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The Mrl Function Of The K-Out-Of-N System With Nonidentical Components In The System Level

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Summary.A k-out-of-n system is working if at least k of its n components are operating. The system breaks down at the time of the $(n-k+1)$th component failure. Since all components start working at the same time, this approach leads to a kind of redundancy called active redundancy of $n-k$ components. Important particular cases of k-out-of-n system are parallel and series systems corresponding to k=1 and $k=n$, respectively. In this paper, we consider the mean residual life (MRL) function of a parallel and k-out-of-n systems consisting of n components having independent and nonidentically distributed lifetimes. We provide new representations of the MRL function for such systems. The MRL functions of systems consisting of components having exponential and power distributed lifetimes are presented. Also we introduce a numerical example to study the effect of increasing the system level and various parameters on the mean residual life of the systems. Further, the relation between the mean residual life for the system and the mean residual life of its components is investigated.

Key words: Mean residual life function, Parallel system, k-out-of-n-system, Permanents

1. Introduction

When the variables are independent but not assumed to be identically distributed, the usage of the permanents provides an effective technique to handle the case of order statistics from nonidentical parents.

Consider S_n as the set of permutations of $1, 2, \ldots, n$. If A is an $n \times n$ matrix, the permanent of A, denoted by $PerA$, is defined as:

$$PerA = \sum_{\pi \in S_n} \prod_{i=1}^{n} a_{i\pi(i)}$$

where the summation extends over all permutations of $\{1, 2, \ldots, n\}$.

The permanent does not change when the rows or columns of the matrix are permuted. And also, the permanent admits a Laplace expansion along any row or column of the matrix. Thus if we denote by $A(i, j)$ the matrix obtained by deleting row i and column j of the $n \times n$ matrix A, then for $i, j = 1, 2, \ldots, n$

$$PerA = \sum_{j=1}^{n} a_{ij} PerA(i, j)$$

and

$$PerA = \sum_{i=1}^{n} a_{ij} PerA(i, j).$$

If $a_1, a_2, \ldots,$ are column vectors, then

$$\left[\underbrace{a_1}_{i_1}, \underbrace{a_2}_{i_2}, \ldots, \right]$$

will denote the matrix obtained by taking i_1 copies of a_1, i_2 copies of a_2 and so on (Bapat and Beg, 1989).

Vaughan and Venables (1972) have shown that the density of $X_{r::n}$ is conveniently expressed in terms of permanents, when $X_1, X_2, \ldots, X_n$ are order statistics of the independent random variables with absolutely continuous distribution functions $F_1, F_2, \ldots, F_n$ and densities $f_1, f_2, \ldots, f_n$ respectively. The distribution function of $X_{r:n} (1 \leqslant r \leqslant n)$ is given by Bapat and Beg (1989)

$$P(X_{r:n} \leq x) = \sum_{i=r}^{n} \frac{1}{i!(n-1)!} Per \begin{bmatrix} F_1(x) & 1 - F_1(x) \\ \vdots & \vdots \\ \underbrace{F_n(x)}_{i} & \underbrace{1 - F_n(x)}_{n-i} \end{bmatrix}$$

where $-\infty < x < \infty$ and $PerA$ denotes the permanent of a square matrix A; the permanent is defined just like the determinant, except that all signs in the expansion are positive. A simple argument shows (David, 1981) that

$$P(X_{r:n} \leqslant x) = \sum_{i=r}^{n} P(iX_1, \dots, X_n \leqslant x)$$
$$= \sum_{i=r}^{n} \sum_{j_1, \dots j_n} \prod_{l=1}^{i} F_{j_l}(x) \prod_{l=i+1}^{n} [1 - F_{j_l}(x)],$$

where the summation extends over all permutations $j_1, \dots, j_n$ of $1, \dots, n$ for which $j_1 < \dots < j_i$ and $j_{i+1} < \dots < j_n$.

Most of the fault-tolerant systems such as parallel and k-out-of-n systems consist of nonidentical components. This type of structures finds wide applications in both industrial and technical areas. For the improvement of the reliability of the operation of such complex technical systems the implementation of the structural redundancy is widely used by the method of the k-out-of-n reservation. In this study we provide the results and examples on mean residual life function for k-out-of-n systems consisting of n independent and nonidentical distributed components.

2. The Mrl Functıon Of k-Out-Of-n System

Asadi and Bayramoglu (2006) have studied the MRL function of k-out-of-n system under the condition that at time t all the components are working, i.e. $X_{1\,:\,n} > t$. In the following theorem we propose the MRL function assuming that $X_1, X_2, \dots, X_n$ are independent but nonidentically distributed random variables with distribution function F_i and survival function $\overline{F}_i = 1 - F_i$. Let also $X_{1\,:n} \leqslant X_{2:n} \leqslant \dots \leqslant X_{n:n}$ be the ordered lifetimes of the components. $X_{k:\,n}$, $k = 1, 2, \dots, n$, represents the lifetime of $(n-k+1)$-out-of-n system.

Definition 1 The mean residual life function of the k-out-of-n system under the condition that all components alive at time t, is

$$H_{(n)}^{k}(t) = E(X_{k\,:\,n} - t \mid X_{1\,:\,n} > t),\ k = 1, 2, \dots, n. \tag{1}$$

Theorem 1 If $H_{(n)}^{k}(t)$ is the MRL of the parallel system defined as Eq.(1), then for $\overline{F}(t) > 0$, k=1,2,...,n and t>0

$$H_{(n)}^{k}(t) = \frac{\sum_{i=0}^{k-1} \binom{n}{i}}{n! \prod_{i=1}^{n} \bar{F}_i(t)} \int_t^\infty Per \left[\underbrace{\bar{F}(t) - \bar{F}(x)}_{i} \quad \underbrace{\bar{F}(x)}_{n-i} \right] \tag{2}$$

Proof. If S denotes the survival function of conditional random variable $X_{k\,:\,n} - t \mid X_{1\,:\,n} > t$ then for $x > 0$,

$$S(x|t) = P\left(X_{k:n} - t | X_{1:n} > t\right)$$

$$= \frac{P(X_{k:n} > x + t, X_{1:n} > t)}{P(X_{1:n} > t)}$$

$$(3) \qquad = \frac{\sum_{i=0}^{k-1} \sum_{j_1,\ldots,j_n} \prod_{l=1}^{i} \left[F_{j_l}(x+t) - F_{j_l}(t)\right] \prod_{l=i+1}^{n} \overline{F}_{j_l}(x+t)}{\prod_{i=1}^{n} \overline{F}_i(t)}$$

$$= \frac{\sum_{i=0}^{k-1} \sum_{j_1,\ldots,j_n} \prod_{l=1}^{i} \left[\overline{F}_{j_l}(t) - \overline{F}_{j_l}(x+t)\right] \prod_{l=i+1}^{n} \overline{F}_{j_l}(x+t)}{\prod_{i=1}^{n} \overline{F}_i(t)}$$

Hence the full sum is recognizable as the permanent of a matrix, so $S(x \mid t)$ has the expression as follows.

$$(4) \qquad \frac{\sum_{i=0}^{k-1} \frac{1}{i!(n-1)!} Per \left[\underbrace{\bar{F}(t) - \bar{F}(x+t)}_{i} \quad \underbrace{\bar{F}(x+t)}_{n-i} \right]}{\prod_{i=1}^{n} \overline{F}_i(t)}$$

Given that all the components of the system are working at time $t > 0$, the MRL function of the system is

$$H_{(n)}^{k}(t) = \int_{0}^{\infty} S(x|t)dx$$

$$= \frac{\sum_{i=0}^{k-1} \frac{1}{i!(n-1)!} Per \left[\underbrace{\bar{F}(t) - \bar{F}(x)}_{i} \quad \underbrace{\bar{F}(x)}_{n-i} \right] dx}{\prod_{i=1}^{n} \overline{F}_i(t)}, \text{ k=1,2,...,n}$$

Thus the proof is completed.

The motivation for this structure can be given as an example of the high priority freight train, which is structured as a 3-out-of-4 system consisting of four locomotives (Nelson, 1982). The train is delayed only if two or more locomotives fail. It is assumed that the four locomotives in a train fail independently and times to failure for locomotives are distributed as nonidentical exponential distribution. The MRL of such a system is,

$$\begin{aligned} H^2_{(4)}(t) &= \frac{1}{\lambda_1+\lambda_2+\lambda_3}+\frac{1}{\lambda_1+\lambda_3+\lambda_4}+\frac{1}{\lambda_1+\lambda_2+\lambda_4} \\ &+\frac{1}{\lambda_2+\lambda_3+\lambda_4}-\frac{3}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}. \end{aligned}$$

It is clear that the MRL of the system is a decreasing function of failure rates λ_1, λ_2, λ_3, λ_4 as expected.

2.1. The MRL Function of a Parallel System Having n Components All Alive at Time t

Consider a parallel system with independent and nonidentically distributed components each following the distribution function F_i and survival function (reliability function) $\overline{F}_i = 1 - F_i$, $i = 1, 2, \ldots, n$. When the system is put into operation at time t, all components are working. Let also $X_{1:n} \leqslant X_{2:n} \leqslant \cdots \leqslant X_{n:n}$ be the ordered lifetimes of the components. The consideration of the mean residual life function of this system leads us to the following definition.

Definition 2 The MRL function of a system under the condition all components alive at time t, i.e., $X_{1:n} > t$, is

$$\phi_n(t) = E(X_{n:n} - t | X_{1:n} > t) = E(X_{n:n} | X_{1:n} > t) - t. \tag{5}$$

In Theorem 2 we obtain a representation formula for mean residual life function of a parallel system under condition that all components are survived.

Theorem 2 Let $\phi_n(t)$ be the mean residual life function of a system having a parallel structure and consisting of n independent and nonidentically distributed components with distribution function F_i, $i = 1, 2, \ldots, n$, respectively. Given that all components of the system are working at time $t > 0$ then,

$$\phi_n(t) = \frac{1}{\prod_{i=1}^{n} \overline{F}_i(t)} \frac{1}{(n-1)!} \int_t^{\infty} x Per \left[\underbrace{f(x)}_{1} \quad \underbrace{F(x) - F(t)}_{n-1} \right] dx - t. \tag{6}$$

Proof. We have,

$$P(X_{n:n} \leqslant x | X_{1:n} > t)$$

$$= \frac{P(X_1 \leqslant x, \ldots, X_n \leqslant x, X_1 > t, \ldots, X_n > t)}{P(X_1 > t, \ldots, X_n > t)}. \tag{7}$$

From Eq.(7) we get

$$P(X_{n\,:\,n} \leqslant x \mid X_{1\,:\,n} > t) = \frac{\prod\limits_{i=1}^{n} [F_i(x) - F_i(t)]}{\prod\limits_{i=1}^{n} \overline{F}_i(t)}. \tag{8}$$

Differentiating Eq.(8) with respect to x we obtain the probability density function of conditional random variable $(X_{n\,:\,n}|X_{1:n} > t)$ as

$$\frac{1}{\prod\limits_{i=1}^{n} \overline{F}_i(t)} \sum_{k=1}^{n} f_k(x) \prod_{i \neq k} [F_i(x) - F_i(t)]\,. \tag{9}$$

Using the identity Eq.(9), in Eq.(5) we have,

$$\Psi_n(t) = \frac{1}{\prod\limits_{i=1}^{n} \overline{F}_i(t)} \int_t^{\infty} x \sum_{k=1}^{n} f_k(x) \prod_{i \neq k} [F_i(x) - F_i(t)]\; dx - t.$$

An argument shows that

$$\sum_{k=1}^{n} f_k(x) \prod_{i \neq k} [F_i(x) - F_i(t)] = \sum_{j_1,\ldots,\,j_n} f_{j_1}(x) \prod_{l=2}^{n} [F_{j_l}(x) - F_{j_l}(t)]\,,$$

where the summation extends over all permutations $j_1, \ldots j_n$ of $1, \ldots, n$ for which j_1 and $j_2 < \ldots < j_n$. The result now follows from the definition of the permanent:

$$\frac{1}{(n-1)!} Per \begin{bmatrix} f_1(x) & F_1(x) - F_1(t) \\ \vdots & \vdots \\ \underbrace{f_n(x)}_{1} & \underbrace{F_n(x) - F_n(t)}_{n-1} \end{bmatrix} \tag{10}$$

Given that all components of the system are working at time $t > 0$, we obtain

Thus the proof is completed.

Example 1. Let $F_i(x)$, $i = 1, 2, \ldots, n$ be the exponential distribution function;

$$F_i(x) = \begin{cases} 1 - e^{-\lambda_i x} & x \geq 0, \lambda_i > 0 \\ 0 & x < 0 \end{cases}$$

Then, using Eq.(6) one can show that for $i = 1,\ 2,\ 3$, the MRL function of a system containing three components has the following form:

$$
\begin{aligned}
\phi_3(t) &= \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3}-\frac{1}{\lambda_1+\lambda_2}-\frac{1}{\lambda_1+\lambda_3}\\
&\quad -\frac{1}{\lambda_2+\lambda_3}+\frac{1}{\lambda_1+\lambda_2+\lambda_3}.
\end{aligned}
$$

Note that the MRL of a system having independent and nonidentical exponential components does not depend on t. When the values of $\lambda_i \geqslant 1$ then the MRL of the system decreases. The MRL function of a system containing n components has the form

$$
\phi_n(t) = \sum_{i=1}^{n}(-1)^{i+1} \sum_{1\leqslant j_1<\ldots<j_i\leqslant n} \frac{1}{\sum\limits_{k=1}^{i} \lambda_{j_k}}. \tag{11}
$$

Example 2. Let $F_i(x)$, $i = 1, 2, \ldots, n$, be the power distribution function;

$$
F_i(x) = \begin{cases} 1-(1-x)^{\theta_i} & 0<x<1 \\ 0 & otherwise \end{cases}
$$

Then, the MRL function of a system containing three components has the following form:

$$
\begin{aligned}
\phi_3(t) = (1-t)\Bigg[1 &- \frac{\theta_1}{\theta_1+1}-\frac{\theta_2}{\theta_2+1}-\frac{\theta_3}{\theta_3+1}+\frac{\theta_1+\theta_2}{\theta_1+\theta_2+1}\\
&+\frac{\theta_1+\theta_3}{\theta_1+\theta_3+1}+\frac{\theta_2+\theta_3}{\theta_2+\theta_3+1}-\frac{\theta_1+\theta_2+\theta_3}{\theta_1+\theta_2+\theta_3+1}\Bigg], 0<t<1.
\end{aligned}
$$

In Fig.1, we have presented the graph of the MRL function of a system containing three components in which the lifetime of the components are assumed to be power distribution with different parameter values. It is seen that the MRL function is a decreasing function of parameters θ_1, θ_2 and θ_3.

As a result, the MRL function of a system containing n components is,

$$
\phi_n(t) = (1-t)\left[1-\left(\sum_{i=1}^{n}(-1)^{i+1} \sum_{1\leqslant j_1<\ldots<j_i\leqslant n} \sum_{k=1}^{i} \frac{\theta_{j_k}}{\theta_{j_k}+1}\right)\right] \tag{12}
$$

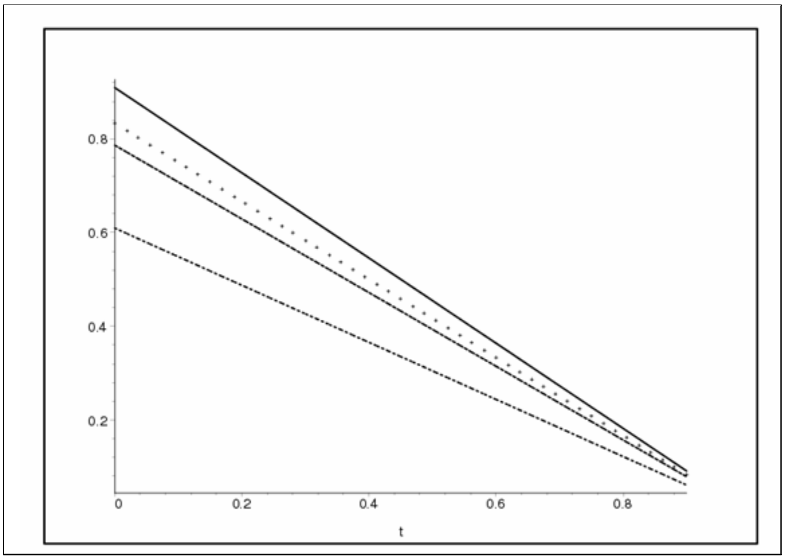


Fig.1. The MRL of a parallel system with n=3 power distributed components with the parameters $(\theta_1,\, \theta_2,\, \theta_3) = (0.5,\, 0.8,\, 0.3)$, $(0.5,\, 0.8,\, 1), (0.5,\, 1,\, 2), (1,\, 2,\, 3)$ respectively.

Asadi and Bayramoglu (2005) have given an extension of the $\phi_n(t)$ as assuming that $X_1,\, X_2, \ldots,\, X_n$ are independent, identically distributed random variables with distribution function F and survival function $\overline{F} = 1 - F$. They defined the MRL function of a system, under the condition that $X_{r:n} > t$, i.e., $(n - r + 1)$, $r = 1,\, 2, \ldots,\, n$, components of the system are still working as

$$M^r_{(n)}(t) = E(X_{n\,:\,n} - t \mid X_{r\,:\,n} > t),\; r = 1,\, 2, \ldots,\, n. \tag{13}$$

Then for $\overline{F}(t) > 0$, $r = 1,\, 2, \ldots,\, n$ and t>0

$$M^r_{(n)}(t) = \frac{\sum\limits_{i=0}^{r-1} \begin{pmatrix} n \\ j \end{pmatrix} \phi^i(t) \sum\limits_{j=1}^{n-i} (-1)^{j+1} \begin{pmatrix} n-i \\ j \end{pmatrix} M_j(t)}{\sum\limits_{i=0}^{r-1} \begin{pmatrix} n \\ i \end{pmatrix} \phi^i(t)}, \tag{14}$$

where $M_j(t) = \dfrac{\int\limits_t^\infty \overline{F}^j(x)dx}{\overline{F}^j(t)}$, $\quad \phi(t) = \dfrac{F(t)}{\overline{F}(t)}$.

In the following theorem we define the MRL function of a parallel system, assuming that $X_1,\, X_2, \ldots,\, X_n$ are independent but nonidentically distributed random variables with distribution function F_i and survival function $\overline{F}_i = 1 - F_i, i = 1, 2, \ldots, n$, under the condition that $X_{r:n} > t$, i.e., $(n - r + 1)$, $r = 1, 2, \ldots, n$, components of the system are still working

Theorem 3 Let $M^r_{(n)}(t)$ be the mean residual life function of a parallel system consisting of n independent and nonidentically distributed components. Then for $t > 0$ and $\overline{F}(t) > 0$.

$$(15) \qquad M^r_{(n)}(t) = \frac{\sum\limits_{i=0}^{r-1} \sum\limits_{j_1,\ldots,j_n} \prod\limits_{l=1}^{i} F_{j_l}(t) \sum\limits_{k=1}^{n-i} (-1)^{k+1} \prod\limits_{l=1}^{k} \overline{F}_{j_{(l+i)}}(x+t) \prod\limits_{l=k+1}^{n-i} \overline{F}_{j_{(l+i)}}(t)}{\sum\limits_{i=0}^{r-1} \frac{1}{i!(n-1)!} Per \left[\underbrace{F(t)}_{i} \;\; \underbrace{\bar{F}(t)}_{n-i} \right]}$$

Proof. It is clear that

$$S(x|t) = \frac{P(X_{n:n} > x + t,\, X_{r:n} > t)}{P(X_{r:n} > t)}$$

$$(16) \qquad = \frac{\sum\limits_{i=0}^{r-1} \sum\limits_{j_1,\ldots,j_n} \prod\limits_{l=1}^{i} F_{j_l}(t) \sum\limits_{k=1}^{n-i} (-1)^{k+1} \prod\limits_{l=1}^{k} \overline{F}_{j_{(l+i)}}(x+t) \prod\limits_{l=k+1}^{n-i} \overline{F}_{j_{(l+i)}}(t)}{\sum\limits_{i=0}^{r-1} \sum\limits_{j_1,\ldots,j_n} \prod\limits_{l=1}^{i} F_{j_l}(t) \prod\limits_{l=i+1}^{n} \left[1 - F_{j_l}(t)\right].}.$$

The full sum in the denominator is recognizable as the permanent of a matrix, so $S(x \mid t)$ has the form
For $r = 1, 2, \ldots, n$ and $t > 0$,

$$M^r_{(n)}(t) = \int\limits_0^\infty S(x|t)dx$$

$$(17) \qquad = \frac{\sum\limits_{i=0}^{r-1} \sum\limits_{j_1,\ldots,j_n} \prod\limits_{l=1}^{i} F_{j_l}(t) \sum\limits_{k=1}^{n-i} (-1)^{k+1} \prod\limits_{l=1}^{k} \overline{F}_{j_{(l+i)}}(x+t) \prod\limits_{l=k+1}^{n-i} \overline{F}_{j_{(l+i)}}(t)}{\sum\limits_{i=0}^{r-1} \frac{1}{i!(n-1)!} Per \left[\underbrace{F(t)}_{i} \;\; \underbrace{\bar{F}(t)}_{n-i} \right]}$$

Therefore the proof is completed.

3. Numerical Results

This section introduces a numerical example for the various value of k of n components of system. Further, the relation between the mean residual life for the system and the mean residual life of its components is investigated as in the study by Sarhan and Abouammoh (2001). They have investigated the reliability of nonrepairable k-out-of-n systems with nonidentical components subjected to independent and common shocks and the relationship between the failure rate of the system and that of its components.

Let us consider an airplane that has three engines. Furthermore, suppose that the design of the aircraft is such that at least two engines are required to function for the aircraft to remain airborne. This means that the engines are reliability-wise in a k-out-of-n configuration, where k=2 and n=3. More specifically, they are in a 2-out-of-3 configuration. It is assumed that the engines X_i, i=1,2,3, are Weibull distributed with parameters (α_i, λ_i), respectively (Petit and Turnbull, 2001).

This distribution was selected based on the common usage in engineering, versatility and to reduce the complexity of the data analysis. The two parameter Weibull distribution is a time dependent distribution that is also one of the most useful probability distributions in reliability. It can be used to model both increasing, and decreasing failure rates. α is referred to as the shape parameter. If α is less than one, the mean residual life function is increasing over time. If α is greater than one, the mean residual life function is decreasing over time. If α is equal to one, the mean residual life function is constant over time, that is the exponential distribution.

The time to failure X of an engine is said to be Weibull distributed with parameters α_i>0 and λ_i>0 for i=1, 2, 3 if the distribution function is given by

$$F(t) = \begin{cases} 1 - e^{-(\lambda_i t)^{\alpha_i}} & t > 0 \\ 0 & otherwise \end{cases} . \tag{18}$$

It is assumed that the scale parameter λ is identical for all components lifetimes. It is also assumed that the working of the components is independent of one another. The mean residual life of an engine at age t is the average remaining life among those engines which have survived until time t. The mean residual life function of the 2-out-of-3 system under the condition that all components alive at time $t > 0$ is

$$H^2_{(3)}(t) = \frac{\sum_{i=0}^{1} \binom{3}{i}}{3! \prod_{i=1}^{3} \bar{F}_i(t)} \int_t^{\infty} Per \left[\underbrace{e^{-(\lambda_i t)^{\alpha_i}} - e^{-(\lambda_i x)^{\alpha_i}}}_{i} \quad \underbrace{e^{-(\lambda_i x)^{\alpha_i}}}_{3-i} \right] dx. \tag{19}$$

Fig.2a-e shows the mean residual life function of this system with different shape parameter α_i, i=1, 2, 3 of lifetime distribution of components when:

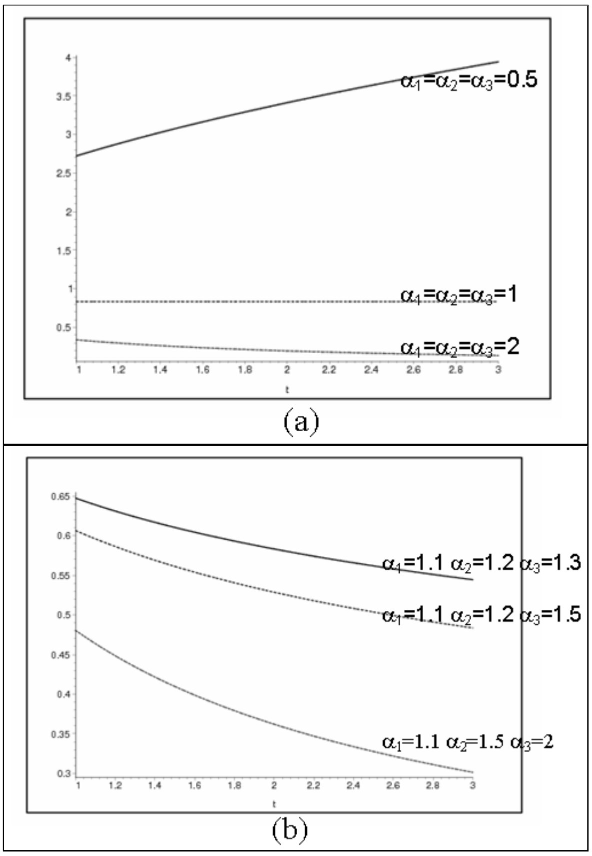
All components have identically distributed i.e. $\alpha_1 = \alpha_2 = \alpha_3$.

All components have a linear decreasing mean residual life function i.e. α_i>1.

All components have a linear increasing mean residual life function i.e. α_i<1.

The first component has a increasing mean residual life function while the rest two components have an decreasing mean residual life function.

The first and second components have a increasing mean residual life function while the third one has an decreasing mean residual life function.



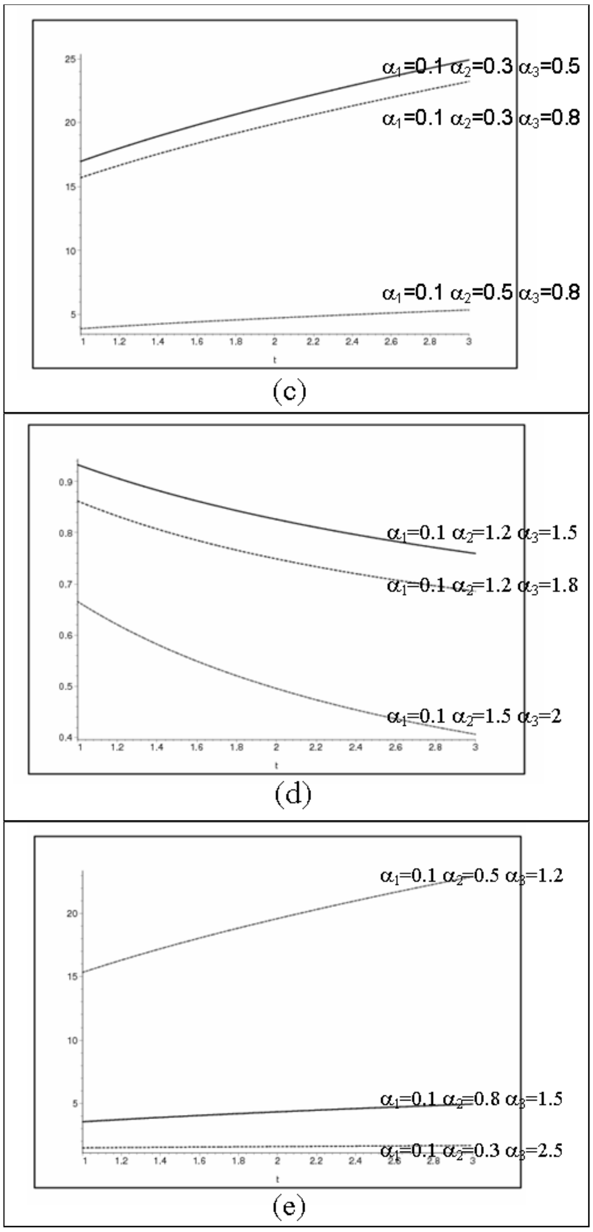


Fig.2. The MRL Curves of the 2-out-of-3 System with Weibull Distributed components (λ=1).

11 n=3	$\alpha_1, \alpha_2, \alpha_3$	t=1	t=2	t=3	t=4	t=5
k=1	1.00, 1.00, 1.00	1.83	1.83	1.83	1.83	1.83
	0.50, 0.50, 0.50	8.39	9.91	11.07	12.06	12.92
	2.00, 2.00, 2.00	0.65	0.40	0	0	0
	1.10, 1.20, 1.30	1.38	1.28	1.21	1.16	1.13
	1.10, 1.50, 2.00	1.12	0.99	0.92	0.88	0.75
	0.30, 0.50, 0.80	25.82	31.04	35.07	38.48	41.52
	0.50, 1.20, 1.50	4.23	4.98	5.58	6.09	6.55
	0.50, 1.50, 2.00	4.12	4.89	5.50	6.02	6.39
	0.50, 0.80, 1.20	4.63	5.41	6.02	6.54	7.00
	0.50, 0.80, 2.50	4.55	5.35	5.98	6.51	6.98
k=2	1.00, 1.00, 1.00	0.83	0.83	0.83	0.83	0.83
	0.50, 0.50, 0.50	2.72	3.41	3.94	4.39	4.78
	2.00, 2.00, 2.00	0.34	0.19	0	0	0
	1.10, 1.20, 1.30	0.65	0.58	0.55	0.52	0.50
	1.10, 1.50, 2.00	0.48	0.36	0.30	0.26	0.24
	0.30, 0.50, 0.80	2.78	3.45	3.97	4.41	4.80
	0.50, 1.20, 1.50	0.78	0.72	0.68	0.64	0.62
	0.50, 1.50, 2.00	0.59	0.45	0.38	0.33	0.30
	0.50, 0.80, 1.20	1.18	1.29	1.37	1.43	1.48
	0.50, 0.80, 2.50	0.98	1.08	1.18	1.26	1.33
k=3	1.00, 1.00, 1.00	0.33	0.33	0.33	0.33	0.33
	0.50, 0.50, 0.50	0.89	1.17	1.38	1.56	1.71
	2.00, 2.00, 2.00	0.15	0.08	0	0	0
	1.10, 1.20, 1.30	0.27	0.24	0.22	0.21	0.20
	1.10, 1.50, 2.00	0.20	0.13	0.10	0.08	0.07
	0.30, 0.50, 0.80	0.75	0.91	1.03	1.11	1.19
	0.50, 1.20, 1.50	0.30	0.25	0.22	0.20	0.19
	0.50, 1.50, 2.00	0.22	0.15	0.11	0.09	0.07
	0.50, 0.80, 1.20	0.40	0.41	0.41	0.41	0.40
	0.50, 0.80, 2.50	0.22	0.12	0.07	0.05	0.03

Table 1. The Mean Residual Life at Different System Level.

Based on the Fig.2, the following conclusions are possible. It is seen from Fig.2(a) that the components having constant mean residual life functions, i.e $\alpha_1 = \alpha_2 = \alpha_3$=1, the mean residual life function of the 2-out-of-3 system is constant. When all components identically distributed and α_i>1, the system has decreasing MRL function, otherwise the system has increasing MRL function. When either all components have a linear decreasing mean residual life function (b), i.e. α_i>1, or two components have linear decreasing mean residual life function (d), the system has a linear decreasing. As the values of α_i get larger, the values of mean residual life decrease (b). Either all components have linear increasing mean residual life function (c), i.e. α_i<1, or two components have linear increasing mean residual life function (e), the system has increasing mean residual life function.

In Table 1, a particular case with n=3 and k=1, 2, 3 is analyzed numerically to study the effect of increasing the system level and various parameters on the mean residual life of the system. The mean residual life of the k-out-of-3 configuration was calculated versus different parameters of required units. All the computations were done using Maple 5.1.

4. Conclusions

In this paper, we have provided new representations of the MRL function for the parallel system and k-out-of-n system. Also we introduce a numerical example to study the effect of increasing the system level and various parameters on the mean residual life of the systems.

A parallel system is equivalent to a 1-out-of-3 system, i.e. the k is equal to 1, while a series system is equivalent to a 3-out-of-3 system, i.e. the k is equal to 3. The system structure changes from a parallel structure to a 2-out-of-3 structure, then to a series structure. In other words, the system structure changes from strong to weak as the system level increases. So it is necessary to provided redundant equipment in a parallel structure, in cases where the failure of the system is not acceptable. When the components have constant mean residual life functions, i.e. coming from the exponential distribution; the mean residual life function of the system is constant in all the system level. Since the system structure changes from strong to weak as system levels increases, the mean residual life decreases for all parameters.

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