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Atmospheric oscillations in late-type stars – I. Non-linear response to excitation by acoustic wave energy spectra

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ABSTRACT

The main aim of this paper is to perform first self-consistent numerical computation of the response of stellar atmospheres to the propagation of acoustic waves specified by realistic acoustic wave energy spectra. In the numerical approach, stellar atmospheres are stratified, non-isothermal and plane-parallel, and only their magnetic-free regions are considered. The resulting atmospheric heating is calculated and the time sequence of atmospheric velocities computed by a time-dependent hydrodynamic numerical code is analysed. All computations are done adiabatically and Fourier analysis is used in order to determine the oscillatory properties of stellar atmospheres. The numerical approach is supplemented by an analytical treatment in which three different local acoustic cut-off frequencies are selected from the literature; their values in the stellar atmospheric models are calculated and compared to the numerical results. The theoretical results obtained clearly show that atmospheric oscillations do exist in late-type stars and that their origin and physical properties are similar to those observed in the solar atmosphere. The oscillation frequency of stellar atmospheric oscillations ranges from 7.5 mHz for F5V stars to 16.0 mHz for M0V stars. The relevance of this theoretically predicted range of stellar oscillation in solar-like stars to the recent data obtained by the NASA space mission *Kepler* is discussed.

Key words: shock waves – methods: numerical – stars: atmospheres – stars: late-type – stars: oscillations – stars: solar-type.

1 INTRODUCTION

Observations of the Sun show that the solar photosphere and chromosphere have their own dominant modes of oscillations. The observational evidence for the existence of the photospheric oscillations was found in the early 1960s when the solar 5-min oscillations (also known as *p*-modes) were first identified by Leighton, Noyes & Simon (1962). It has since become clear that the observed pattern of solar global oscillations is caused by the superposition of a large number of acoustic waves trapped beneath the solar surface (Ulrich 1970; Leibacher & Stein 1971; Deubner & Gough 1984; Libbrecht 1988; Christensen-Dalsgaard et al. 1996; De Pontieu, Erdélyi & De Moortel 2005).

The main oscillations of the solar chromosphere are typically identified with 3-min ($\nu = 5.5$ mHz) oscillations. Observations of Ca II H&K, Hα and the Ca II infrared triplet lines show that the 3-min chromospheric oscillations range from 2 to 5 min inside non-magnetic or weak magnetic regions (supergranulation cells); however, in magnetic regions located at the boundaries of supergranules (the magnetic network), the oscillations range from 6 to 15 min (Orrall 1966; Beckers & Schulz 1972; Giovanelli, Harvey & Livingston 1978; Dame 1983, 1984; Deubner 1991; Rutten & Uitenbroek 1991; Lites, Rutten & Kalkofen 1993; Kalkofen 1997; Curdt & Heinzel 1998; Judge, Tarbell & Wilhelm 2001; McAteer et al. 2002, 2003; Tritschler, Schlichenmaier & Bellot Rubio 2005). The origin of these oscillations can be understood as a response of the solar chromosphere to different waves existing in this atmospheric region. The waves can either be trapped if wave cavities exist in the solar chromosphere (e.g. Deubner 1998) or they can be propagating if there are no such wave cavities (e.g. Carlsson & Stein 1998). As originally suggested by Fleck & Schmitz (1991), in the latter case freely propagating acoustic waves excite atmospheric oscillations at the acoustic cut-off frequency; it must also be noted that the same waves may be responsible for the observed chromospheric heating (e.g. Ulmschneider, Musielak & Fawzy 2001; Fawzy et al. 2002a,b,c).

The previous analytical studies (Fleck & Schmitz 1993; Kalkofen et al. 1994; Schmitz & Fleck 1995; Sutmann, Musielak & Ulmschneider 1998) showed that the free atmospheric oscillations are always present, independent of the form of the initial disturbance that caused them, and they decay in time as $t^{-3/2}$ if the

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frequency of the driving acoustic waves is not equal to the acoustic cut-off frequency. On the other hand, the so-called forced atmospheric oscillations do not decay in time if the wave source drives them continuously. For the case when the wave frequency is exactly equal to the acoustic cut-off frequency, the free and forced oscillations are the same and they do not decay in time. Similar results were obtained for the atmospheric oscillations inside magnetic flux tubes embedded in the solar atmosphere. Responses of magnetic flux tubes to propagating longitudinal tube waves and pulses (Musielak & Ulmschneider 2003a) and to the propagating transverse tube waves and pulses (Hasan & Kalkofen 1999; Musielak & Ulmschneider 2003b) were investigated and it was demonstrated that the atmospheric oscillations are also driven by such waves at the cut-off frequency corresponding to each wave.

In their numerical approach, Sutmann & Ulmschneider (1995a) studied the linear response of a plane-parallel solar atmosphere to adiabatic small amplitude wave excitation by using monochromatic waves and acoustic wave energy spectra of different shapes. The non-linear response of the solar atmosphere to large amplitude adiabatic wave excitations was investigated by Sutmann & Ulmschneider (1995b). They found upon exciting the solar atmosphere with monochromatic waves that a critical frequency $\nu_{\rm cr}$ controls the behaviour of the resonance decaying rate: if the atmosphere is excited with frequencies $\nu < \nu_{\rm cr}$, the resonance decays rapidly; on the other hand if the atmosphere is excited with frequencies larger than $\nu_{\rm cr}$, a persistent resonance occurs. Exciting the atmosphere with a wave spectrum resulted in a behaviour in which $\nu > \nu_{\rm cr}$. They used different shapes of the acoustic wave spectra and found that the resonant frequencies do not depend on the wave shape and the generated resonance frequencies are $\nu = 6$–7 mHz at the top of the solar chromosphere.

Based on the above observational and theoretical results obtained for the Sun, one would expect a similar kind of atmospheric oscillations to be present in atmospheres of solar-type stars. There have been reports of detection of stellar *p*-mode oscillations in several main-sequence stars (Brown et al. 1991; Brown & Gilliland 1994; Bouchy & Carrier 2001; Matthews et al. 2004; Claudi et al. 2005; Mosser et al. 2005); however, some of these observations seemed to be inconclusive (e.g. Bedding et al. 2005; Regulo & Roca Cortés 2005). There have also been numerous attempts to detect solar-like oscillations in late-type stars. Stellar atmospheric oscillations associated with stellar flares have been detected (Andrews 1989; Houdebine et al. 1993; Mathioudakis et al. 2003, 2006). Moreover, the NASA space mission *Kepler* has observed solar-like oscillations in more than 2000 solar-like stars (e.g. Chaplin et al. 2011a,b; Huber et al. 2011).

The main purpose of this paper is to perform first self-consistent numerical computation of the excitation of atmospheric oscillations in late-type stars by the realistic acoustic wave energy spectra generated in convective zones of these stars. Stellar atmospheres with temperature gradients resulting from the acoustic wave heating are considered, and the process of filtering of the energy spectra through these atmospheres is investigated numerically. Frequencies of atmospheric oscillations are calculated for different atmospheric heights in stars of different effective temperatures and gravities. We also select from the literature three different local acoustic cut-off frequencies, which were obtained analytically, compute their values in our stellar atmospheric models, and compare them to our numerical results. The purpose of this comparison is to identify the acoustic cut-off that better represents the natural frequency of stellar atmospheres. The relevance of our results to the previous observations reported by Bedding et al. (2007) and to the recent data obtained by the NASA space mission *Kepler* is also discussed.

Our paper is organized as follows: the problem is formulated in Section 2 and the method used to solve the problem is also described there; in Section 3, we present the obtained results and discuss them; our conclusions are given in Section 4.

2 FORMULATION AND METHOD

We now describe our numerical approach and computation of the acoustic wave energy spectra and fluxes generated in stellar convection zones. We also discuss our selection of acoustic cut-off frequencies.

2.1 Initial atmosphere models

Updated stellar parameters for late-type stars are taken from Gray (2005). These values, notably $T_{\rm eff}$ and $\log g$ (see Table 1), serve as the basis for the present study.

The computations start by constructing a grey radiative equilibrium plane-parallel atmosphere for a given effective temperature $T_{\rm eff}$ and gravity g, using a temperature correction method by Cuntz, Rammacher & Ulmschneider (1994). A non-grey radiative equilibrium model atmosphere, which includes non-local thermodynamic equilibrium (NLTE) H^- and the Mg II k line, has been constructed and the resulting atmosphere has a temperature gradient that decreases outward. The bottom of the model is taken at height $z = 0$ km, where the external optical depth is $\tau_{5000} = 1$.

2.2 Boundary conditions and hydrodynamics computation

The models of the acoustically heated chromospheres are calculated by using the numerical code given by Rammacher & Ulmschneider (2003). This code allows computation of one-dimensional time-dependent wave propagation in stellar atmospheres while incorporating the treatment of hydrogen ionization by solving the time-dependent statistical rate equations. The time-dependent hydrodynamic equations are solved for an atmospheric slab by using the method of characteristics; furthermore, the thermodynamical relationships as well as the Rankine–Hugoniot relations across the shocks are also solved in a self-consistent time-dependent manner. We neglect the radiative cooling for the current computations.

The bottom boundary condition is a piston which by specifying the velocity introduces waves into the atmosphere. The actual acoustic wave energy spectra generated by turbulent motions in stellar convection zones are used in our numerical simulations. In

Table 1. The parameters used in the current study for the late-type stars with different effective temperatures $T_{\rm eff}$, gravity $\log g$ and the acoustic wave energy flux $F_{\rm ac}$ computed for each star.

Star	$T_{\rm eff}$ (K)	$\log g$	$F_{\rm ac}$ (erg cm^{-2} s^{-1})
F5V	6528	4.292	5.24E8
G0V	5943	4.360	1.63E8
G5V	5657	4.443	8.50E7
K0V	5282	4.497	3.43E7
K5V	4487	4.651	3.32E6
M0V	3850	4.783	3.35E4

our Lagrangian code, this is the only boundary condition that is necessary.

The top boundary condition is a transmitting boundary. Upon the propagation of the waves in an outer atmosphere with a decreasing density, shocks can form with different strengths which depend on the input mechanical wave energy and wave period. Shocks are treated as discontinuities. The input mechanical energy fluxes were taken from the realistic computation of the acoustic wave generation. The input mechanical energy flux also affects the shock formation heights: it has been found that when increasing the input mechanical energy flux, one has stronger shocks which form at lower heights of the atmospheres (see Fawzy, Ulmschneider & Cuntz 1998; Fawzy 2010).

We also want to point out that our calculations of the generation of acoustic wave energy spectra and fluxes in stellar convection zones (see Section 2.4) are performed independently from our simulations of the wave propagation and shock formation. The main implication of this treatment is that Reynolds stress fluctuations, which are the main sources of acoustic waves in our wave generation calculations, are not included into the hydrodynamic computations (see above). This may lead to some inconsistencies in the lowest photospheric layers of our models, where strong overshooting may be present as implied by some numerical results (e.g. Rogers, Glatzmaier & Jones 2006; Wedemeyer-Bohm, Lagg & Norlund 2009, and references therein). The problem is out of the scope of the present paper but it will be considered elsewhere.

2.3 Fourier analysis

In our code we use a Lagrangian representation in which the grid points move with the gas (mass) elements. For the Fourier analysis of the velocity fluctuations at different heights of the atmosphere we should have Eulerian grid points (at fixed heights) in both space and time, so we interpolate the velocities in both space and time by the method of weighted parabolas (Ulmschneider et al. 1977). For our calculation of the fast Fourier transform, the number of samples N has been fixed to 2048 points, and the sampling time interval (τ) is chosen in such a way to get a frequency resolution of $\Delta\nu = 0.0005$ Hz according to $\Delta\nu = 1/(\tau N)$.

2.4 Generation of acoustic waves in late-type stars

To calculate the acoustic wave energy spectra and fluxes generated at the top of stellar convection zones, we follow the procedure described by Ulmschneider, Theurer & Musielak (1996). The procedure is based on the classical mixing-length theory and allows computation of the acoustic wave energy spectra and fluxes for a broad range of stars with different effective temperatures, surface gravities and metal abundances.

Following the approach developed by Musielak et al. (1994), acoustic waves are generated by turbulent convection, and turbulence is modelled by an extended Kolmogorov spectrum with a modified Gaussian frequency factor. In general, a broad spectrum of acoustic waves is generated by turbulent convection (Lighthill 1952; Stein 1968; Musielak et al. 1994); however, only a small portion of the wave energy is reflected back towards the solar interior (Goldreich & Kumar 1990). The remaining part can propagate freely to the solar chromosphere where it may lead to local heating by forming shocks (Ulmschneider et al. 2001; Fawzy et al. 2002a,b,c).

The calculations of the generation of acoustic wave energy spectra and fluxes performed by Musielak et al. (1994) and Ulmschneider et al. (1996) were based on the standard mixing-length theory used by Bohn (1984), who followed the original formulation presented by Cox & Giuli (1968). In this formulation, the convective velocities and energy fluxes are calculated by specifying the mixing-length parameter $\alpha = l_{\rm mix}/H_p$, where $l_{\rm mix}$ is the mixing length. The value of α used by Cox & Giuli (1968) was such that numerical agreement with the results obtained by Böhm-Vitense (1958), who used $l_{\rm mix} = H_p$, was found. Different versions of the mixing-length theory require different values of α (e.g. Gough 1976, 1977; Marcus, Press & Teukolski 1983; Bohn 1984; Houdek & Gough 1998). It was shown that such values can be obtained by numerical simulations.

Steffen (1993) found in his time-dependent solar numerical convection computations that the maximum convective velocity $v_{\rm CMax} \approx 2.8\,{\rm km\,s^{-1}}$ is reached at $\tau_{5000} \approx 50$ and that this value can be reproduced by the mixing-length theory with a mixing-length parameter of $\alpha \approx 2$. Similarly, recent state-of-the-art solar convection zone simulations by Stein et al. (2009a,b) encompassing the scale of supergranules concluded that the behaviour of solar convection is consistent with a mixing-length parameter of about $\alpha = 1.8$. These are important results as they validate the mixing-length theory and also give approximate values of α required for such validations.

In the previous studies by Musielak et al. (1994) and Ulmschneider et al. (1996), three different values of α were considered, namely, 1.0, 1.5 and 2.0. In this paper, our approach is different as we take $\alpha = 1.8$ for all considered stars. The computed stellar acoustic wave energy fluxes are given in Table 1. We also computed the wave energy fluxes for $\alpha = 2.0$ and obtained fluxes that were 1.5 larger than those calculated for $\alpha = 1.8$. The values of our stellar acoustic wave energy fluxes computed with $\alpha = 2.0$ are in good agreement with those obtained by Houdek & Gough (1998).

2.5 Selection of acoustic cut-off frequencies

We want to connect the results of our numerical calculations with the concept of acoustic cut-off frequency originally introduced by Lamb (1908, 1932) and later applied to the solar atmosphere by Unno & Kato (1964), Whitaker (1963), Deubner & Gough (1984) and Brown, Mihalas & Rhodes (1986) and to other inhomogeneous media (e.g. Musielak, Musielak & Mobashi 2006, and references therein). In order to do so, we select from the literature three different acoustic cut-off frequencies and compute their values at different heights of our stellar atmospheric models. Then, we compare these values to the numerically calculated frequency of stellar atmospheric oscillations. The comparison allows us to determine which acoustic cut-off better represents the natural oscillations of non-isothermal stellar atmospheres.

It has been demonstrated (Lamb 1908, 1932) that acoustic waves with frequencies equal to, or lower than, the acoustic cut-off frequency cannot freely propagate in an isothermal atmosphere but instead they are evanescent. It must be noted that the cut-off frequency is derived from the dispersion relation obtained for acoustic waves propagating in an isothermal atmosphere and, therefore, it is a global quantity (the same in the entire atmosphere). The acoustic cut-off frequency, $\omega_{\rm a}$, is given by the ratio of sound speed, $c_{\rm s}$, to twice the density, H_ρ, or pressure, H_p, scale height, with all these quantities being constant in an isothermal atmosphere, $H_\rho = H_p = H = $ const. Hence, we can write $\omega_{\rm a} = c_{\rm s}/2H = $ const. In reality, it is known that the solar and stellar atmospheres are not isothermal and that the existing temperature gradient leads to $c_{\rm s} = c_{\rm s}(z)$ and

$H_\rho(z) \neq H_p(z)$, which means that the acoustic cut-off frequency becomes a function of atmospheric height.

In the previous studies of the modification of the acoustic cut-off frequency by local temperature gradients existing in the solar convection zone, photosphere and chromosphere, the effects of temperature gradients on the acoustic cut-off frequency have been accounted for by considering the so-called local dispersion relation approach. In this approach, the dispersion relation is assumed to be satisfied locally in the solar atmosphere and local values of the sound speed, $c_s(z)$, where z is an atmospheric height, and the scale height $H_p(z)$ are used to calculate $\omega_a(z)$ whose variations with height (Brown et al. 1986) are given by

$$\Omega_{\mathrm{cut},1}(z) \equiv \omega_a(z) = \frac{c_s(z)}{2H_p(z)}. \qquad (1)$$

We adopt this formula, compute its value in our models and compare it with the results of our numerical simulations.

An important extension of Lamb's acoustic cut-off frequency was reported by Gough (1993), who derived the acoustic cut-off for an arbitrarily stratified atmosphere of a real gas including the perturbation of the gravitational potential and the effect of spherical geometry. A similar treatment was presented earlier by Christensen-Dalsgaard, Cooper & Gough (1983) and applied to the Sun by Christensen-Dalsgaard & Frandsen (1983). A simplified version of the original Gough (1993) derivation, neglecting the perturbation in the gravitational potential (i.e. applying the Cowling approximation) and effects of spherical geometry, was given by Deubner & Gough (1984; see their equation 2.4) who obtained

$$\Omega_{\mathrm{cut},2}(z) = \frac{c_s(z)}{2H_\rho}\left[1 - 2\frac{\mathrm{d}H_\rho(z)}{\mathrm{d}z}\right]^{1/2}, \qquad (2)$$

which is the most commonly used expression for the acoustic cut-off frequency in helio- and asteroseismology. Hence, we also adopt this formula in our computations and compare its value to our numerical results.

A rigorous method to determine local acoustic cut-off frequencies in non-isothermal media was developed by Musielak et al. (2006). The method requires casting the acoustic wave equations in their standard forms and finding the critical frequencies $\Omega_{c,u}$ and $\Omega_{c,p}$ for the wave velocity u and the wave pressure p, respectively. Once the critical frequencies are obtained, the oscillation theorem is used to determine the corresponding turning-point frequencies $\Omega_{t,u}$ and $\Omega_{t,p}$. The resulting acoustic cut-off is a local quantity and it is chosen by taking the larger turning-point frequency. The method has been recently extended to include gravity (see Musielak, Price & Routh 2011).

According to Musielak et al. (2006, 2011) the critical frequencies are defined as

$$\Omega^2_{c,u}(z) = [\omega_a(z) + \omega_s(z)]^2 + 2\omega_a(z)\omega_s(z) - c_s(z)\omega'_s(z) \qquad (3)$$

and

$$\Omega^2_{c,p}(z) = [\omega_a(z) + \omega_s(z)]^2 - 2\omega_a(z)\omega_s(z) + c_s(z)\omega'_s(z), \qquad (4)$$

where $\omega_a(z)$ is given by equation (1) and

$$\omega_s(z) = \frac{\mathrm{d}c_s(z)}{\mathrm{d}z} \qquad (5)$$

and

$$\omega'_s(z) = \frac{\mathrm{d}^2 c_s(z)}{\mathrm{d}z^2}. \qquad (6)$$

Table 2. Values of the local acoustic cut-off frequencies $\nu_{\mathrm{cut},1}$, $\nu_{\mathrm{cut},2}$ and $\nu_{\mathrm{cut},3}$ computed at the middle atmospheric height of the initial stellar atmosphere models.

Star	Height (km)	$\nu_{\mathrm{cut},1}$ (mHz)	$\nu_{\mathrm{cut},2}$ (mHz)	$\nu_{\mathrm{cut},3}$ (mHz)
F5V	2000	4.17	4.10	4.30
G0V	1700	5.01	5.01	5.28
G5V	1400	6.10	6.00	6.53
K0V	1200	7.00	6.78	7.50
K5V	800	10.30	10.07	11.09
M0V	530	14.70	14.69	16.87

The turning-point frequencies (Musielak et al. 2006, 2011) are defined as

$$\Omega^2_{t,u}(z) = \Omega^2_{c,u}(z) + \frac{1}{4}\left[\int^z \frac{\mathrm{d}\tilde{z}}{c_s(\tilde{z})}\right]^{-2} \qquad (7)$$

and

$$\Omega^2_{t,p}(z) = \Omega^2_{c,p}(z) + \frac{1}{4}\left[\int^z \frac{\mathrm{d}\tilde{z}}{c_s(\tilde{z})}\right]^{-2}, \qquad (8)$$

and the local acoustic cut-off frequency $\Omega_{\mathrm{cut}}(z)$ is given by

$$\Omega_{\mathrm{cut},3}(z) = \max[\Omega_{t,u}(z), \Omega_{t,p}(z)]. \qquad (9)$$

The physical meaning of this cut-off frequency is that its value at a given atmospheric height determines the frequency that acoustic waves must have in order to be propagating at this height.

For our comparison to numerical results obtained in Section 3, we select the local acoustic cut-off frequencies $\Omega_{\mathrm{cut},1}$, $\Omega_{\mathrm{cut},2}$ and $\Omega_{\mathrm{cut},3}$, and compute their corresponding values $\nu_{\mathrm{cut},1}$, $\nu_{\mathrm{cut},2}$ and $\nu_{\mathrm{cut},3}$ given in mHz at the middle atmospheric heights of the initial stellar atmosphere (see Table 2). The presented results show that $\nu_{\mathrm{cut},3} > \nu_{\mathrm{cut},1} \geq \nu_{\mathrm{cut},2}$ and that the differences between $\nu_{\mathrm{cut},3}$ and the other two cut-offs become more prominent for cooler stars.

3 RESULTS AND DISCUSSION

3.1 Frequencies of atmospheric oscillations

Since late-type stars have a similar atmospheric structure to the Sun, we expect to find similar types of atmospheric oscillations in the atmospheres of these stars.

Our computations cover stars with effective temperatures ranging from $T_{\mathrm{eff}} = 3850$ K (spectral type: M0V) to 6528 K (spectral type: F5V) and with gravities in the range $\log g = 4.29$–4.78; the detailed parameters are listed in Table 1. The initial atmospheres of these stars extend up to 4000, 3300, 2700, 2300, 1500 and 1060 km for F5V, G0V, G5V, K0V, K5V and M0V, respectively. The time sequences of velocities resulting from the propagating acoustic waves are taken at the middle atmospheric heights of these stars, namely around 2000, 1700, 1400, 1200, 800 and 530 km for F5V, G0V, G5V, K0V, K5V and M0V, respectively.

We introduced adiabatic wave spectra of frequencies covering about 60 mHz and mechanical energy fluxes listed in Table 1 through the plane-parallel atmospheres of these stars at height $z = 0$. The computation time extends up to 2500 s, which is enough for the background atmosphere to reach hydrostatic equilibrium. Similar behaviour to that observed for the solar case is found, namely, the exciting wave spectra components decrease with height and a resonance component is formed at Ω_{num}, which is determined from

Table 3. Values of the numerically determined frequency of the atmospheric oscillations ν_{num} and the local acoustic cut-off frequencies $\nu_{cut,1}$, $\nu_{cut,2}$ and $\nu_{cut,3}$ computed at the middle atmospheric height of the stellar atmosphere models heated by acoustic waves.

Star	Height (km)	ν_{num} (mHz)	$\nu_{cut,1}$ (mHz)	$\nu_{cut,2}$ (mHz)	$\nu_{cut,3}$ (mHz)
F5V	2000	7.5	1.53	5.76	7.97
G0V	1700	8.5	1.58	7.27	10.09
G5V	1400	13.0	2.08	8.86	12.32
K0V	1200	14.0	2.13	10.23	14.28
K5V	800	15.0	3.96	11.32	15.20
M0V	530	16.0	6.72	15.39	18.68

our numerical simulations (see Table 3). All stars with different spectral types show similar behaviour.

As listed in Table 3, the resonance components peak at about 7.5 and 16.0 mHz for the F5V and M0V stars, respectively, showing an increase by a factor of about 2.1 between the two stars. Hot stars show resonance oscillations with longer periods, which decrease with decreasing effective temperature. The velocity fluctuations at the stellar surface and middle atmosphere of a G0V star are shown in Fig. 1, while Fig. 2 shows filtering of the original acoustic wave energy spectra (left-hand panels) and after the waves propagate up to the middle of non-isothermal stellar atmospheres (right-hand panel).

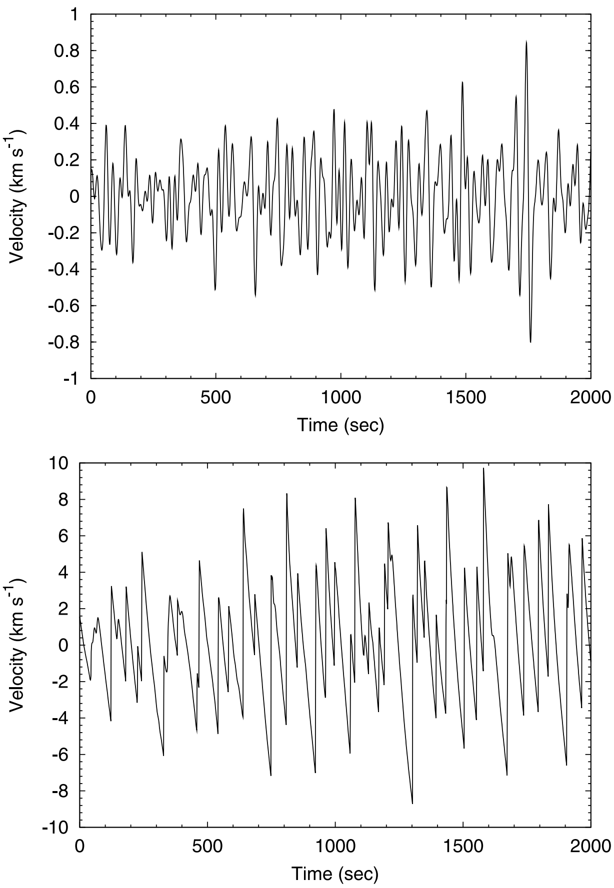

Figure 1. The gas velocity fluctuations at the stellar surface (upper panel) and the middle atmosphere (lower panel) of a G0V-type star.

Finally, Fig. 3 shows the outward temperature increase caused by the formation of strong shocks in atmospheres of stars of different spectral types.

3.2 Comparison of numerical and analytical results

We now compare the numerically determined frequency ν_{num} of atmospheric oscillations to the local acoustic cut-off frequencies $\nu_{cut,1}$ (see equation 9), $\nu_{cut,2}$ (see equation 2) and $\nu_{cut,3}$ (see equation 1), which are computed at the middle atmospheric height of the stellar atmosphere models heated by acoustic waves (see Table 3). The main reason that the comparison is made at the middle of the chromosphere is that according to our numerical simulations the oscillations reach the maximum power at this height (see Fig. 5e).

The results presented in Table 3 show that the values of $\nu_{cut,3}$ are slightly higher than, or comparable to, ν_{num} (except for the M0V star). This means that $\Omega_{cut,3}$ can be identified with the natural frequency of non-isothermal stellar atmospheres of F5V, G0V, G5V, K0V and K5V stars. The values of $\nu_{cut,2}$ are typically lower than ν_{num}, except for the M0V star for which they are comparable. Hence, $\Omega_{cut,2}$ better approximates the natural frequency of the non-isothermal atmosphere of this star. It is also seen that the values of $\nu_{cut,1}$ are significantly different from ν_{num}, which implies that $\Omega_{cut,1}$ cannot be identified as the natural frequency in any of the considered stars.

In Table 3, the three cut-offs are evaluated only at the middle atmospheric height of each star. However, variations of the cut-offs with height are shown for F5V and K0V stars in Fig. 4. The most prominent features of the results presented in these figures are very strong spikes, which occur in the middle and upper stellar atmospheres and are caused by strong shocks formed by the propagating acoustic waves. The cut-offs $\Omega_{cut,2}$ and $\Omega_{cut,3}$ are mainly affected by the shocks because they depend on the density, pressure and temperature gradients in the atmospheres; as expected the effect of the gradients on $\Omega_{cut,1}$ is minor. The spikes make it difficult to determine the actual local values of both $\Omega_{cut,2}$ and $\Omega_{cut,3}$, hence, some values of these two cut-offs given in Table 3 must be taken with caution. Nevertheless, general trends in the behaviour of the three cut-offs can clearly be seen. First of all, the shape of the curves representing $\Omega_{cut,2}$ and $\Omega_{cut,3}$ is similar in most of the atmospheres; however, there are some narrow regions where the observed spikes are significantly different. Secondly, $\Omega_{cut,3}$ seems to be larger than $\Omega_{cut,2}$ throughout the entire atmospheres. Thirdly, the values of all three cut-offs are similar in the lower parts of the atmospheres.

It must be also mentioned that in the regions of the atmospheres where the shocks dominate, there are some atmospheric layers where $\Omega_{cut,2}$ becomes imaginary; the obvious reasons are steep temperature gradients caused by these shocks (see Fig. 3). In our plots of the cut-offs shown in Fig. 4, we removed all layers in which imaginary $\Omega_{cut,2}$ was obtained. Since both $\Omega_{cut,1}$ and $\Omega_{cut,3}$ are always real, the imaginary values of $\Omega_{cut,2}$ are the main disadvantage in using this cut-off frequency in the considered stellar atmospheres. To resolve this problem, stellar atmospheres with the 'averaged temperature variations' with height must be considered, which is out of the scope of this paper.

3.3 Height dependence of atmospheric oscillations

One of the important questions is whether late-type stars show atmospheric oscillations at all heights in stellar atmospheres or not. Detailed analysis of the velocity power spectra at different heights in stellar atmospheres shows that the resonance oscillations are mostly

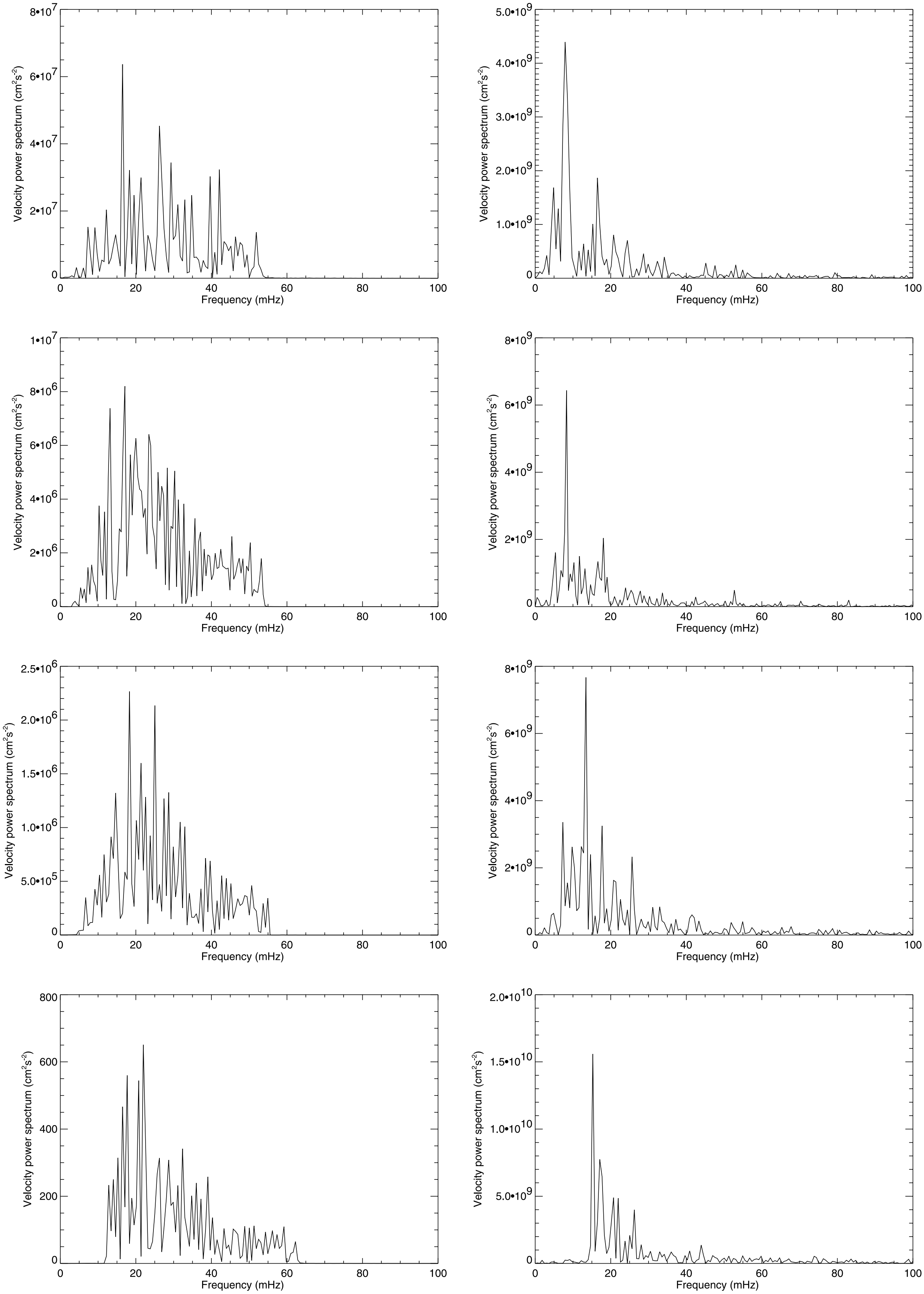

Figure 2. Acoustic wave energy spectra at height $z = 0$ (left-hand column) and in the middle chromosphere (right-hand column). The results are for F5V, G0V, K0V and M0V stars, respectively (from top to bottom).

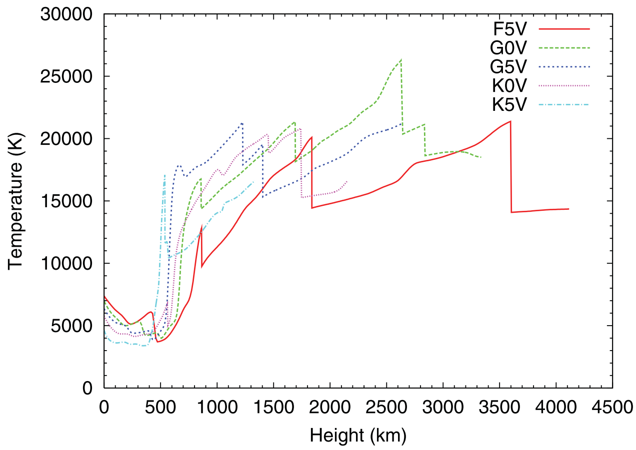

Figure 3. Height dependence of temperature for late-type stars after the hydrostatic equilibrium is reached. The snapshot clearly shows formation of strong shocks which lead to an outward temperature increase.

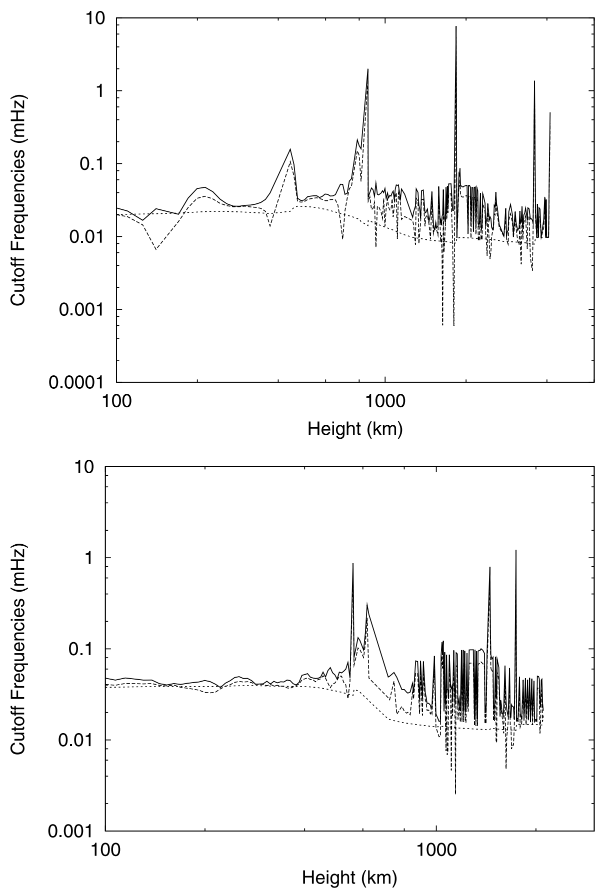

Figure 4. The local acoustic cut-off frequencies $\Omega_{cut,1}$ (dotted line), $\Omega_{cut,2}$ (dashed line) and $\Omega_{cut,3}$ (solid line) are plotted versus atmospheric height for a F5V star (upper panel) and for a K0V star (lower panel). Note that the resulting spikes are caused by strong shocks formed by the propagating acoustic waves in the atmospheres of these stars.

detected in the chromosphere and that they reach their maximum in the middle chromosphere. Fig. 5 presents an example of a G5V star: the atmosphere extends up to a height of 2750 km and the atmospheric oscillation has its maximum at about 1400 km.

3.4 Discussion

In the previous analytical (e.g. Kalkofen et al. 1994; Sutmann et al. 1998) and numerical (e.g. Sutmann & Ulmschneider 1995a) studies of the excitation of oscillations in an isothermal solar atmosphere, it was shown that the atmosphere responds to the propagating acoustic waves with oscillations, which decay in time as $t^{-3/2}$. It was also demonstrated that in order to sustain these oscillations, the acoustic waves must be continuously generated. The main conclusion reached by these authors was that the propagating acoustic waves can be responsible for the excitation of the observed 3-min solar chromospheric oscillations (e.g. McAteer et al. 2002, 2003; Tritschler et al. 2005).

The results presented in this paper are consistent with those previously obtained by the analytical and numerical studies. We significantly extended the previous studies by incorporating realistic temperature gradients in stellar atmospheres and by performing numerical calculations of the excitation of atmospheric oscillations for late-type stars with spectral types ranging from F5V to M0V. Based on our numerical results, we conclude that atmospheric oscillations driven by propagating acoustic waves are common phenomena in late-type stars. We used our results to predict the range of frequencies of these oscillations in stellar atmospheres of solar-like stars.

Among the three local acoustic cut-off frequencies ($\Omega_{cut,1}$, $\Omega_{cut,2}$ and $\Omega_{cut,3}$) selected from the literature, $\Omega_{cut,3}$ compared well to the numerically computed frequencies of stellar atmospheric oscillations for all considered stars except M0V. This means that $\Omega_{cut,3}$ well approximates the natural frequency of the atmospheres of these stars. For the M0V star, a better approximation is given by $\Omega_{cut,2}$. However, $\Omega_{cut,1}$ does not approximate the natural frequency in any of the considered stars. This is rather surprising as the previous studies performed by Christensen-Dalsgaard et al. (1983), Deubner & Gough (1984), Brown et al. (1986), Gough (1993) and others have demonstrated that the cut-off frequencies $\Omega_{cut,1}$ and $\Omega_{cut,2}$ have been successfully used in helio- and asteroseismology. The discrepancy between those previous results and the results presented in this paper can be explained by the fact that while $\Omega_{cut,1}$ and $\Omega_{cut,2}$ well approximate the acoustic cut-off frequency inside stars and in their photospheres (see Table 2), a more refined approach is needed (see Section 2.5) in order to obtain the correct cut-off frequency for stellar chromospheres, especially if strong shocks formed by the propagating acoustic waves are present (see Table 3).

From an observational point of view, it is important to point out that the NASA space mission *Kepler* has observed solar-like oscillations in more than 2000 solar-type stars (e.g. Chaplin et al. 2011a,b; Huber et al. 2011). Even though these oscillations are the visible manifestations of standing acoustic waves in the stellar interiors, the results obtained in this paper may help in establishing connections between the internal and external (chromospheric) stellar oscillations. It must also be mentioned that there is evidence for oscillations with amplitudes of 2.5 times the solar oscillations and a mode lifetime of 2.3 d, which is similar to the solar data, observed in β Hydri (G2IV) by Bedding et al. (2007). The observed frequencies of these oscillations were modelled by Brandao et al. (2011).

The results obtained in this paper are valid only for stars of luminosity class V; however, they can easily be rescaled to the subgiant G2IV, at least, for our initial model as the heated atmosphere model would require the acoustic wave energy flux for this star. After interpolating in Table 2 for the spectral type G2 and taking into account the difference in gravity, which directly affects the acoustic cut-off frequency, the resulting cut-off frequency for β Hydri is approximately 1.5 mHz, which seems to be consistent with the

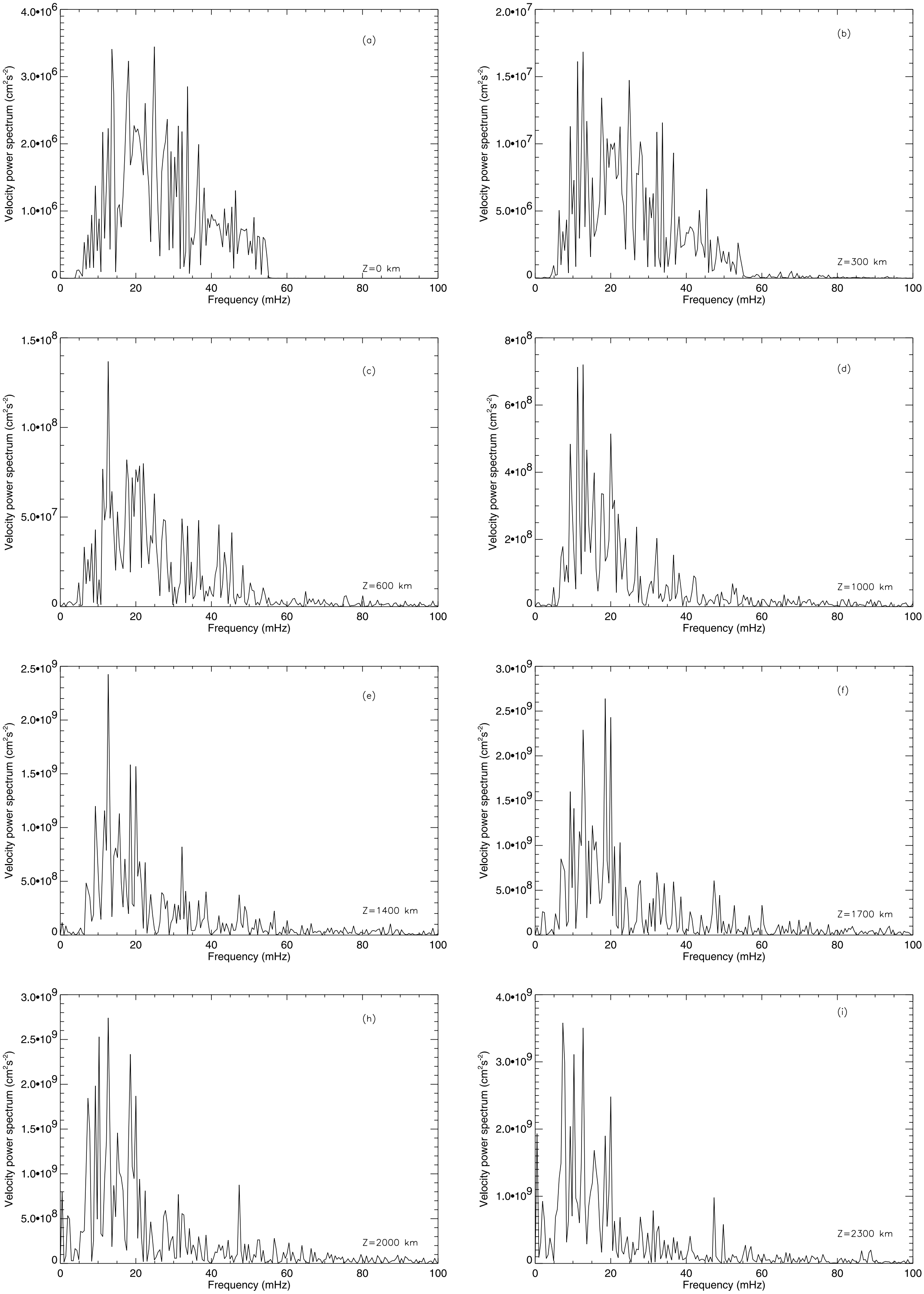

Figure 5. Velocity power spectra as a function of height for a G5V star.

observational data (Bedding et al. 2007) and the theory (Brandao et al. 2011).

The above result is consistent with the now well-established linear relationship between the frequency of the maximum global oscillation amplitudes and the (adiabatic) acoustic cut-off frequency (Stello et al. 2010; Huber et al. 2011). This shows that the theoretical approach presented in this paper is well suited to perform more detailed theoretical studies of this relationship.

4 CONCLUSIONS

We studied numerically the excitation of atmospheric oscillations in late-type stars by the acoustic wave energy spectra generated in the convective zones of these stars. The process of filtering of these energy spectra throughout stellar atmospheres was investigated and frequencies of atmospheric oscillations were calculated for different atmospheric heights in late-type stars. We predicted the existence of atmospheric oscillations in these stars and demonstrated that frequencies of stellar atmospheric oscillations range from 7.5 mHz for F5V stars to 16.0 mHz for M0V stars. This shows that the oscillation frequencies strongly depend on stellar effective temperatures and gravities. It is important to mention here that the increase in the velocity power spectrum in the middle of stellar chromospheres is a good indicator of strong oscillations, and that a shock overtaking is a possible cause of this increase.

From the available literature, we selected three local acoustic cut-off frequencies: $\Omega_{\rm cut,1}$ considered by Brown et al. (1986), $\Omega_{\rm cut,2}$ originally introduced by Deubner & Gough (1984) and Gough (1993), and $\Omega_{\rm cut,3}$ recently proposed by Musielak et al. (2006, 2011). Our comparison of these cut-offs to the frequency of atmospheric oscillations determined from numerical computations shows that $\Omega_{\rm cut,3}$ can be identified with the natural frequency of the non-isothermal atmospheres of F5V, G0V, G5V, K0V and K5V stars. However, $\Omega_{\rm cut,2}$ better approximates the natural frequency of the M0V star. Our results also show that $\Omega_{\rm cut,1}$ does not approximate the natural frequency in any of the considered stars.

Our theoretical results clearly indicate that atmospheric oscillations do exist in late-type stars and that their origin and physical properties are similar to those observed in the solar atmosphere. The observations of β Hydri (G2IV) by Bedding et al. (2007) and solar-like oscillations reported in more than 2000 solar-type stars by the NASA space mission *Kepler* (e.g. Chaplin et al. 2011a,b; Huber et al. 2011) do confirm the existence of atmospheric oscillations in late-type stars; however, they become important in establishing connections between the theory and these very recent observational data.

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