Cost-Based Domain Filtering for Stochastic Constraint Programming*

Roberto Rossi[1], S. Armagan Tarim[2], Brahim Hnich[3], and Steven Prestwich[1]

Cork Constraint Computation Centre - CTVR, University College, Cork, Ireland
{r.rossi,s.prestwich}@4c.ucc.ie
Department of Management, Hacettepe University, Ankara, Turkey
armagan.tarim@hacettepe.edu.tr
Faculty of Computer Science, Izmir University of Economics, Turkey
brahim.hnich@ieu.edu.tr

Abstract. Cost-based filtering is a novel approach that combines techniques from Operations Research and Constraint Programming to filter from decision variable domains values that do not lead to better solutions [7]. Stochastic Constraint Programming is a framework for modeling combinatorial optimization problems that involve uncertainty [19]. In this work, we show how to perform cost-based filtering for certain classes of stochastic constraint programs. Our approach is based on a set of known inequalities borrowed from Stochastic Programming — a branch of OR concerned with modeling and solving problems involving uncertainty. We discuss bound generation and cost-based domain filtering procedures for a well-known problem in the Stochastic Programming literature, the static stochastic knapsack problem. We also apply our technique to a stochastic sequencing problem. Our results clearly show the value of the proposed approach over a pure scenario-based Stochastic Constraint Programming formulation both in terms of explored nodes and run times.

1 Introduction

Constraint Programming (CP) [1] has been recognized as a powerful tool for modeling and solving combinatorial optimization problems. CP provides global constraints offering concise and declarative modeling capabilities and efficient domain filtering algorithms. These algorithms remove combinations of values which cannot appear in any consistent solution. Cost-based filtering is an elegant way of combining techniques from CP and Operations Research (OR) [7]. OR-based optimization techniques are used to remove from variable domains values that cannot lead to better solutions. This type of domain filtering can be

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combined with the usual CP-based filtering methods and branching heuristics, yielding powerful hybrid search algorithms. Cost-based filtering is a novel technique that has been the subject of significant recent research.

Stochastic Constraint Programming (SCP) [19] is an extension of CP, in which there is a distinction between decision variables, which we are free to set, and stochastic (or observed) variables, which follow some probability distribution. SCP is designed to handle problems in which uncertainty comes into play. Uncertainty may take different forms: data about events in the past may not be known exactly due to measuring or difficulties in sampling, and data about events in the future may simply not be known with certainty.

In this work we propose a novel approach to performing cost-based filtering for certain classes of stochastic constraint programs. Our approach is based on a well-known inequality borrowed from Stochastic Programming [4], a branch of OR that is concerned with modeling constraint satisfaction/optimization problems under uncertainty. We implemented this approach for two problems in which uncertainty plays a role. In both cases we obtained significant improvements with respect to a pure SCP formulation both in terms of explored nodes and run times.

The rest of the paper is structured as follows. In Section 2 we give the necessary formal background. In Section 3 we review relevant inequalities from Stochastic Programming. In Section 4, we introduce global optimization chance constraints. We describe our empirical results in Section 5 and review related works in Section 6. Finally, we conclude and outline our future work in Section 7.

2 Formal Background

A *Constraint Satisfaction Problem* (CSP) [1] is a triple $\langle V, C, D \rangle$, where $V = \{V_1, \ldots, V_n\}$ is a set of decision variables, D is a function mapping each element of V to a domain of potential values, and C is a set of constraints stating allowed combinations of values for subsets of variables in V. A *solution* to a CSP is an assignment to every variable of a value in its domain, such that all of the constraints are satisfied. We may also be interested in finding a feasible solution that maximizes (minimizes) the value of a given objective function over a subset of the variables. With no loss of generality, we restrict our discussion to maximization problems.

Optimization-oriented global constraints embed an optimization component, representing a proper relaxation of the constraint itself, into a global constraint [7]. This component provides three pieces of information: (a) the optimal solution of the relaxed problem; (b) the optimal value of this solution representing an upper bound on the original problem objective function; (c) a *gradient function* $\mathbf{grad}(V,v)$, which returns for each variable-value pair (V,v) an optimistic evaluation of the profit obtained if v is assigned to V. These pieces of information are exploited both for propagation purposes and for guiding the search.

In [19], a *stochastic CSP* is defined as a 6-tuple $\langle V, S, D, P, C, \theta \rangle$, where V is a set of decision variables and S is a set of stochastic variables, D is a function