	5	6		
<b>Beginning Time (min)</b>	275	2,420	7,480	-	-
<b>Completion Time (min)</b>	2,375	7,420	8,100	-	-
<b>Due Date (min)</b>	3,600	7,800	8,100	-	-
<b>Earliness (min)</b>	1,225	380	0	1,605	-
<b>Tardiness (min)</b>	0	0	0	-	0
<b>Idle Time (min)</b>	275				
<b>Total Setup Time (min)</b>	105				

### ***5.3 Comparison of Mathematical Model and Heuristic Method***

To be able to compare the success of the mathematical model and the heuristic method, same 10-jobs-sized and 15-jobs-sized problems are solved by using both methods. The objective function value results, difference between the objective function values and closeness of the results are stated in Table 31 and Table 32.

Table 31. Comparison between the mathematical model and the heuristic method for 10-jobs-sized examples.

<b>Instance</b>	<b>Objective Function Value of the Mathematical Model</b>	<b>Objective Function Value of the Heuristic Method</b>	<b>Deviation from the Optimal (%)</b>
1	938.32	938.32	0.00
2	678.30	681.70	0.50
3	865.99	882.99	1.93
4	1,060.47	1,569.03	32.41
5	803.94	811.59	0.94
6	753.10	892.87	15.65
7	705.72	705.72	0.00
8	549.84	549.84	0.00
9	870.50	1,456.20	40.22
10	648.56	648.56	0.00

According to Table 31, for 10-jobs-sized examples, it can be stated that the deviation of the objective function values of the heuristic method from that of the mathematical model vary between 0% and 40.22% with an average of 9.17%.

Table 32. Comparison between the mathematical model and the heuristic method for 15-jobs-sized examples.

<b>Instance</b>	<b>Objective Function Value of the Mathematical Model</b>	<b>Objective Function Value of the Heuristic Method</b>	<b>Deviation from the Optimal (%)</b>
1	702.75	742.70	5.38
2	625.66	625.66	0.00
3	474.45	561.75	15.54
4	753.70	753.70	0.00
5	480.45	567.15	15.29

According to Table 32, for 15-jobs-sized examples, it can be stated that the deviation of the objective function values of the heuristic method from that of the mathematical model vary between 0% and 15.54% with an average of 7.24%.

According to Tables 31 and 32, some instances have higher deviation values due to having a loose scheduling. The proposed heuristic gives lower deviation values when the scheduling is not loose. In other words, if the due dates are close, the idle time of the lines are low; so, the scheduling is not loose. The same pattern is also observed in Lee and Pinedo (1997).

## CHAPTER 6: CONCLUSION

In this study, a real-life unrelated parallel machine scheduling problem with sequence-dependent setup times and machine eligibility restriction is studied and it is applied in Euro Gıda. The objective of the study is to minimize weighted sum of the total tardiness, the total earliness, the total idle time of the labelling lines and the total setup time. To solve this problem, a mathematical model and a heuristic method are proposed. The datasets used in the study is directly supplied from the real data of Euro Gıda. Although the topic is common in the literature, the complexity and being a real-life problem differentiates the study from the other studies in the literature.

According to the performed computational experiments it can be stated that the mathematical model is successful and rapid for small-sized problems whereas it is slow to solve large-sized problems. Therefore, a heuristic method which is more appropriate for this real-life problem in terms of application is developed. The developed heuristic method solves the real-life problem 91.79% close to the mathematical model. Hence, the developed heuristic method is acceptable and applicable.

For future research, a decision support system for the automation of the heuristic method can be developed. Currently, the approach includes several manual steps resulting in high processing time. This manual work can also lead to some errors during its application in real life. Moreover, with the aim of idle time minimization, an additional step that includes increasing job batch sizes to make stock for the next customer order, can be included in the method. However, this should be performed by considering the optimization of not making undesired excess stock, using warehouse space inefficiently and stock holding cost.

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