RELIABILITY OF WEIGHTED $k - out - of - n : G$ SYSTEMS WITH m TYPE OF COMPONENTS

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This is to certify that we have read this thesis and that in our opinion it is fullu adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

RELIABILITY OF WEIGHTED $k - out - of - n : G$ SYSTEMS WITH m TYPE OF COMPONENTS

Tezel, Seçil

M.S. in Applied Statistics
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In this thesis, weighted $k - out - of - n : G$ systems consisting of more than one type of component are studied. A general formula for computing the system reliability weighted $k - out - of - n : G$ system which has m types of components is proposed. Reliability of different systems has been computed. Optimal values of the number of components in each group are determined under a minimum required reliability by minimizing the total acquisition cost. Furthermore, numerical examples are included.

Keywords: Order statistics, reliability analysis, weighted $k-out-of-n$ systems.

ÖZET

m TİP BİLEŞENE SAHİP AĞIRLIKLI $n'den - k'l$ı SİSTEMLERİN GÜVENİNİRLİĞİ

Tezel, Seçil

Uygulamalı İstatistik Yüksek Lisans Programı
Tez Danışmanı: Prof. Dr. G. Yazgı Tütüncü
Ocak, 2020

Bu tezde, birden fazla tip bileşenden oluşan ağırlıklı n'den-k'lı sistemler çalışıldı. m tane tip bileşene sahip ağırlıklı n'den-k'lı sistemlerin güvenirlilik analizi için genel bir yöntem önerildi. Farklı yapılara sahip sistemler için güvenirlilik gösterildi. Her gruptaki bileşen sayısının optimum değerleri, toplam edinme maliyetini en aza indirerek gerekli minimum güvenilirlik altında belirlendi. Ayrıca, sayısal örnekler de dahil edilmiştir.

Anahtar Kelimeler: Sıra istatistikleri, güvenirlilik analizi, ağırlıklı $n'den - k'l$ı sistemler.

Dedicated to my family...

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CHAPTER 1

INTRODUCTION

Reliability engineering emerged in the late 1940s and early 1950s. It was first applied to communication and transportation systems. Most of the reliability studies in the past were limited to the analysis of the performance aspects of the systems. In the past half century, more studies have been conducted on reliability analysis. The main purpose of reliability has been to find the best option to improve system reliability. The best way to do this is to increase the reliability of components in the system or to use standby redundancy. There are a number of measures that investigate the performance of a system, such as reliability, availability, and system efficiency. In this paper, we will focus on improving system reliability to analyze the effects of components which have different weights on system reliability. Furthermore, we will investigate the optimum value of the number of components in each type minimizing the total cost which help us to make the best design of the system.

Reliability is the probability of a system to perform at least for a predefined time interval when the stated conditions are satisfied. Hence, in describing the reliability of a given system, it is needed to specify the component's failure process, the system structure which describes how the components are associated, and the state in which the system is described to be failed.

Order statistics and their properties are presented in Chapter 2. The definition of reliability, the type of systems such as Binary and Multi-State, the basic definitons about coherent system and standby systems are given in Chapter 3. $k - out - of - n$ systems, their working principle and the reliability of this system with a cold standby component are pointed out in Chapter 4. Weighted $k - out - of - n$ systems, their working principle and the reliability of this system with two types of components and also adding a cold standby component into the model are presented in Chapter 5. Finally in Chapter 6, the reliability analysis of weighted $k-out-of-n$ systems with m types of components and optimal values of the number of components in each type are investigated under a minimum required reliability of by minimizing the total acquisition cost. Also, some numerical examples are included.

CHAPTER 2

ORDER STATISTICS

A function which assigns each outcome in a sample space S with a real value is called a random variable. X denotes a random variable and x denotes the specific value that a random variable X may take.

If the random variables $X_1, \ldots, X_n$ are arranged in order of magnitude and then written as

$$X_1 \leq \ldots \leq X_n,$$

we call X_i the i^{th} order statistic $(i = 1, \ldots, n)$. In many paper X_i are assumed to be statistically independent and identically distributed.

2.1 Distribution of a Single Order Statistic

We suppose that $X_1, \ldots, X_n$ are n identical and independent variables, each with cumulative distribution function (c.d.f.) $F(x)$. Let $F_{(r)}(x)$ denote the c.d.f. of the r^{th} order statistics X_r where $i = 1, \ldots, n$. Then the c.d.f. of the largest order statistics X_n is given by

$$F_{(n)}(x) = P\{X_{(n)} \leq x\} = P\{\text{all } X_i \leq x\} = F^n(x)$$

Likewise we have

$$F_{(1)}(x) = P\{X_{(1)} \leq x\} = 1 - P\{X_{(1)} > x\}$$

$$= 1 - P\{\text{all } X_i > x\} = 1 - [1 - F(x)]^n$$

The distribution function of r^{th} order statistic is

$$F_{(r)}(x) = P\{X_{(r)} \leq x\} = P\{\text{at least r of the } X_i \text{ are less than or equal to x}\}$$

$$= \sum_{i=r}^{n} \binom{n}{i} F^i(x)[1 - F(x)]^{n-i} \tag{2.1}$$

If F is absolutely continuous with probability density function (pdf) $f(x) = F'(x)$, then the distribution function of r^{th} order statistic is

$$F_{(r)}(x) = \frac{1}{B(r, n-r+1)} \int_0^{F(x)} t^{r-1}(1-t)^{n-r} dt \tag{2.2}$$

$$= \frac{n!}{(r-1)!(n-r)!} \int_0^{F(x)} t^{r-1}(1-t)^{n-r} dt$$

2.2 Joint Distribution of Two or More Order Statistics

The joint density function of $X_{(r)}$ and $X_{(s)}$ $(1 \leq r < s \leq n)$ is denoted by $f_{(r)(s)}(x, y)$. It follows that for $x \leq y$

$$f_{(r)(s)}(x, y) = \frac{n!}{(r-1)!(s-r-1)!(n-s)!} F^{r-1}(x) f(x) \times [F(y) - F(x)]^{s-r-1} f(y)[1 - F(y)]^{n-s} \tag{2.3}$$

Generalizations are now clear. Thus the joint pdf of $X_{(n_1)}, \ldots, X_{(n_k)}$ $(1 \leq n_1 < \ldots < n_k \leq n;\ 1 \leq k \leq n)$ is for $x_1 \leq \ldots \leq x_k$,

$$f_{(n_1)\ldots(n_k)}(x_1, \ldots, x_k) = \frac{n!}{(n_1-1)!(n_2-n_1-1)!\ldots(n-n_k)!}$$
$$\times F^{n_1-1}(x_1) f(x_1)[F(x_2) - F(x_1)]^{n_2-n_1-1} f(x_2) \times \ldots \times [1 - F(x_k)]^{n-n_k} f(x_k)$$

The joint c.d.f. $F_{(r)(s)}(x,y)$ of X_r and X_s may be obtained by integration of (2.3) as well as by a direct argument valid also in the discrete case. We have for $x < y$

$$F_{(r)(s)}(x,y) = P\{\text{at least r } X_i \leq x, \text{ at least s } X_i \leq y\}$$
$$= \sum_{j=s}^{n}\sum_{i=r}^{j} P\{\text{exactly i } X_i \leq x, \text{ exactly j } X_i \leq y\}$$
$$= \sum_{j=s}^{n}\sum_{i=r}^{j} \frac{n!}{i!(j-i)!(n-j)!} F^i(x)[F(y)-F(x)]^{j-i}[1-F(y)]^{n-j}$$

Also for $x \geq y$ the inequality $X_s \leq y$ implies $X_r \leq x$, so that

$$F_{(r)(s)}(x,y) = F_{(s)}(y)$$

Let $X_1,\ldots,X_{n_1}$ be random variables with joint cumulative distribution function (c.d.f.) $F(x_1,x_2,\ldots,x_{n_1})$ univariate marginal c.d.f. $F(x)$ and probability density function (p.d.f.) $f(x)$ and $Y_1,\ldots,Y_{n_2}$ be random variables with joint continuous joint c.d.f. $G(y_1,y_2,\ldots,y_{n_2})$ having univariate marginal $G(x)$ and probability density function (p.d.f.) $g(x)$. We assume that these two collections of random variables independently distributed. Denote by $\{W_1,\ldots,W_n\}$ the $n = n_1 + n_2$ random variables combined from $n_1 Xs$ and $n_2 Ys$.

Denote the distribution function of order statistic $W_{r:n}$ by $H_r(w)$. Then according to the theory of order statistics one can write

$$H_r(w) = P(W_{r:n} \leq w) = P(at\ least\ r\ of\ W_1,\ldots,W_n \leq w)$$
$$= \sum_{i=r}^{n} P(exactly\ i\ of\ W_1,\ldots,W_n \leq w).$$

Bairamov and Parsi (2011) derived the distribution of $W_{r:n}$ as follows

$$H_{(r)}(x) = P(W_{r:n} \leq x)$$

$$= \sum_{i=r}^{n} \sum_{j=max(0,n_1+i-n)}^{min(i,n_1)} \binom{n_1}{j} \binom{n_2}{i-j}$$

$$\times F(x)^j (1-F(x))^{n_1-j} G(x)^{i-j} (1-G(x))^{n_2-i+j}$$

The p.d.f. of $T_{r:n}$ is given by

$$h_{(r)}(x) = \sum_{i=max(0,n_1+r-1-n)}^{min(r-1,n_1-1)} \binom{n_1}{1} \binom{n_1-1}{i} \binom{n_2}{r-1-i}$$

$$\times F(x)^i (1-F(x))^{n_1-1-i} G(x)^{r-1-i} (1-G(x))^{n_2-r+i+1} f(x)$$

$$+ \sum_{i=max(0,n_1+r-n)}^{min(r-1,n_1)} \binom{n_2}{1} \binom{n_1}{i} \binom{n_2-1}{r-1-i}$$

$$\times F(x)^i (1-F(x))^{n_1-i} G(x)^{r-1-i} (1-G(x))^{n_2-r+i} g(x)$$

Let show it by considering the expected value of the largest order statistic c.d.f. $F(x)$.

$$E(X_{(n)}) = \int_{-\infty}^{\infty} nx[F(x)]^{n-1} dF(x)$$

2.3 Residual Lifetimes of Remaining Components from I.I.D. Random Variables

The residual lifelengths of the remaining functioning components following the k^{th} failure in the system is defined by Bairamov and Arnold (2008). Also, they debate the joint distribution of these exchangeable independence of the residual lifelengths.

Consider $(n-k+1)-out-of-n$ system which will function succesfully until k on the components have failed. Accordingly, if $L_1, L_2, \ldots, L_n$ are denoted by the lifetimes of the individual components, then the lifetime of the $(n-k+1)-out-of-n$ system will be represented by k^{th} order statistic $L_{k:n}$. After $(n-k+1)-out-of-n$ system fails, it is often reasonable to stop the system and rescue the working components for using them in other systems.

On the other hand, if the system works without a break the common procedure is to use standby components to prevent the failure of the system. Hence the system will continue to work with the remaining components together with the standby components.

In the modeling of failure times for components of the system with independent and identically distributed (i.i.d.) components, we assume that the failure of one component does not affect the working of the remaining ones. The classical theory of $(n-k+1)-out-of-n$ systems assumes that the n lifetimes $L_1, L_2, \ldots, L_n$ of the components of the system are i.i.d. with common absolutely continuous distribution function F and corresponding density f. Hence, the time of the first failure will be the first order statistic $L_{1:n}$ and the following times between failures can be identified with the difference of $L_{i:n} - L_{i-1,n}$, $i = 2, 3, \ldots, n$.

Even under the usual assumption that the original lifetimes were independent and identically distributed, it appears that the residual lifetimes of the working components will be replaceable, but not independent. They will be conditionally independent given the time of k^{th} failure, but we are not supposing to know the time when system's failure, we just know it has stopped working because k failures have happened. If we put the freed components into a new system, we will need to consider that the lifetime of the components in this new system are i.i.d. but this time they are dependent.

For any $k \in 1, 2, \ldots, n$ we will use the notation $L_1^{(k)}, L_2^{(k)}, \ldots, L_{n-k}^{(k)}$ to denote the residual lifetimes of the $n-k$ components still working at the time of the k^{th} failure. For each k, we may define

$$L_{1:n-k}^{(k)} = min(L_1^{(k)}, L_2^{(k)}, \ldots, L_{n-k}^{(k)})$$

Upon reflection, it is evident that these $L_{1:n-k}^{(k)}$'s simply represent an alternative description of the gaps of the order statistics of the original sample $L_1, L_2, \ldots, L_n$. Therefore,

$$L_{k+1:n} - L_{k:n} = L_{1:n-k}^{(k)}$$

and

$$L_{k-1:n} = L_{1:n} + L_{1:n-1}^{(1)} + L_{1:n-2}^{(2)} + \ldots + L_{1:n-k}^{(k)}$$

If we are given $L_{k:n} = x$, then the conditional distribution of the subsequent order statistics $L_{k+1:n}, \ldots, L_{n:n}$ is the same as the distribution of order statistics of a sample of size $n-k$ from the distribution of F truncated below at x. If we denote by $Y_i^{(k)}$, $i = 1, 2, \ldots, n-k$ the randomly ordered values of $L_{k+1:n}, \ldots, L_{n:n}$, the given $L_{k:n} = x$, these $Y_i^{(k)}$'s will be independent and identically distributed with common survival function $\bar{F}(x+y)/\bar{F}(x)$. The residual lifetimes after k failures, $L_1^{(k)}, \ldots, L_{n-k}^{(k)}$, may be represented as

$$L_i^{(k)} = Y_i^{(k)} - L_{k:n}$$

where $i = 1, 2, \ldots, n-k$.

2.4 Residual Lifetimes of Remaining Components from Mixed Random Variables

In this section, we show the residual lifetimes of the remaining components from two differenf independent sets combined together.

Let $L_1^{(1)}, \ldots, L_{n_1}^{(1)}$ be independent and identically distributed random variables with cumulative distribution function $F_1(t)$. Also, let $L_1^{(2)}, \ldots, L_{n_2}^{(2)}$ be independent and identicaaly distributed random variables with cumulative distributon function $F_2(t)$. Assume that these two collections of random variables are independent of each other and they denote lifetimes of two different types of components.

Let us denote by $L_1, \ldots, L_n$ the $n = n_1 + n_2$ lifetimes of components in a system combined from n_1 of $L^{(1)}$'s and n_2 of $L^{(2)}$'s. The r^{th} order statistics of the combined sample is denoted by $L_{r:n}$, where $r = 1, \ldots, n$.

Let M be a random variable showing the number of failed components of type 1 at the time of r^{th} failure. If we are given $L_{r:n} = x$ and $M = m$, then the conditional distribution of the subsequent order statistics from the first sample $L_{m+1:n_1}^{(1)}, \ldots, L_{n_1:n_1}^{(1)}$ is the same as the distribution of order statistics of a sample of size $n_1 - m$ from the distribution of F truncated below at x.

Similarly, the conditional distribution of the subsequent order statistics from the second sample $L_{r-m+1:n_2}^{(2)}, \ldots, L_{n_2:n_2}^{(2)}$ is the same as the distribution of order statistics of a sample of size $n_2 - r + m$ from the distribution F_2 truncated below at x.

If we denote respectively the remaining lifetimes of the remaining components from the first sample and second sample as $L_i^{(1),r}$, where $i = 1, \ldots, n_1 - m$, and $L_j^{(2),r}$, where $j = 1, \ldots, n_2 - r + m$ and the randomly ordered values as $L_{m+1:n_1}^{(1)}, \ldots, L_{n_1,n_1}^{(1)}$ and $L_{r-m+1:n_2}^{(2)}, \ldots, L_{n_2,n_2}^{(2)}$, then given $L_{r:n} = x$ and $M = m$,

$L_i^{(1),r}$ will be i.i.d. with common survival function $\bar{F}_1(x+t)/\bar{F}_1(x)$ and $L_j^{(2),r}$ will again be independent and identically distributed with common survival function $\bar{F}_2(x+t)/\bar{F}_2(x)$.

CHAPTER 3

SYSTEM RELIABILITY AND TYPES OF SYSTEMS

In this chapter, we investigate system reliabilities and types of system. Why do we need Reliability Modeling? Reliability is generally used in many different fields and in many different disciplines. System reliability analysis is applied for many purposes. One of these objectives is to identify maintenance times that are of great importance for the system to function smoothly and in accordance with its function.

Another objective of the reliability analysis is to analyze the current failure conditions of production lines or systems and to make changes in line placement in the light of this analysis.

3.1 System Reliability

Reliability is the possibility of a system to perform at least for a predefined time interval when the stated conditions are satisfied. As a result, the possibility that a system will work effectively as desired is called "system reliability." The probability of failure is called unreliability. System reliability is an amount of how well a system finishes its plan goal. A system is designed to perform one or more operations.

In order to evaluate the reliability analysis of the system, it is necessary to determine the following situations.

1. The rules which keep the system functioning

2. The relationship between the system components

3.2 The type of systems

In this section, we will investigate the type of systems. Systems are generally divided into two parts such as Binary and Multi-State systems. Then we give special cases such as series, parallel and a mix of both and their reliabilities. Moreover, we define the Coherent Systems and Standby System. In this study, we do not interest these type of systems so we just give explanations of them and this paper do not include further information about them.

3.2.1 Binary Systems

When the system and its components have only two states such as working and failed, then these types of systems are called binary systems. The status of each component or system is a discrete random variable, as it has two values that indicate working and failure conditions.

Let a system consists of n components. If x_i denotes the state of the *ith* component in the system. Then

$$x_i = \begin{cases} 1 & \text{if ith component functions} \\ 0 & \text{if ith component fails} \end{cases}$$

for $i = 1, 2, ..., n$.

Then, vector $x = (x_1, x_2, \ldots, x_n)$ represents the states of all components and it is called the component state vector.

Let ϕ denote the state of system, then it can be defined as

$$\phi(x_1, x_2, ..., x_n) = \begin{cases} 1 & \text{if system functions} \\ 0 & \text{if system fails} \end{cases}$$

The function $\phi(\vec{x})$, which is called the structure function of system, is a function of states of components.

3.2.1.1 Series Configuration

A series system which has n components is the easiest structures. All n components must be performing to provide system operation. In other saying, the system does not work when any one of the n components fails.

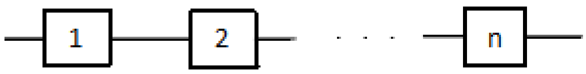

Figure 3.1: A series system configuration

Therefore, the reliability of a series system is

$$\begin{aligned} R &= P(\text{all components operate successfully}) \\ &= P(A_1 \cap A_2 \cap \ldots \cap A_n) \\ &= \prod_{i=1}^{n} P(A_i) \end{aligned}$$

where $P(A_i)$, $1 \leq i \leq n$, denote the probability of event A_i that component i operates succesfully during the intended periof of time. Then the reliability of component i is $p_i = P(A_i)$.

Since component operate independently, the reliability of a series system can be written as

$$R = \prod_{i=1}^{n} p_i$$

A series system structure function as follows

$$\phi(\vec{x}) = \prod_{i=1}^{n} x_i = min(x_1, x_2, \ldots, x_n)$$

The lifetime of a series system is the minimum of component lifetimes.

3.2.1.2 Parallel Configuration

In a parallel structure consisting of n components, the system is successfull if any one of the n components is working.

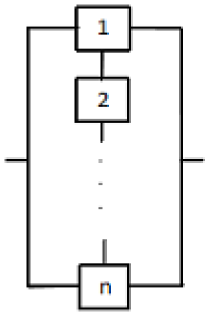

Figure 3.2: A parallel system configuration

Thus, the reliability of a parallel system is

$$R = P(A_1 \cup A_2 \cup \ldots \cup A_n)$$

$$= 1 - P(\overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n})$$

$$= 1 - \prod_{i=1}^{n} P(\overline{A_i})$$

$$= 1 - \prod_{i=1}^{n} [1 - P(A_i)]$$

Since component failures are independent, the reliability of a parallel system can be written as

$$R = 1 - \prod_{i=1}^{n} (1 - p_j)$$

where p_i, $1 \leq i \leq n$, denote the reliability of component x_i.

A parallel system structure function as follows

$$\phi(\vec{x}) = 1 - \prod_{i=1}^{n}(1 - x_i) = max(x_1, x_2, \ldots, x_n)$$

The lifetime of a parallel system is the maximum of component lifetimes.

According to Barlow and Proschan, the structure function of a parallel system can be written as

$$\phi(\vec{x}) = \coprod_{i=1}^{n} x_i$$

3.2.1.3 Series-Parallel Configuration

Believe that a system which consisting of k subsystems related in parallel, with subsystem i consisting of n_i component in series for $i = 1, \ldots, k$. Such a system is called a $series - parallel$ system.

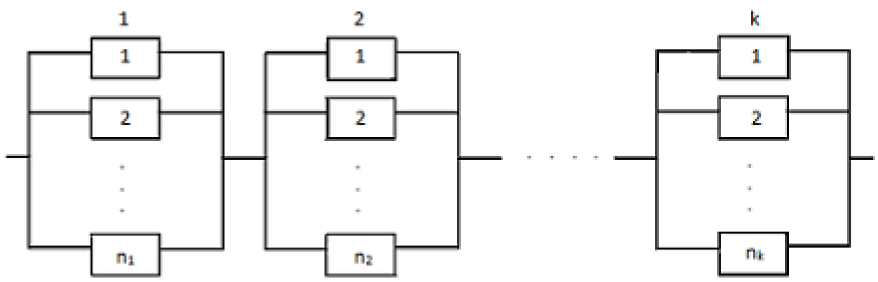

Figure 3.3: A series-parallel system configuration

Let R_i be the reliability of subsystem i and p_{ij} the reliability of component j, $1 \leq j \leq n_i$, in subsystem i. Then

$$R_i = \prod_{j=1}^{n_i} p_{ij},$$

and the system reliability is

$$R = 1 - \prod_{i=1}^{k}(1 - R_i).$$

When we apply two equations together, then

$$R = 1 - \prod_{i=1}^{k}(1 - \prod_{j=1}^{n_i} p_{ij}).$$

The system relibility as follows if the components are identical in each pattern

$$R = 1 - \prod_{i=1}^{k}(1 - p_i^{n_i})$$

where p_i is the reliability of each component in the subsystem, $i = 1, \ldots, k$.

3.2.1.4 Parallel-Series Configuration

Suppose a system consisting of k subsystems in series and subsystem i, $1 \leq i \leq k$, in turn consists of n_i components in parallel. Such a system is called a $parallel - series$ system.

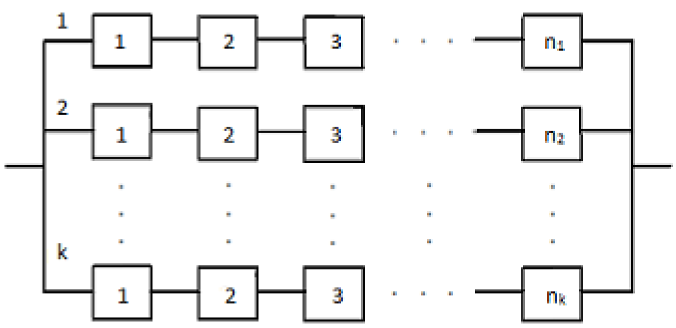

Figure 3.4: A parallel-series system configuration

Let R_i be the reliability of subsystem i and p_{ij} the reliability of component j, $1 \leq j \leq n_i$, in subsystem i. Then

$$R_i = 1 - \prod_{j=1}^{n_i}(1 - p_{ij}),$$

and the system reliability is

$$R = \prod_{i=1}^{k} R_i.$$

When we apply two equations together, then

$$R = \prod_{i=1}^{k}[1 - \prod_{j=1}^{n_i}(1 - p_{ij})].$$

If all components in subsystem i are identical, then p_{ij} is the same for $j = 1, \ldots, n_i$. Let p_{ij} for $i = 1, \ldots, k$ and $j = 1, \ldots, n_i$. Then the system reliability is

$$R = \prod_{i=1}^{k}(1 - q_i^{n_i}),$$

where $q_i = 1 - p_i$ is the failure probability of a component in subsystem i.

3.2.2 Multi-State Systems

All engineering systems are designed to meet the requirements of a specific environment. Some of these systems can perform their tasks at various distinct levels of efficiency, also called performance ratios. Systems with a limited number of performance ratios are called multi-state systems (MSS). The multi-state system consists of elements which may in turn be multi-state. Components are the smallest structural unit of a system. It cannot be further subdivided. This does not mean that an item cannot be made of parts. Reliability analysis is considered an independent unit and means that components are not analyzed for reliability performances.

A binary system is the easiest case of an multi-state system having two individual states (perfect functioning and complete failure).

There are many distinctive situations in which a system should be measured to be a multi-state system:

1. Systems consisting of different units that have a cumulative effect on all system performances should be considered as MSS. The performance ratio of such a system depends on the availability of the units. Different numbers of available units can create different levels of task performance.

   Examples of this situation are $k - out - of - n$ systems. These systems consist of n identical binary units. It can have $n+1$ states depending on the number of units available. System performance is assumed to be proportional to the number of units available. It is assumed that the performance ratios corresponding to more than $k - 1$ units available are acceptable. When the cumulative system performances of different units are different, different combinations of existing units can provide different performance ratios for the entire system. In this case, the number of MSS states increases significantly.

2. The performance ratio of the components that make up a system may vary due to different environmental conditions. Decrease in MSS performance may also be caused by component failures.

   The performance ratings of the components can range from perfect functioning to complete failure. Malfunctions that cause a decrease in component performance are called partial failures. After partial failures, components may not be able to fully perform their tasks.

In this paper, we do not study Multi-State system. So, we do not give any further information about this system.

3.2.3 Coherent Systems

Definition. Let a system consist of n components. The component $i = 1, 2, \ldots, n$ is said to be irrelevant if and only if

$$\phi(1_i, \vec{x}) = \phi(0_i, \vec{x})$$

for any component state vector $\vec{x}$.

In other words, a component is irrelevant if the state of the system does not change by the state of this component. Otherwise, the component is known to be relevant.

Definition. A system with structure function $\phi(\vec{x})$ is coherent if and only if $\phi(\vec{x})$ is nondecreasing in each argument x_i for $1 \leq i \leq n$ and every component is relevant (Kuo and Zuo, 2003).

In other words a system is coherent if the following conditions are satisfied.

1. $\phi(\vec{0}) = 0$
   All components are failed, then the system is failed.
2. $\phi(\vec{1}) = 1$
   All components work, then the system works.
3. If $x < y$, then $\phi(\vec{x}) \leq \phi(\vec{y})$
   Development of any component does not decrease the productivity of the system.
4. For every component i, there exists a component state vector such that the state of component i commands the state of the system.

The reliability of a coherent system consisting of n components can be defined as the probability that system functions

$$R = P(\phi(\vec{x}) = 1).$$

Reliability of the ith component of this system is defined as the probability that ith component functions

$$P(x_i = 1) = p_i$$

for $i = 1, 2, ..., n$.

3.2.4 Standby Systems

A system which has parallel with n components operates in one of the n component works. The rest of the components work simultaneously on the system. Instead of $n-1$ redundant components, another redundancy, called standby redundancy, can be used. They are used to improve the reliability of the system. In this case, the active components may be replaced or additional components may be added as replacement components. A detection and switching mechanism is used to control the operation of the active components. When an active component fails, a backup component is run.

There are different types of standby: cold standby, warm standby and hot standby.

1. Hot standby components have the same failure rate as the active component. Thus, they are also called active redundant components.
2. A cold standby component has a zero failure rate. In other words when it is standby, it does not fail.
3. Warm standby components have a failure rate that is between the failure rates of a cold standby components and a hot standby components

When the sensing and switching mechanism is perfect, that is, the standby component is activated as soon as the active component fails. When the last component fails in active operation, the system fails.

To understand the concept we assume that a system has two components. First one is an active component, the other one is a standby component. When the life of an active component is over, a standby component is switched into operation. The system's life is over when the life of a standby component is failed.

The system's lifetime is equal to the sum of the lifes of an active component and a cold standby component.

The lifetime of the system is denoted by L, the lifetime of the active component is denoted by L_1 and the lifetime of the standby component is denoted by L_2. Then,

$$L = L_1 + L_2$$

There are two cases for the system to work until the time t. The first one is, an active component works until time t. The second one is an active component fails at time x $(0 \leq x \leq t)$ and a cold standby component is put on operation and works between time x and time t. So, the reliability of the system is

$$\begin{aligned} P(L > t) &= P(L_1 > t) + \int_0^t P(L_2 > (t - x)) f(x) dx \\ &= 1 - F(t) + \int_0^t (1 - G(t - x)) f(x) dx \end{aligned}$$

where $F(t)$ denote the failure rate distribution of the active component, also $G(t)$ denote the failure rate distribution of the standby component. The probability density function of the active component with lifetime L_1 is denoted by $f(t)$.

CHAPTER 4

$K-OUT-OF-N$ SYSTEMS

In this chapter, we investigate the reliability of $k-out-of-n$ and consecutive $k-out-of-n$ systems. There are many studies on the literature about reliability analysis. Many papers include finding survival function, failure rate function and mean residual life function. The reliability analysis of $k-out-of-n$ systems have been discussed in the literature (see, for example, Zhang and Lan (1998), Cheng and Zhang (2001), Sheu and Chang (2001), Navarro and Hernandez (2008), Navarro and Rychlik (2010), Zhang and Yang (2010), Raqab and Rychlik (2011), Eryılmaz (2011)).

Some papers on the reliability analysis of consecutive $k-out-of-n$ systems are Chiang and Niu (1981), Hwang (1982), Kuo et al. (1994), Chao et al.(1994), Kuo et al. (1990), Zuo (1993), Zhang and Lan (1998), Cheng and Zhang (2001), Sheu and Chang (2001).

4.1 k-out-of-n systems

The defitinions of $k-out-of-n:G$ and $k-out-of-n:F$ systems as follows

Definition. An n component system that works (or is "good") if and only if at least k of the n components work (or are good) is called a $k-out-of-n:G$ system (Kuo and Zuo, 2003).

Definition. An n component system that fails if and only if at least k of the n components fail is a called a $k-out-of-n:F$ system (Kuo and Zuo, 2003).

According to these two definitions, a $k-out-of-n:G$ system is equal to an $(n-k+1)-out-of-n:F$ system.

Both parallel and series systems are especial cases of the $k-out-of-n$

system. A series system is equal to a $1-out-of-n:F$ system and to and $n-out-of-n:G$ system while a parallel system is equal to an $n-out-of-n:F$ system and to a $1-out-of-n:G$ system.

The reliability of $k-out-of-n:G$ system as follows

$$R=\sum_{i=k}^{n}\binom{n}{i}p^i(1-p)^{n-i}$$

where p denotes the reliability of each component.

When all of the components are independent and identical, the reliability of a general $k-out-of-n$ system can be written as

$$R=\sum_{i=k}^{n}p^i(1-p)^{n-i}$$

where p is the reliability of the component.

Then, the structure function of $k-out-of-n:F$ system is

$$\phi(\vec{x})=\begin{cases}1 & \sum_{i=1}^{n}x_i>n-k\\ 0 & \sum_{i=1}^{n}x_i\leq n-k\end{cases}$$

4.2 Consecutive k-out-of-n Systems

Definition. Suppose that n components are linearly(circularly) connected if and only if at least k consecutive components fail. The linear(circular) consecutive $k-out-of-n:F$ system called this types of structure (Kuo and Zuo, 2003).

Definition. For the system to work if and only if at least k consecutive components work, the system structure is called the linear consecutive $k-out-of-n:G$ system (Kuo and Zuo, 2003).

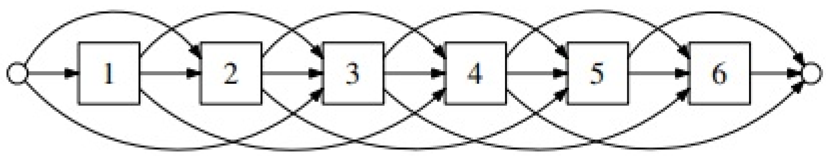

Figure 4.1: A linear consecutive $3-out-of-6:F$ system (Kuo and Zuo, 2003)

The consecutive $k-out-of-n$ system contain the series and the parallel system as especial cases. For example, when $k=1$, the linear consecutive $k-out-of-n:F$ system changes to the series system.

When $k=n$, the linear consecutive $k-out-of-n:F$ system becomes the parallel system.

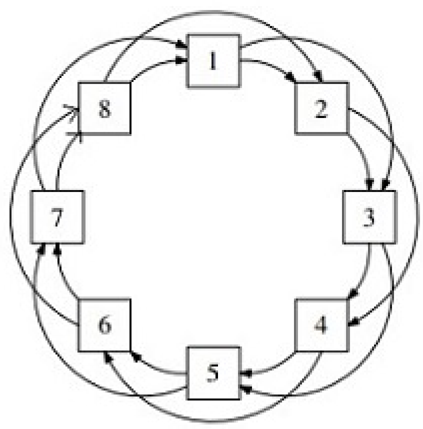

Figure 4.2: A circular consecutive $2-out-of-8:F$ system (Kuo and Zuo, 2003)

The reliability of consecutive $k-out-of-n:F$ system is shown as the following

$$R(k,n)=\sum_{j=0}^{n} N(j,k,n)p^{n-j}(1-p)^{j}$$

where $N(j,k,n)$ denotes the number of ways to arrange j failed component in a line such as no k or more failed components consecutively. Also, p denotes component reliability in a system with independent and identically distributed components.

Then, the structure function of consecutive $k-out-of-n:F$ system is

$$\phi(\vec{x}) = \prod_{i=1}^{n-k+1} (1 - \prod_{j=1}^{i+k-1} (1 - x_j))$$

4.3 $k - out - of - n$ Systems with a Cold Standby Component

The reliability analysis of systems is associated to the order statistics related with components' lifetimes. If $X_1, \ldots, X_n$ mean the lifetimes of components, then the lifetime of $k - out - of - n : G$ system without a standby component relating to the order statistic $X_{n-k+1:n}$, the $(n - k + 1)$th smallest among $X_1, \ldots, X_n$. Therefore, the distribution theory of order statistics has a significant role in the analysis of the systems.

In a $k - out - of - n : G$ system with a single cold standby component, when the system fails, that is at the time when the $(n - k + 1)$th failure occur, a standby component with lifetime L_c is put into operation. Then, the lifetime of $k - out - of - n : G$ system with single cold standby component can be represented as

$$L = X_{n-k+1:n} + min(X_{n-k+2:n} - X_{n-k+1:n}, L_c)$$

for $k = 2, \ldots, n$ and $L = X_{n:n} + L_c$ for $k = 1$, where $X_{1:n} < \ldots < X_{n:n}$ are ordered lifetimes of active components.

Standby component is related to the performance of a system when it is activated. So, the system's performance is influenced by the performance of the standby component only after the random time $X_{n-k+1:n}$.

First of all, we define the mean residual life function which is following

$$\varphi_1(t) = E(L - t \mid T > t) \tag{4.1}$$

$$\varphi_2(t) = E(L - t \mid X_{n-k+1:n} > t) \tag{4.2}$$

$$\varphi_3(t) = E(L - t \mid X_{1:n} > t) \tag{4.3}$$

for $t \geq 0$.

The usual mean residual life function is defined as equation 4.1 and it gives no information if the standby component is in operation at time t. Because if $\{L > t\}$, then we either have $\{X_{n-k+1:n} > t\}$ or $\{X_{n-k+1:n} \leq t,\ X_{n-k+1:n} + min(X_{n-k+2:n} - X_{n-k+1:n}, L_c) > t\}$ which implies that at time t the standby component may be or may not be active.

Oppositely, in the function defined by equation 4.2, it is known that the system functions with active components at time t and standby component is invest in operation after time t.

The function defined by equation 4.3 represents the mean residual life function under the condition that all components work at time t.

Let $X_1, \ldots, X_n$ be independent and identically distributed with common continuous c.d.f. F. For an independent standby component with c.d.f. $G(x) = P\{L_c \leq x\}$.

Then, the reliability of the system is given

$$R = E(L - t \mid X_{1:n} > t) = \sum_{m=0}^{n-k} \sum_{j=0}^{m} \binom{n}{m} \binom{m}{j} (-1)^j M_{n-m+j}(t)$$

$$+ \frac{1}{B(n-k+1,k)} \frac{1}{\bar{F}^n(t)} \int_0^\infty \int_t^\infty \bar{F}^{k-1}(s+x)$$

$$\times (F(x) - F(t))^{n-k} \bar{G}(s) dF(x) ds$$

where $B(a,b)$ denotes the Beta function defined by $B(a,b) = \frac{r(a)r(b)}{r(a+b)}$ for $a, b > 0$, M is a random variable showing the number of failed components at the time of rth failure.

CHAPTER 5

WEIGHTED $K - OUT - OF - N$ SYSTEMS

5.1 Notations

We will use the following notations throughout this chapter:

Let X_i means the state of the ith component, where $X_i = 1$ if the component functions, and $X_i = 0$ if the component fails.

$n:$ the number of the components in the system

$C = 1, 2, ..., n:$ the index set of all components

C_1 :the set of components with weight w_1

C_2 :the set of components with weight w_2

n_1 :the number of components in C_1

n_2 :the number of components in C_2

w_c :the weight of the cold standby component

$k:$ minimum required weight/capacity for the functioning of the system

F_1 :the common lifetime distribution of the components in C_1

$\overline{F}_1$:survival function of the components in C_1

F_2 :the common lifetime distribution of the components in C_2

$\overline{F}_2$:survival function of the components in C_2

$L_{r:n}$:rth order statistics $r \in C$

M :random variable showing the number of failed components from C_1 at the time of rth failure

$L_k^{n_1,n_2} \mid L_{r:n}, M$:remaining lifetime of the system given $L_{r:n}$ and M

R: the reliability of the system at time t

In reliability problems, generally all components of the system are built to have equal weight in terms of their contribution to system performance. In real life engineering systems, the contribution of the components of the system to the performance of the system may be different. In many real-life problems, each of the components that make up a system makes different contributions to the system. Therefore, the operation of such a system will depend on the contribution of each individual component to the operation of the system, rather than just the components that make up the system. This contribution / performance / benefit and so on. values will be called as weight in the study. It is clear that the component that contributes more to system performance will have a greater weight.

In this chapter, we studied general weighted $k-out-of-n$ systems and their reliabilities. Also, we investigated another studies about of that systems consist of different types of components.

In order to investigate such cases Wu and Chen (1994) studied a more general model than $k-out-of-n:G$ system and called it weighted $k-out-of-n:G$ system. In this system, the components may have different integer weights and the system functions if the total weight of functioning components is at least threshold k. The reliability of weighted $k-out-of-n:G$ systems can be computed by using recursive formula Chen and Yang (2005), Eryılmaz and Tütüncü (2009), Higashiyama (2001). There are many studies that have been proposed about the dynamic analysis of weighted $k-out-of-n:G$ systems such as Eryılmaz (2011), Samaniego and Shaked (2007). Because of the structure of weighted $k-out-of-n:G$ systems, the components have different reliability which makes computation more harder. Other studies that have been done in this area are Cui and Xie (2005), Eryilmaz (2012), Kochar and Xu (2010), Navarro, Samaniego and Balakrishnan (2011). Eryılmaz and Sarıkaya (2014) studied a special type of weighted $k-out-of-n:G$ system which has only two types of components having different weights and reliabilities.

5.2 *Weighted* $k-out-of-n$ *Systems*

Definition. A weighted k-out-of-n:G system, which has n components, each with its own positive integer weight, works if the total weight of working components is at least k.

Definition. A weighted k-out-of-n:F system, which has n components, each with its own positive integer weight, fails if the total weight of working components is less than k.

The reliability of a weighted $k-out-of-n:G$ system as follows

$$R(t) = P\left(\sum_{i=1}^{n} w_i X_i(t) \geq k\right)$$

where $i = 1, 2, ..., n$.

5.3 *Weighted* $k-out-of-n:G$ *Systems with Two Types of Components*

First of all, we show that the assumptions for the system under study.

The model statements are enumerated below:

1. The system consisting of n independent binary components.
2. The components of the system are categorized into two groups regarding to their weight/capacity.
3. The system is assumed to work if the total weight of all functioning components exceed a prespecified threshold.

Eryılmaz and Sarıkaya (2014) derived the following equation. Then, the reliability of weighted $k - out - of - n : G$ systems with two types of components can be denoted by the probability as follows

$$R = P(L_k^{n_1,n_2} > t) = \sum_{\substack{w_1 i + w_2 j \geq k \\ 0 \leq i \leq n_1, 0 \leq j \leq n_2}} \binom{n_1}{i} \overline{F}_1(t)^i F_1(t)^{n_1 - i} \binom{n_2}{j} \overline{F}_2(t)^j F_2(t)^{n_2 - j} \tag{5.1}$$

5.4 Weighted $k - out - of - n : G$ Systems with Two Types of Components and a Cold Standby Component

We first give the main statements for the system over study.

The model statements are enumerated below:

1. The system consisting of n independent binary state components and an independent binary state cold standby component.
2. The components are categorized into two groups regarding to their weight/capacity. In addition, there exists a single cold standby component with distinct capacity/weight and reliability.
3. The system works if the total weight of the operating components exceeds a prespecified threshold.

Franko, Tütüncü and Eryılmaz (2017) derived the following equation. Then, the reliability of the system can be denoted by the probability as follows

$$R = P(L_{k-w_c}^{n_1-m,n_2-r+m} > t - x \mid L_{r:n} = x, M = m)$$

$$= \sum_{\substack{w_1 i + w_2 j \geq k \\ 0 \leq i \leq n_1 - m, 0 \leq j \leq n_2 - r + m}} \binom{n_1 - m}{i} \left(\frac{\overline{F}_1(t)}{\overline{F}_1(x)} \right)^i \left(1 - \frac{\overline{F}_1(t)}{\overline{F}_1(x)} \right)^{n_1 - m - i}$$

$$\times \binom{n_2 - r + m}{j} \left(\frac{\overline{F}_2(t)}{\overline{F}_2(x)} \right)^j \left(1 - \frac{\overline{F}_2(t)}{\overline{F}_2(x)} \right)^{n_2 - r + m - j}$$

CHAPTER 6

WEIGHTED $K - OUT - OF - N : G$ SYSTEMS WITH M TYPES OF COMPONENTS

In this chapter, weighted $k - out - of - n : G$ systems consisting of m type of components have been considered. We first give our assumptions and notations to create the reliability model. Then we propose a new model for reliability of these systems. Furthermore, we investigate the optimal value of the number of components. Finally, we give numerical examples to understand this new model.

6.1 Assumptions

Main assumptions which are used for modeling a weighted $k-out-of-n : G$ system consisting of m types of components are given. The model statements are listed below:

1. The system consisting of n independent binary components.
2. The components of the system are categorized into m groups regarding to their weight/capacity.
3. The system is assumed to work if the total weight of all functioning components exceeds a prespecified threshold.

6.2 Notations

We will use the following notations:

$n :$ the number of the components in the system

$C = 1, 2, ..., n :$ the index set of all components

C_j :the set of components with weight w_j, $j = 1, 2, ..., m$

n_j :the number of components in C_j, $j = 1, 2, ..., m$

k : minimum required weight/capacity for the functioning of the system

p_j :the reliability of the components in C_j, $j = 1, 2, ..., m$

$\overline{p}_j$:the unreliability of the components in C_j, $j = 1, 2, ..., m$

6.3 Reliability of Weighted $k - out - of - n : G$ Systems with m Types of Components

Eryılmaz and Sarıkaya (2014) derived the reliability of weighted $k - out - of - n : G$ system with two types of components. We generalize this equation 5.1, then we propose the following theorem.

Theorem 6.1 *Let weighted $k - out - of - n$ systems consisting of m types of components. Then the reliability of this types of systems is given following*

$$R = P(L_k^{n_1,n_2,...,n_m} > t) = \sum_{\substack{w_1 i_1 + w_2 i_2 + ... + w_m i_m \geq k \\ 0 \leq i_1 \leq n_1, 0 \leq i_2 \leq n_2, ..., 0 \leq i_m \leq n_m}} \binom{n_1}{i_1} p_1(t)^{i_1} \overline{p}_1(t)^{n_1 - i_1}$$

$$\times \ldots \times \binom{n_m}{i_m} p_m(t)^{i_m} \overline{p}_m(t)^{n_m - i_m}$$

Many real engineering systems have different types of components. So, this theorem is more important to apply on real engineering problems.

Proof.

$$| C_1 |= n_1, | C_2 |= n_2, \ldots, | C_m |= n_m$$

$$C = C_1 \cup C_2 \cup \ldots \cup C_m$$

$$C_1 \cap C_2 \cap \ldots \cap C_m = \varnothing$$

Let X_i means the state of i^{th} component, where $X_i = 1$ if the component functions and $X_i = 0$ if the component fails. Then, the reliability of the system can be shown as by the probability

$$R = P(w_1 Y_1 + w_2 Y_2 + \ldots + w_m Y_m \geq k)$$

where $Y_j = \sum\limits_{i \in C_j} X_i$, $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$.

By reason of the components are independent and reliability of the components is p_j, $j = 1, 2, \ldots, m$.

$$P(X_i = 1) = p_j, i \in C_j$$

The random variables $Y_1, Y_2, \ldots, Y_m$ follow multinomial distributions with parameters $(n_1, p_1), \ldots, (n_m, p_m)$.

$$R = P(w_1 \sum_{i \in C_1} X_i + w_2 \sum_{i \in C_2} X_i + \ldots + w_m \sum_{i \in C_m} X_i \geq k)$$

$$R = \sum_{\substack{w_1 i_1 + \ldots + w_m i_m \geq k \\ 0 \leq i_1 \leq n_1, \ldots, 0 \leq i_m \leq n_m}} P(Y_1 = i_1) P(Y_2 = i_2) \ldots P(Y_m = i_m)$$

$$= \sum_{\substack{w_1 i_1 + \ldots + w_m i_m \geq k \\ 0 \leq i_1 \leq n_1, \ldots, 0 \leq i_m \leq n_m}} \binom{n_1}{i_1} p_1(t)^{i_1} \overline{p}_1(t)^{n_1 - i_1} \times \ldots \times \binom{n_m}{i_m} p_m(t)^{i_m} \overline{p}_m(t)^{n_m - i_m}$$

□

Example 6.1. In the following table, we compute the reliability of the weighted-k-out-of-n:G system when $m = 2$.

Table 6.1: Reliability of weighted $k - out - of - n : G$ system where $m = 2$.

n_1	n_2	w_1	w_2	p_1	p_2	k	R
3	7	1	2	0.95	0.97	10	0.9999
5	5	1	2	0.95	0.97	10	0.9995
3	7	1	2	0.95	0.97	15	0.9578
5	5	1	2	0.95	0.97	15	0.6644
8	7	1	2	0.95	0.97	15	0.9999
10	5	1	2	0.95	0.97	15	0.9989
8	7	1	2	0.95	0.97	20	0.9193
10	5	1	2	0.95	0.97	20	0.5141

Example 6.2. In the following table, we compute the reliability of the weighted-k-out-of-n:G system when $m = 3$.

Table 6.2: Reliability of weighted $k - out - of - n : G$ system where $m = 3$.

n_1	n_2	n_3	w_1	w_2	w_3	p_1	p_2	p_3	k	R
3	2	5	1	2	3	0.95	0.97	0.93	15	0.9942
2	5	3	1	2	3	0.95	0.97	0.93	15	0.9949
3	2	5	1	2	3	0.95	0.97	0.93	20	0.6892
2	5	3	1	2	3	0.95	0.97	0.93	20	0.6890
5	7	3	1	2	3	0.95	0.97	0.93	20	0.9986
6	5	4	1	2	3	0.95	0.97	0.93	20	0.9974
5	7	3	1	2	3	0.95	0.97	0.93	25	0.9009
6	5	4	1	2	3	0.95	0.97	0.93	25	0.8805

6.4 Optimal Value of The Number of Components in Each Type

In this section, we investigate the optimal value of the number of components in each type minimizing the total cost of the system subject to a minimum system reliability requirement.

Let c_i denote the acquisition cost of one element in the i^{th} group. If r_0 is the minimum required reliability for the system, then the problem can be formulated as

$$\text{Minimize}: \; n_1 c_1 + n_2 c_2 + \ldots + n_m c_m$$

$$\text{Subject to}: \; R(n_1, n_2, \ldots, n_m) \geq r_0$$

$$n_1 + n_2 + \ldots + n_m = n$$

$$n_i \geq 0, i = 1, 2, \ldots, m$$

Example 6.3. In the following table, we compute the reliability of the weighted-k-out-of-n:G system when $m = 2$ and we show the total costs of each component.

Let $n = 6$, $c_1 = 2$, $c_2 = 3$ and the minimum required reliability is $r_0 = 0.95$.

Table 6.3: The total cost of each component where $m = 2$

n_1	n_2	w_1	w_2	p_1	p_2	k	R	Total Cost
0	6	1	2	0.95	0.97	6	0.9999	18
1	5	1	2	0.95	0.97	6	0.9997	17
2	4	1	2	0.95	0.97	6	0.9993	16
3	3	1	2	0.95	0.97	6	0.9967	15
4	2	1	2	0.95	0.97	6	0.9878	14
5	1	1	2	0.95	0.97	6	0.9480	13
6	0	1	2	0.95	0.97	6	0.7350	12

Thus, the minumum reliability is given belongs to $n_1 = 4$ and $n_2 = 2$. Then, the optimal value of the system is found to be $n_1 = 4$ and $n_2 = 2$ and the total cost is 14.

Example 6.4. In the following table, we compute the reliability of the weighted-k-out-of-n:G system when $m = 3$ and we show the total costs of each component.

Let $n = 4$, $c_1 = 3$, $c_2 = 2$, $c_3 = 1$ and the minimum required reliability is $r_0 = 0.98$.

Table 6.4: The total cost of each component where $m = 3$

n_1	n_2	n_3	w_1	w_2	w_3	p_1	p_2	p_3	k	R	Total Cost
0	0	4	1	2	3	0.95	0.97	0.93	5	0.9987	4
0	1	3	1	2	3	0.95	0.97	0.93	5	0.9859	5
0	2	2	1	2	3	0.95	0.97	0.93	5	0.9874	6
0	3	1	1	2	3	0.95	0.97	0.93	5	0.9914	7
0	4	0	1	2	3	0.95	0.97	0.93	5	0.9948	8
1	0	3	1	2	3	0.95	0.97	0.93	5	0.9859	6
1	1	2	1	2	3	0.95	0.97	0.93	5	0.9848	7
1	2	1	1	2	3	0.95	0.97	0.93	5	0.9264	8
1	3	0	1	2	3	0.95	0.97	0.93	5	0.9126	9
2	0	2	1	2	3	0.95	0.97	0.93	5	0.8649	8
2	1	1	1	2	3	0.95	0.97	0.93	5	0.8998	9
2	2	0	1	2	3	0.95	0.97	0.93	5	0.8491	10
3	0	1	1	2	3	0.95	0.97	0.93	5	0.7973	10
3	1	0	1	2	3	0.95	0.97	0.93	5	-	11
4	0	0	1	2	3	0.95	0.97	0.93	5	-	12

Thus, the minumum reliability is given belongs to $n_1 = 1$, $n_2 = 1$, $n_3 = 2$. Then, the optimal value of the system is found to be $n_1 = 1$, $n_2 = 1$, $n_3 = 2$ and the total cost is 7.

CHAPTER 7

CONCLUSION AND FUTURE WORK

In this thesis, a method for computing the system reliability of weighted $k - out - of - n : G$ systems consisting of m type of components is presented. Generally in reliability problems, all components of a system are supposed to have equivalent weights. But in most of real life engineering systems, the impact made by the component to the performing of the whole system might be different. To increase the reliability of the system may be an important engineering problem. The main application of our study that theorem we proposed is to make a good design of the system. Therefore, we investigate the optimum value of the number of components in each type minimizing the total cost which help us to make the best design of the system. So this study is more useful to investigate the real life systems. As a future work, these findings can be used on the data set from getting the real life such as wing turbine applications and a standby component can be added into the model.

APPENDICES

Appendix 1

Codes of Mathematica when $m = 2$

$p1 = 0.95;$

$p2 = 0.97;$

$w1 = 1;$

$w2 = 2;$

$k = 6;$

$r0 = 0.98;$

$list = Reap[Do[If[w1 * i1 + w2 * i2 \geq k,$

$Sow[\{i1, i2\}]], \{i1, 0, n1\}, \{i2, 0, n2\}]][[2, 1]];$

$For[a = 1; top = 0, a \leq Length[list], a + +, i1 = list[[a, 1]]; i2 = list[[a, 2]];$

$P[n1\_, n2\_] =$

$Binomial[n1, i1] * (p1)^{i1} * (1 - p1)^{(n1-i1)} * Binomial[n2, i2] * (p2)^{n2} * (1 - p2)^{(n2-i2)};$

$top = top + P[n1, n2]];$

$n1 = 4;$

$n2 = 2;$

$c1 = 2;$

$c2 = 3;$

$If[top \geq r0, cost = n1 * c1 + n2 * c2, cost = 0];$

$Print["reliability = ", top, "", "totalcost = ", cost]$

Appendix 2

Codes of Mathematica when $m = 3$

$p1 = 0.95;$

$p2 = 0.97;$

$p3 = 0.93;$

$w1 = 1;$

$w2 = 2;$

$w3 = 3;$

$k = 5;$

$r0 = 0.98;$

$list = Reap[Do[If[w1 * i1 + w2 * i2 + w3 * i3 \geq k, Sow[\{i1, i2, i3\}]],$

$\{i1, 0, n1\}, \{i2, 0, n2\}, \{i3, 0, n3\}]][[2, 1]];$

$For[a = 1; top = 0, a \leq Length[list], a++, i1 = list[[a, 1]]; i2 = list[[a, 2]];$

$i3 = list[[a, 3]]; P[n1\_, n2\_, n3\_] = Binomial[n1, i1] * (p1)^{i1} * (1 - p1)^{(n1-i1)}$

$Binomial[n2, i2] * (p2)^{i2} * (1-p2)^{(n2-i2)} * Binomial[n3, i3] * (p3)^{i3} * (1-p3)^{(n3-i3)};$

$top = top + P[n1, n2, n3]];$

$n1 = 1;$

$n2 = 0;$

$n3 = 3;$

$c1 = 3;$

$c2 = 2;$

$c3 = 1;$

$If[top \geq r0, cost = n1 * c1 + n2 * c2 + n3 * c3, cost = 0];$

$Print["reliability = ", top, "", "totalcost = ", cost]$

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