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On Modification of Weak Structures via Hereditary Classes

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abstract: In this paper, we introduce and study the properties of some new kinds of generalized closed subsets with respect to a weak structure modified by a hereditary class. Then some already established results are generalized. Also, we define a new kind of continuity depending on the new class of generalized closed subsets.

Key Words: weak structure; hereditary class; $\mathcal{H}g\phi(w)$-closed set; $\mathcal{H}g\phi(w,w')$-continuous.

Contents

1. Introduction

Á. Császár [3] introduced a new notion of structures called weak structure. Also, he defined some structures and operators under more general conditions. Since then, many other authors have studied the important properties of the new structure (see [8,4,7,1,6,10]). The notion of weak structure can replace in many situations minimal structures, generalized topology and general topology. Zahran et al. [10] showed that the construction leading from a generalized topology and a hereditary class introduced by Á. Császár [2] remains valid, together with a lot of applications, if the generalized topology is replaced by a weak structure. For more properties of weak structures we refer to [9,5].

Throughout this paper, w will denote a weak structure WS on X and $\mathcal{H} \neq \emptyset$ a hereditary class on X (i.e., if $A \in \mathcal{H}$ and $B \subset A$ implies $B \in \mathcal{H}$). A set $A \subset X$ is said to be w-open (briefly, $A \in \mathcal{O}(w)$) iff $A \in w$ and w-closed iff $X \setminus A \in \mathcal{O}(w)$. For $A \subset X$, i_wA and c_wA are defined as the union of all w-open subsets of A and the intersection of all w-closed sets containing A, respectively. It is known that i_w is restricting, monotone, and idempotent. The map c_w is enlarging, monotone and idempotent.

A subset $A \in \pi(w)$ (resp., $\beta(w)$, $\sigma(w)$, $\rho(w)$, $\alpha(w)$) if $A \subset i_wc_w(A)$ (resp., $A \subset c_wi_wc_w(A)$, $A \subset c_wi_w(A)$, $A \subset i_wc_w(A) \cup c_wi_w(A)$, $A \subset i_wc_wi_w(A)$) (see [3]

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for details). A subset $A \subset X$ is called $\phi(w)$-open (resp. $\phi(w)$-closed) iff $A \in \phi(w)$ (resp. $X \setminus A \in \phi(w)$) where $\phi = \mathcal{O}$, π, β, σ, ρ, α.

A subset $A \subset X$ is said to be a $g\phi(w)$-closed set iff $c_{\phi(w)}(A) \subset U$ whenever $A \subset U$ and U is $\phi(w)$-open where $\phi = \mathcal{O}$, π, β, σ, ρ, and α.

2. $\mathcal{H}g\phi(w)$-closed sets

Definition 2.1. *Let w be a WS, and $\mathcal{H}$ be a hereditary class on X. A subset $A \subset X$ is said to be generalized $\phi(w)$-closed with respect to the hereditary class $\mathcal{H}$ (briefly, $\mathcal{H}g\phi(w)$-closed) if and only if $c_{\phi(w)}(A) \setminus U \in \mathcal{H}$, whenever $A \subset U$ and $U \in \phi(w)$.*

A subset $B \subset X$ is said to be generalized $\phi(w)$-open with respect to the hereditary class $\mathcal{H}$ (briefly, $\mathcal{H}g\phi(w)$-open) if and only if $X \setminus B$ is $\mathcal{H}g\phi(w)$-closed.

Remark 2.2. *Every $g\phi(w)$-closed set is $\mathcal{H}g\phi(w)$-closed, but the converse need not be true in general, as this may be seen from the following example.*

Example 2.3. *Let $X = \{a, b, c, d\}$ and $w = \{\emptyset, \{d\}, \{a, b\}, \{b, c\}, \{a, b, d\}\}$ be a WS on X. If a hereditary class $\mathcal{H} = \{\emptyset, \{a\}, \{b\}, \{c\}\}$ and $A = \{b, d\}$ are given, it is clear that the subset A is $\mathcal{H}g\mathcal{O}(w)$-closed but not $g\mathcal{O}(w)$-closed in X.*

Remark 2.4. *If $\mathcal{H}$ is the power set $\mathcal{P}(X)$ of X, then all subsets of X are $\mathcal{H}g\phi(w)$-closed, and if $\mathcal{H} = \{\emptyset\}$, then every $\mathcal{H}g\phi(w)$-closed set is also $g\phi(w)$-closed.*

Proposition 2.1. *Let X be a set, w be a WS on X. If $\mathcal{H}$ and $\mathcal{H}'$ are two hereditary classes on X with $\mathcal{H} \subset \mathcal{H}'$, then every $\mathcal{H}g\phi(w)$-closed(open) subset of X is also $\mathcal{H}'g\phi(w)$-closed(open).*

Proof: Clear. □

Theorem 2.5. *Let w be a WS and $\mathcal{H}$ be a hereditary class on X. If $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$, a set A is $\mathcal{H}g\phi(w)$-closed in X if and only if $F \subset c_{\phi(w)}(A) \setminus A$ and F being $\phi(w)$-closed in X imply $F \in \mathcal{H}$.*

Proof: Suppose that A is $\mathcal{H}g\phi(w)$-closed, $F \subset c_{\phi(w)}(A) \setminus A$ and F is $\phi(w)$-closed. Then $A \subset X \setminus F$. By our assumption, $c_{\phi(w)}(A) \setminus (X \setminus F) \in \mathcal{H}$. But $F \subset c_{\phi(w)}(A) \setminus (X \setminus F)$ and hence $F \in \mathcal{H}$.

Conversely, assume that $F \subset c_{\phi(w)}(A) \setminus A$ and F being $\phi(w)$-closed in X imply that $F \in \mathcal{H}$. Suppose $A \subset U$ and $U \in \phi(w)$. Then $c_{\phi(w)}(A) \setminus U = c_{\phi(w)}(A) \cap (X \setminus U)$ is a $\phi(w)$-closed set in X, that is contained in $c_{\phi(w)}(A) \setminus A$. By assumption, $c_{\phi(w)}(A) \setminus U \in \mathcal{H}$. This implies that A is $\mathcal{H}g\phi(w)$-closed. □

Remark 2.6. *The union of two $\mathcal{H}g\phi(w)$-closed sets need not be $\mathcal{H}g\phi(w)$-closed in general as shown by the following example.*

Example 2.7. *Let $X = \{a, b, c, d\}$, and $w = \{\emptyset, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b\}\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{c\}\}$. Then, $A_1 = \{a\}$ and $A_2 = \{c, d\}$ are $\mathcal{H}g\pi(w)$-closed sets in X, but their union $A_1 \cup A_2 = \{a, c, d\}$ is not $\mathcal{H}g\pi(w)$-closed since $c_{\pi(w)}(A_1 \cup A_2) \setminus \{a, c, d\} = \{b\}$ is not in $\mathcal{H}$.*

Remark 2.8. *The intersection of two $\mathcal{H}g\phi(w)$-closed sets need not be $\mathcal{H}g\phi(w)$-closed in general as shown by the following example.*

Example 2.9. *Let $X = \{a, b, c\}$ with a WS $w = \{\emptyset, \{b\}\}$. If $A = \{a, b\}$, $B = \{b, c\}$ and $\mathcal{H} = \{\emptyset\}$, then A and B are $\mathcal{H}g\mathcal{O}(w)$-closed but the intersection $A \cap B = \{b\}$ is not $\mathcal{H}g\mathcal{O}(w)$-closed.*

Theorem 2.10. *If A is $\mathcal{H}g\phi(w)$-closed and $A \subset B \subset c_{\phi(w)}(A)$, then B is $\mathcal{H}g\phi(w)$-closed.*

Proof: For any WS w and hereditary class $\mathcal{H}$ on X, suppose A is $\mathcal{H}g\phi(w)$-closed and $A \subset B \subset c_{\phi(w)}(A)$ and $B \subset U$ and $U \in \phi(w)$. Then $A \subset U$. Since A is $\mathcal{H}g\phi(w)$-closed, we have $c_{\phi(w)}(A) \setminus U \in \mathcal{H}$. Now $B \subset c_{\phi(w)}(A)$. This implies that $c_{\phi(w)}(B) \setminus U \subset c_{\phi(w)}(A) \setminus U \in \mathcal{H}$. Hence B is $\mathcal{H}g\phi(w)$-closed. □

Corollary 2.11. *If $i_w(A) \subset B \subset A$ and if A is $\mathcal{H}g\phi(w)$-open in X, then B is $\mathcal{H}g\phi(w)$-open in X.*

Theorem 2.12. *Let $A \subset Y \subset X$ and suppose that A is $\mathcal{H}g\phi(w)$-closed in X. Then A is $\mathcal{H}g\phi(w)$-closed relative to the subspace Y of X, with respect to the hereditary class $\mathcal{H}|_Y = \{F \subset Y : F \in \mathcal{H}\}$.*

Proof: Let $A \subset U \cap Y$ and $U \in \phi(w)$ in X, then $A \subset U$. Since A is $\mathcal{H}g\phi(w)$-closed, we have $c_w(A) \setminus U \in \mathcal{H}$. Now $(c_w(A) \cap Y) \setminus (U \cap Y) = (c_w(A) \setminus U) \cap Y \in \mathcal{H}$, whenever $A \subset U \cap Y$ and $U \in \phi(w)$. Hence A is $\mathcal{H}g\phi(w)$-closed relative to the subspace Y. □

Theorem 2.13. *Let A be an $\mathcal{H}g\phi(w)$-closed, F be a $\phi(w)$-closed, and $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$ then $A \cap F$ is an $\mathcal{H}g\phi(w)$-closed set in X.*

Proof: Let $A \cap F \subset U$ and $U \in \phi(w)$. Then $A \subset U \cup (X \setminus F)$. Since A is $\mathcal{H}g\phi(w)$-closed, we have $c_{\phi(w)}(A) \setminus (U \cup (X \setminus F)) \in \mathcal{H}$. Now, $c_{\phi(w)}(A \cap F) \subset c_{\phi(w)}(A) \cap F = (c_{\phi(w)}(A) \cap F) \setminus (X \setminus F)$. Therefore,

$$\begin{aligned} c_{\phi(w)}(A \cap F) \setminus U &\subset \big(c_{\phi(w)}(A) \cap F\big) \setminus (U \cap (X \setminus F)) \\ &\subset c_{\phi(w)}(A) \setminus (U \cap (X \setminus F)) \in \mathcal{H}. \end{aligned}$$

Hence $A \cap F$ is $\mathcal{H}g\phi(w)$-closed in X. □

Theorem 2.14. *For $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$, a set A is $\mathcal{H}g\phi(w)$-open in X if and only if $F \setminus U \subset i_{\phi(w)}(A)$ for some $U \in \mathcal{H}$, whenever $F \subset A$ and F is $\phi(w)$-closed.*

Proof: Suppose A is $\mathcal{H}g\phi(w)$-open. Suppose $F \subset A$ and F is $\phi(w)$-closed. We have $X \setminus A \subset X \setminus F$. By assumption, $c_{\phi(w)}(X \setminus A) \subset (X \setminus F) \cup U$ for some $U \in \mathcal{H}$. This implies $X \setminus ((X \setminus F) \cup U) \subset X \setminus (c_{\phi(w)}(X \setminus A))$ and hence $F \setminus U \subset i_{\phi(w)}(A)$.

Conversely, assume that $F \subset A$ and F is $\phi(w)$-closed imply $F \setminus U \subset i_{\phi(w)}(A)$ for some $U \in \mathcal{H}$. Consider a $\phi(w)$-open set G such that $X \setminus A \subset G$. Then $X \setminus G \subset A$. By assumption, there exists a set $U \in \mathcal{H}$ such that $(X \setminus G) \setminus U \subset i_{\phi(w)}(A) = X \setminus c_{\phi(w)}(X \setminus A)$. This gives that $X \setminus (G \cup U) \subset X \setminus c_{\phi(w)}(X \setminus A)$. Then, $c_{\phi(w)}(X \setminus A) \subset G \cup U$ for some $U \in \mathcal{H}$. This shows that $c_{\phi(w)}(X \setminus A) \setminus G \in \mathcal{H}$. Hence $X \setminus A$ is $\mathcal{H}g\phi(w)$-closed. □

Remark 2.15. *Note that Theorems 2.13 and 2.14 remain valid also for $\phi = \mathcal{O}$ if w is closed under union, that is w is a generalized topological space.*

Theorem 2.16. *If $A \subset B \subset X$, A is $\mathcal{H}g\phi(w)$-open relative to B and B is $\mathcal{H}g\phi(w)$-open relative to X with respect to an ideal $\mathcal{H}$, then A is $\mathcal{H}g\phi(w)$-open in X, for $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$.*

Proof: Suppose $A \subset B \subset X$, A is $\mathcal{H}g\phi(w)$-open relative to B and B is $\mathcal{H}g\phi(w)$-open relative to X. Suppose $F \subset A$ and F is $\phi(w)$-closed in X. Since A is $\mathcal{H}g\phi(w)$-open relative to B, by Theorem 2.14, $F \setminus U_1 \subset i_{\phi(w)(B)}(A)$ for some $U_1 \in \mathcal{H}$, where $i_{\phi(w)(B)}(A)$ denotes the $\phi(w)$-interior of A with respect to B. This implies there exists a $\phi(w)$-open set G_1 such that $F \setminus U_1 \subset G_1 \cap B \subset A$. Since B is $\mathcal{H}g\phi(w)$-open, $F \subset B$ and F is $\phi(w)$-closed; we have $F \setminus U_2 \subset i_{\phi(w)}(B)$ for some $U_2 \in \mathcal{H}$. This implies there exists a $\phi(w)$-open set G_2 such that $F \setminus U_2 \subset G_2 \subset B$ for some $U_2 \in \mathcal{H}$. Now $F \setminus (U_1 \cup U_2) \subset (F \setminus U_1) \cap (F \setminus U_2) \subset G_1 \cap G_2 \subset G_1 \cap B \subset A$. Then $F \setminus (U_1 \cup U_2) \subset i_{\phi(w)}(A)$ for some $U_1 \cup U_2 \in \mathcal{H}$ and hence A is $\mathcal{H}g\phi(w)$-open in X. □

Remark 2.17. *The following example shows that the previous theorem does not hold when $\mathcal{H}$ is just a hereditary class, not an ideal.*

Example 2.18. *Let $X = \{a, b, c, d\}$, and $w = \{\emptyset, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b\}\}$ and $\mathcal{H} = \{\emptyset, \{c\}, \{d\}\}$. Then, $B = \{a, c, d\}$ is $\mathcal{H}g\sigma(w)$-open in X, and the set $A = \{c, d\}$ is $\mathcal{H}g\sigma(w)$-open in B. However, A is not $\mathcal{H}g\sigma(w)$-open in X.*

Theorem 2.19. *For $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$, a set A is $\mathcal{H}g\phi(w)$-closed in X if and only if $c_{\phi(w)}(A) \setminus A$ is $\mathcal{H}g\phi(w)$-open.*

Proof: Assume $F \subset c_{\phi(w)}(A) \setminus A$ and F be $\phi(w)$-closed. Then $F \in \mathcal{H}$. This implies that $F \setminus U = \emptyset$, for some $U \in \mathcal{H}$. Clearly, $F \setminus U \subset i_{\phi(w)}(c_{\phi(w)}(A) \setminus A)$. By Theorem 2.14, $c_{\phi(w)}(A) \setminus A$ is $\mathcal{H}g\phi(w)$-open.

For the sufficiency, suppose $A \subset G$ and $G \in \phi(w)$ in X. Then $c_{\phi(w)}(A) \cap (X \setminus G) \subset c_{\phi(w)}(A) \cap (X \setminus A) = c_{\phi(w)}(A) \setminus A$. By hypothesis, $(c_{\phi(w)}(A) \cap (X \setminus G)) \setminus U \subset i_{\phi(w)}(c_{\phi(w)}(A) \setminus A) = \emptyset$ for some $U \in \mathcal{H}$. This implies that $c_{\phi(w)}(A) \cap (X \setminus G) \subset U \in \mathcal{H}$ and hence $c_{\phi(w)}(A) \setminus G \in \mathcal{H}$. Thus, A is $\mathcal{H}g\phi(w)$-closed. □

Definition 2.20. *Let w be a WS on X, the sets A and B are said to be $\phi(w)$-separated if $c_{\phi(w)}(A) \cap B = \emptyset$ and $A \cap c_{\phi(w)}(B) = \emptyset$.*

Theorem 2.21. *Let w be a WS and $\mathcal{H}$ be an ideal on X. If $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$, A and B are $\phi(w)$-separated, and $\mathcal{H}g\phi(w)$-open sets in X, then $A \cup B$ is $\mathcal{H}g\phi(w)$-open.*

Proof: Suppose A and B are $\phi(w)$-separated and $\mathcal{H}g\phi(w)$-open sets in X and F be a $\phi(w)$-closed subset of $A \cup B$. Then $F \cap c_{\phi(w)}(A) \subset A$ and $F \cap c_{\phi(w)}(B) \subset B$. By assumption, $(F \cap c_{\phi(w)}(A)) \setminus U_1 \subset i_{\phi(w)}(A)$ and $(F \cap c_{\phi(w)}(B)) \setminus U_2 \subset i_{\phi(w)}(B)$ for some $U_1, U_2 \in \mathcal{H}$. That is $((F \cap c_{\phi(w)}(A)) \setminus i_{\phi(w)}(A)) \in \mathcal{H}$ and $(F \cap c_{\phi(w)}(B)) \setminus i_{\phi(w)}(B) \in \mathcal{H}$. Then $((F \cap c_{\phi(w)}(A)) \setminus i_{\phi(w)}(A)) \cup ((F \cap c_{\phi(w)}(B)) \setminus i_{\phi(w)}(B)) \in \mathcal{H}$. Hence $(F \cap (c_{\phi(w)}(A) \cup c_{\phi(w)}(B)) \setminus (i_{\phi(w)}(A) \cup i_{\phi(w)}(B))) \in \mathcal{H}$. But $F = F \cap (A \cup B) \subset F \cap c_{\phi(w)}(A \cup B)$, and we have

$$\begin{aligned} F \setminus i_{\phi(w)}(A \cup B) &\subset (F \cap c_{\phi(w)}(A \cup B)) \setminus i_{\phi(w)}(A \cup B) \\ &\subset (F \cap c_{\phi(w)}(A \cup B)) \setminus (i_{\phi(w)}(A) \cup i_{\phi(w)}(B)) \in \mathcal{H}. \end{aligned}$$

Hence, $F \setminus U \subset i_{\phi(w)}(A \cup B)$ for some $U \in \mathcal{H}$. This proves that $A \cup B$ is $\mathcal{H}g\phi(w)$-open. □

Corollary 2.22. *Let A and B are $\mathcal{H}g\phi(w)$-closed sets with respect to an ideal $\mathcal{H}$ on X, and suppose $X \setminus A$ and $X \setminus B$ are $\phi(w)$-separated in X. Then $A \cap B$ is $\mathcal{H}g\phi(w)$-closed.*

Remark 2.23. *As the following example shows, the Theorem 2.21 does not hold for the case where $\mathcal{H}$ is only a hereditary class, but not an ideal.*

Example 2.24. *Let $X = \{a, b, c, d\}$, $w = \{\emptyset, \{a, b\}, \{b, c, d\}\}$, and $\mathcal{H} = \{\emptyset, \{c\}, \{d\}\}$. Then the subsets $A = \{c\}$ and $B = \{d\}$ are $\sigma(w)$-separated , and $\mathcal{H}g\sigma(w)$-open sets. On the other hand their union $A \cup B = \{c, d\}$ is not $\mathcal{H}g\sigma(w)$-open since $c_{\sigma(w)}(\{a, b\}) = X$, and $X \setminus \{a, b\} = \{c, d\} \notin \mathcal{H}$ show that $\{a, b\}$ is not $\mathcal{H}g\sigma(w)$-closed.*

3. $\mathcal{H}g\phi(w, w')$-Continuity

The collection of all $\mathcal{H}g\phi(w)$-open sets form a WS on X. If we denote this WS as $O\mathcal{H}g\phi(w)$, then $\mathcal{O}(w) \subset O\mathcal{H}g\phi(w)$.

Definition 3.1. *Let X be a set, w be a WS, and $\mathcal{H}$ be a hereditary class on X. For a subset $A \subset X$, the intersection of all $\mathcal{H}g\phi(w)$-closed sets containing A, is called as the $\mathcal{H}g\phi(w)$-closure of A and denoted by $c^{\mathcal{H}}_{\phi(w)}(A)$.*

Definition 3.2. *Let X be a set, w be a WS, and $\mathcal{H}$ be a hereditary class on X. For a subset $A \subset X$, the union of all $\mathcal{H}g\phi(w)$-open subsets of A, is called as the $\mathcal{H}g\phi(w)$-interior of A and denoted by $i^{\mathcal{H}}_{\phi(w)}(A)$.*

Definition 3.3. *Let X and Y be sets, w and w' are WS's on X and Y, respectively, and $\mathcal{H}$ be a hereditary class on X. Then the function $f : X \to Y$ is said to be $\mathcal{H}g\phi(w, w')$-continuous if and only if $f^{-1}(A)$ is $\mathcal{H}g\phi(w)$-open in X whenever A is $\phi(w')$-open in Y.*

Let X and Y be sets, w and w' are WS's on X and Y, respectively. Then the function $f : X \to Y$ is said to be $\phi(w, w')$-continuous if and only if $f^{-1}(A)$ is $\phi(w)$-open in X whenever A is $\phi(w')$-open in Y.

Theorem 3.4. *Let X and Y be sets, w and w' are WS's on X and Y respectively, and $\mathcal{H}$ be a hereditary class on X. If the function $f : X \to Y$ is $\phi(w, w')$-continuous, then it is also $\mathcal{H}g\phi(w, w')$-continuous.*

Proof: Proof is clear, since any $\phi(w)$-closed set is $\mathcal{H}g\phi(w)$-closed. □

Theorem 3.5. *Let X and Y be sets, w and w' are WS's on X and Y respectively, $\mathcal{H}$ be a hereditary class on X, and $f : X \to Y$ be a function. If f is $\mathcal{H}g\phi(w, w')$-continuous, then $f(c^{\mathcal{H}}_{\phi(w)}(A)) \subset c_{\phi(w')}(f(A))$ for any subset $A \subset Y$.*

Proof: Let $y \in f(c^{\mathcal{H}}_{\phi(w)}(A))$. Then, there exists a point $x \in c^{\mathcal{H}}_{\phi}(A)$ such that $y = f(x)$. On the other hand, let U be a $\phi(w')$-open set containing y. This implies $x \in f^{-1}(U)$, where $f^{-1}(U)$ is $\mathcal{H}g\phi(w)$-open. By definition of closure $f^{-1}(U) \cap A \neq \emptyset$. Then there exists a point $t \in f^{-1}(U) \cap A$ with $f(t) \in U \cap f(A) \neq \emptyset$. This shows that $y \in c_{\phi(w')}(f(A))$. □

Let X and Y be sets, w and w' are WS's on X and Y respectively. Then the function $f : X \to Y$ is said to be $\phi(w, w')$-closed if and only if $f(A)$ is $\phi(w')$-closed in Y whenever A is $\phi(w)$-closed in X.

Theorem 3.6. *Let $f : X \to Y$ be $\phi(w, w')$-continuous and $\phi(w, w')$-closed. If $A \subset X$ is $\mathcal{H}g\phi(w)$-closed in X, then $f(A)$ is $f(\mathcal{H})g\phi(w')$-closed in Y, where $f(\mathcal{H}) = \{f(U) : U \in \mathcal{H}\}$, and $\phi \in \{\pi, \beta, \sigma, \rho, \alpha\}$.*

Proof: Suppose $A \subset X$, A is $\mathcal{H}g\phi(w)$-closed, $f(A) \subset G$ and $G \in \phi(w')$. Then $A \subset f^{-1}(G)$. By definition, $c_{\phi(w)}(A) \setminus f^{-1}(G) \in \mathcal{H}$ and hence $f(c_{\phi(w)}(A)) \setminus G \in f(\mathcal{H})$. Since f is $\phi(w, w')$-closed, $c_{\phi(w')}(f(A)) \subset c_{\phi(w')}(f(c_{\phi(w)}(A))) = f(c_{\phi(w)}(A))$. Then $c_{\phi(w')}(f(A)) \setminus G \subset f(c_{\phi(w)}(A)) \setminus G \in f(\mathcal{H})$ and hence $f(A)$ is $f(\mathcal{H})g\phi(w')$-closed. □

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