The Mean Residual Life Function of a k-out-of-n Structure at the System Level

Majid Asadi and Ismihan Bayramoglu

***Abstract*—In the study of the reliability of technical systems, k-out-of-n systems play an important role. In the present paper, we consider a k-out-of-n system consisting of n identical components with independent lifetimes having a common distribution function F. Under the condition that, at time t, all the components of the system are working, we propose a new definition for the mean residual life (MRL) function of the system, and obtain several properties of that system.**

***Index Terms*—Characterization, generalized Pareto distributions, increasing failure rate distributions, mean residual lifetime, order statistics, parallel systems.**

ACRONYM[1]

MRL	mean residual life
GPD	generalized Pareto distribution
IFR	increasing failure (hazard) rate
DFR	decreasing failure (hazard) rate

NOTATION

$m(t)$	MRL function
$T_{k:n}$	life time of the $(n-k+1)$-out-of-n system
$H_n^k(t)$	MRL of the $(n-k+1)$-out-of-n system
$\bar{F}(t)$	survival function

I. INTRODUCTION

An important method for improving the reliability of a system is to build redundancy into it. A common structure of redundancy is the k-out-of-n system. A k-out-of-n system consists of n components, and functions iff at least k of the components function. In the case where $k = 1$, the system is a parallel system; and in the case of $k = n$, the system is known as a series system. Let $T_1, \ldots, T_n$ denote the lifetimes of n components connected in a system with a k-out-of-n structure. Assume that T_i are i.i.d. random variables with common continuous distribution function F, and survival function (reliability function) $\bar{F} = 1 - F$. Let also $T_{1:n} \le T_{2:n} \le \ldots \le T_{n:n}$ be the ordered lifetimes of the components. Then $T_{k:n}$, $k = 1, 2 \ldots, n$, represents the lifetime of the $(n-k+1)$-out-of-n system. If we denote the survival function of the system, at time t, by $\bar{S}(t)$, we have

$$\bar{S}(t) = P(T_{k:n} \ge t) = \sum_{i=0}^{k-1} \binom{n}{i} F^i(t) \bar{F}^{n-i}(t), \qquad t > 0.$$

Assuming that each component of the system has survived up to time t, the survival function of $T_i - t$, the residual lifetime of the components, given that $T_i > t$, $i = 1, \ldots, n$ is

$$\bar{F}(x|t) = \frac{\bar{F}(t+x)}{\bar{F}(t)}. \tag{1}$$

This (1) is the corresponding conditional survival function of the components at age t. The mean residual life (MRL) function m of each component is equal to

$$m(t) = E(T_i - t | T_i \ge t) = \int_0^\infty \bar{F}(x|t)dx = \frac{\int_t^\infty \bar{F}(x)dx}{\bar{F}(t)}, \quad i = 1, 2, \ldots, n.$$

The MRL function $m(t)$ plays an important role in reliability and survival analysis. It is well known that the MRL function $m(t)$ characterizes the distribution function F uniquely; see, for example, Kotz & Shanbhag [8]. In particular, when T_i are non-negative, for $t > 0$,

$$\bar{F}(t) = \frac{m(0)}{m(t)} e^{-\int_0^t \frac{1}{m(x)} dx}.$$

As the lifetime of a $(n-k+1)-$out-of-n system is $T_{k:n}$, the MRL function of a the system is equal to

$$E(T_{k:n} - t | T_{k:n} > t) = \frac{\int_t^\infty \bar{F}_{k:n}(x)dx}{\bar{F}_{k:n}(t)},$$

where $\bar{F}_{k:n}$ denotes the survival function of $T_{k:n}$. Recently, Li & Chen [10] studied the aging properties of the residual life length of a k-out-of-n system with independent (not necessarily identical) components given that the $(n-k)$th failure has occurred at time $t \ge 0$. They have described also the behavior of several classes of life distributions in terms of the monotonicity of the residual life given the time of the $(n-k)$th failure. Belzunce

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[1]The singular and plural of an acronym are always spelled the same.

et al. [5] define new aging classes, and provide characterizations for a nonparametric class of life distributions based on aging, and variability orderings of the residual life of k-out-of-n systems. For further results on behaviors of aging properties based upon the residual life of k-out-of-n systems, see Li & Zuo [11]. Langberg *et al.* [9] give characterizations of nonparametric classes of distributions by the stochastic ordering of the residual life of the k-out-of-n system, given that the $(n-k)$th failure has occurred at different times. There is a close relationship between the concept dealt with in the present paper, and residual life discussed in the papers [5], [9], [10], [11].

Recently Bairamov *et al.* [3], under the condition that none of the components of the system fails at time t, defined the MRL function of a parallel system as

$$M^1_{(n)}(t) = E(T_{n:n} - t | T_{1:n} > t)$$

and obtained several properties of it. They have also shown that, under some regularity conditions, the survival function $\bar{F}$ can be represented as

$$\bar{F}(t) = \exp\left\{ -\frac{1}{n} \int_0^t \frac{1 + \frac{d}{dx} M^1_{(n)}(x)}{M^1_{(n)}(x) - M^1_{(n-1)}(x)} dx \right\},$$

where $M^1_{(n-1)}(t)$ is the MRL of a parallel system having $n-1$ components. Asadi & Bairamov [1] have given an extension of the $M^1_{(n)}(t)$ as

$$M^k_n(t) = E(T_{n:n} - t | T_{k:n} > t), \qquad k = 1, 2, \ldots, n.$$

The $M^k_n(t)$ defined here is in fact the MRL of the system under the condition that at least $n-k+1$, $k = 1, 2, \ldots, n$ components of the system are working, and the other components have already failed. Several properties of $M^r_n(t)$ are studied in [1].

The aim of the present paper is to give an extension of the definition of the MRL function proposed by Bairamov *et al.* [3], in the case where the system has a k-out-of-n structure, and explore some of its properties. In Section II, we consider a k-out-of-n system, and assume that at time t all the components are working. Under this assumption, we propose a MRL function for the system, and obtain some of its properties.

II. The Mean Residual Life Function of a k-Out-of-n System

In this section, we consider a $(n-k+1)$-*out-of-n* system, and assume that the components of the system have independent lifetimes with common distribution function F & survival function (reliability function) $\bar{F} = 1 - F$. We assume that at time $t > 0$, all the components of the system are working, i.e. $T_{1:n} > t$. Therefore, the residual life time of the system is $T_{k:n} - t | T_{1:n} > t$. If S denotes the survival function of this conditional random variable, then it can be shown that, for $x > 0$,

$$\begin{aligned} S_k(x|t) &= P(T_{k:n} > x + t | T_{1:n} > t) \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \left(\frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^s. \end{aligned} \quad (2)$$

Given that all the components of the system are working at time t, we define the MRL function of the system as

$$\begin{aligned} H^k_n(t) &= E(T_{k:n} - t | T_{1:n} > t) \\ &= \int_0^\infty S_k(x|t) dx \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \int_0^\infty \left(\frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^s dx \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \int_t^\infty \left(\frac{\bar{F}(x)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x)}{\bar{F}(t)} \right)^s dx. \end{aligned} \quad (3)$$

It should be pointed out here that the MRL $H^k_n(t)$ defined above is in fact the MRL of $T_{k:n}$ at the system level.

Remark 1: It should be pointed out here that the distribution of $T_{k:n} - t | T_{1:n} > t$ is in fact the distribution of the kth order statistics of the sample taken from the conditional distribution $T - t | T > t$.

Remark 2: It can be easily seen, using (3), that the MRL function of the system can be represented as

$$H^k_n(t) = \sum_{s=0}^{k-1} \sum_{j=0}^{s} \binom{n}{s} \binom{s}{j} (-1)^j M_{n-s+j}(t)$$

where

$$M_{n-s+j}(t) = \frac{\int_t^\infty \bar{F}^{n-s+j}(x) dx}{\bar{F}^{n-s+j}(t)}$$

denotes the MRL function of a series system consisting of $n - s + j$ components, $j = 0, \ldots, s$, $s = 0, \ldots, k-1$.

In the following example, we obtain the MRL $H^k_n(t)$ for an important family of distributions:

Example 1: Let X be distributed as GPD with survival function

$$\bar{F}(x) = \left(\frac{b}{ax+b} \right)^{\frac{1}{a}+1}, \qquad x > 0,\ b > 0, -1 < a. \quad (4)$$

The GPD, as a family of distributions, includes the exponential distribution when $a \to 0$, the Pareto distribution for $a > 0$, and the power distribution for $-1 < a < 0$. In this case, we have

$$\begin{aligned} M_{n-s+j}(t) &= \frac{\int_t^\infty (ax+b)^{-(1/a+1)(n-s+j)} dx}{(at+b)^{-(1/a+1)(n-s+j)}} \\ &= \frac{at+b}{(a+1)(n-s+j) - a} \end{aligned}$$

Hence, the MRL function of the system is given by

$$H^k_n(t) = \sum_{s=0}^{k-1} \binom{n}{s} \sum_{j=0}^{s} \binom{s}{j} (-1)^j \frac{at+b}{(n-s+j)(a+1) - a},$$

which is a linear function of t. Note that as $a \to 0$, then $H^k_n(t) \to b \sum_{s=0}^{k-1} (1/n - s)$ i.e., the MRL of a system having independent exponential components does not depend on t.

An important question about the MRL function proposed above is whether it characterizes the underlying distribution F uniquely or not. In the following theorem, we show that, when the distribution function F is absolutely continuous, then it can be uniquely determined by $H_n^r(t)$ & $H_{n-1}^{r-1}(t)$.

Theorem 1: Let the components of the system have a common absolutely continuous distribution function F. Let also f, and $\bar{F}$ denote the density, and survival functions corresponding to F, respectively. Then the survival function $\bar{F}$ can be represented in terms of $H_n^r(t)$ & H_{n-1}^{r-1} as

$$\bar{F}(t) = e^{-\frac{1}{n}\int_0^t \eta(x)dx}, \qquad t > 0, r = 1, \ldots, n,$$

where $\eta(x) = (1 + (dH_n^k(x)/dx))/(H_n^k(x) - H_{n-1}^{k-1}(x))$, and we define $H_{n-1}^0(x) = 0$.

Proof: Let us take $\theta_t(x) = \bar{F}(x)/\bar{F}(t)$, for $x > t$. Then, we have

$$\begin{aligned}H_n^k(t) &= \int_t^\infty \sum_{s=0}^{k-1} \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s \, dx \\ &= \int_t^\infty \left[1 - \sum_{s=k}^{n} \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s\right] dx\end{aligned}$$

On taking the derivative of $H_n^k(t)$ with respect to t, we get the equation shown at the bottom of the page, where $r(t) = f(t)/\bar{F}(t)$ denotes the hazard rate of F. This implies that $r(t) = \eta(t)/n$, and hence the proof is complete. ■

Remark 3: We have

$$\begin{aligned}&H_n^k(t) - H_{n-1}^{k-1}(t) \\ &= \int_t^\infty \binom{n-1}{k-1} (\theta_t(x))^{n-k+1} (1-\theta_t(x))^{k-1} \, dx\end{aligned}$$

which shows that the denominator of $\eta(t)$, defined in Theorem 1, is always non-negative. Moreover, If the distribution function F is assumed to be strictly increasing, then we have for $t < x$, $\theta_t(x) < 1$. In this case, the denominator of $\eta(t)$ is always positive.

Remark 4: If we assume, for a $(n-k+1)$-out-of-n system having n independent components with a common distribution F, that

$$H_n^k(t) = \frac{(at+b)}{(a+1)} \sum_{s=0}^{k-1} \frac{1}{n-s}$$

then

$$H_n^k(t) - H_{n-1}^{k-1}(t) = \frac{at+b}{n(a+1)}$$

which in turn implies, using the above theorem, that the underlying distribution F is GPD with a survival function of the form (4). Hence we conclude, based on the result of Example 1, that the MRL function of a system is a linear function of time, iff the common distribution is GPD of the form (4).

In reliability theory, and in the modeling and study of the properties of a lifetime random variable, two important concepts which have widely been studied are IFR & DFR.

A distribution function F is said to be IFR (DFR) if the corresponding hazard rate $r(t) = f(t)/\bar{F}(t)$ is an increasing (decreasing) function of t, where f denotes the density function of F.

We refer the reader to Barlow & Proschan [4] for more details on these Concepts, and other classes of life distributions.

In following theorem, we prove a result showing that, when the components of the system have a common IFR (DFR) distribution, then $H_n^k(t)$ is decreasing (increasing) in t.

Theorem 2: If the components of the system have a IFR (DFR) distribution function F, then $H_n^k(t)$ is decreasing (increasing) in t.

Proof: Let $r(t)$ denote the hazard rate of F. Then, $r(t)$ is increasing (decreasing) iff for $x, t > 0(\bar{F}(x+t)/\bar{F}(t))$ is decreasing (increasing) in t. From this result, it can be easily seen that the survival function $S_k(x|t)$ defined in (2) is decreasing (increasing) in t. This in turn implies that $H_n^k(t)$ is decreasing (increasing) in t, and the proof is complete. ■

The following example gives an important application of the above theorem

Example 2: Let the components of the system have Weibull distribution with survival function

$$\bar{F}(t) = e^{-\left(\frac{t}{\beta}\right)^\alpha}, \qquad t > 0,\ \alpha > 0,\ \beta > 0.$$

Then the MRL $H_n^k(t)$ of the system is decreasing for $\alpha > 1$, and is increasing for $\alpha < 1$.

The following theorem gives a comparison between two $(n-k+1)$-out-of-n systems based on their MRL.

Theorem 3: Let S_1, and S_2 be two $(n-k+1)$-out-of-n systems with independent components. Let the components of S_1, and S_2 have the distribution function F, and G; survival functions $\bar{F}$, and $\bar{G}$; and hazard rates r_F, and r_G, respectively. If, for $t > 0$, $r_F(t) \le r_G(t)$, then $H_n^{1k}(t) \ge H_n^{2k}(t)$, where

$$\begin{aligned}\frac{dH_n^k(t)}{dt} &= -1 - r(t)\left[\int_t^\infty \sum_{s=k}^{n} \binom{n}{s} \left((n-s) \times (\theta_t(x))^{n-s} (1-\theta_t(x))^s - s\,(\theta_t(x))^{n-s+1} (1-\theta_t(x))^{s-1}\right)\right] dx \\ &= -1 - nr(t)\left[\sum_{s=k}^{n} \int_t^\infty \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s \, dx - \sum_{s=k-1}^{n-1} \int_t^\infty \binom{n-1}{s} (\theta_t(x))^{n-s-1} (1-\theta_t(x))^{s-1} \, dx\right] \\ &= -1 + nr(t)\left[H_n^k(t) - H_{n-1}^{k-1}(t)\right],\end{aligned}$$

H_n^{1k}, and H_n^{2k} denote the mean residual lives of S_1, and S_2, respectively.

Proof: The assumption that $r_F(t) \le r_G(t)$ for $t > 0$ implies that for all $0 < t, x$ we have

$$\frac{\bar{F}(t+x)}{\bar{F}(t)} \ge \frac{\bar{G}(t+x)}{\bar{G}(t)}.$$

From this inequality, it can be seen that $H_n^{1k}(t) \ge H_n^{2k}(t)$. ■

III. Some Characterization Results

In this section, we prove some characterization results on GPD.

Theorem 4: Let $T_1, \ldots, T_n$ be i.i.d. non-negative random variables with absolutely continuous distribution function F. Let $T_{1:n}, \ldots, T_{n:n}$ denote the order statistics corresponding to T_i, $i = 1, \ldots, n$. Assume that $\theta(x) = m(x)/m(0)$ where m denotes the mean residual life function of F. Then for $t > 0$, and $k = 1, \ldots, n$

$$\frac{T_{k:n} - t}{\theta(t)} | T_{1:n} > t \overset{d}{=} T_{k:n} \tag{5}$$

iff F is GPD, where d stands for equality in distribution.

Proof: The proof of the 'if' part of the theorem is straightforward, and hence is omitted. To prove the 'only if' part of the theorem, let (5) hold. Then for $x > 0$ we have

$$\begin{aligned} G_n^k(t) &= P\left(\frac{T_{k:n} - t}{\theta(t)} > x | T_{1:n} > t\right) \\ &= \sum_{s=0}^{k-1} \binom{n}{s} (\phi_t(x))^{n-s} (1 - \phi_t(x))^s \\ &= P(T_{k:n} > x) \end{aligned} \tag{6}$$

where $\phi_t(x) = \bar{F}(\theta(t)x + t)/\bar{F}(t)$. Note that

$$\phi_t'(x) = \phi_t(x) \left(r(t) - (x\theta'(t) + 1) r\left(\theta(t)x + t\right)\right)$$

where $r(t) = f(t)/\bar{F}(t)$ denotes the hazard rate of F at t. From this result, on taking the differentiating of both sides of (6), we get

$$\frac{dG_n^k(t)}{dt} = n\left(r(t) - (x\theta'(t) + 1) r\left(\theta(t)x + t\right)\right) \times \left(G_n^k(t) - G_{n-1}^{k-1}(t)\right) = 0.$$

This implies that, because $G_n^k(t) - G_{n-1}^{k-1}(t) = \binom{n-1}{k-1} (\phi_t(x))^{n-k+1} (1 - \phi_t(x))^{k-1} > 0$,

$$r(t) - (x\theta'(t) + 1)\, r\left(\theta(t)x + t\right) = 0,$$

which in turn implies that

$$\bar{F}\left(\theta(t)x + t\right) = \bar{F}(t)\bar{F}(x), \qquad t > 0,\ x > 0. \tag{7}$$

It is shown by Oakes & Dasu [12] that (7) holds iff F is GPD of the form (4). See also [2] for a proof of Oakes & Dasu's result under some weaker conditions. ■

Remark 5: To prove the 'only if' part of the theorem, one does not actually need to assume that θ is the mean residual life function of F divided by mean. It is enough to assume that $\theta(t)$ is a non-negative differentiable function of t. Then, from (4), we can easily see that $\theta(t)$ is equal to $m(t)/m(0)$.

Theorem 5: Let $T_1, \ldots, T_n$ be i.i.d. random variables with absolutely continuous distribution function F. Then for fixed values of k & n

$$E(T_{k+1:n} - T_{k:n} | T_{1:n} > t) = c$$

iff the underlying distribution F is exponential, where c is a positive constant.

Proof: Note that

$$H_n^{k+1}(t) - H_n^k(t) = \int_t^{\infty} \binom{n}{k} \left(\frac{\bar{F}(x)}{\bar{F}(t)}\right)^{n-k} \left(1 - \frac{\bar{F}(x)}{\bar{F}(t)}\right)^k dx$$

Hence the assumption that $E(T_{k+1:n} - T_{k:n} | T_{1:n} > t) = c$ implies

$$\binom{n}{k} \int_t^{\infty} \left(\bar{F}(x)\right)^{n-k} \left(\bar{F}(t) - \bar{F}(x)\right)^k dx = c\left(\bar{F}(t)\right)^n$$

The kth derivative of both sides of this equation with respect to t, after some simplification, gives

$$\int_t^{\infty} \left(\bar{F}(x)\right)^{n-k} dx = c\left(\bar{F}(t)\right)^{n-k}$$

which in turn implies that F has to be exponential. This completes the proof of the theorem. ■

IV. Regression of Order Statistics

In recent years, regression of order statistics aroused interest of many statisticians. It is well known that the best unbiased predictor for the $X_{k+m:n}$, given $X_{k:m}$, with the respect to the squared-error loss, is E($X_{k+m:n} \mid X_{k:n}$). Ferguson [7] considered the problem of classifying the distributions by linearity of regression of E($X_{k+1:n} \mid X_{k:n}$). The problem to characterize the distributions having a linear regression of E($X_{k+m:n} \mid X_{k:n}$) is completely solved by Dembinska & Wesolowski [6]. In this section, we give a relationship between the mean residual life function of a $(n-k+1)$-out-of-n system, proposed in this paper, and the regression of order statistics.

Let $f_{T_{k:n}|T_{1:n}>x}(y)$ be the pdf of a conditional random variable $T_{k:n}|T_{1:n} > x$, and $f_{T_{k+1:n}|T_{1:n+1}=x}(y)$ be the pdf of the conditional random variable $T_{k+1:n}|T_{1:n+1} = x$.

Theorem 6: $f_{T_{k:n}|T_{1:n}>x}(y) = f_{T_{k+1:n}|T_{1:n+1}=x}(y)$ for any $y > x$.

Proof: If it is known that $T_{1:n} > x$, then it is known that all the n components are alive at time x. So they all have the same i.i.d. conditional distribution. Hence $T_{k:n} \mid T_{1:n} > x$ is the kth order statistic of these i.i.d. random variables. On the other hand, if is known that $T_{1:n+1} = x$, then it is known that all the other n components are alive at time x. Hence $T_{k+1:n+1} \mid T_{1:n+1} = x$ is the kth order statistics of these n other i.i.d. random variables, and therefore it has the same distribution as $T_{k:n} \mid T_{1:n} > x$. Then the result follows. ■

Therefore we can write also: If $H_n^k(t) = E(T_{k:n} - t | T_{1:n} > t)$, and $g_{k+1:n+1}(t) = E\{T_{k+1:n+1} | T_{1:n+1} = t\}$, then $H_n^k(t) = g_{k+1:n+1}(t) - t$.

Corollary 1: The distribution function F can be represented by the regression function $g_{k+1:n+1}(t)$ as

$$\bar{F}(t) = \exp\left\{ -\frac{1}{n} \int_0^t \frac{g'_{k+1:n+1}(x)}{g_{k+1:n+1}(x) - g_{k:n}(x)} dx \right\}.$$

Proof: The proof follows from Theorem 6, and the fact that $H_n^k(t) = g_{k+1:n+1}(t) - t$. ■

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