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RESEARCH

Evolution of solar-type activity: acoustic and magnetic energy generation and propagation in β Hydri (G2 IV)

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Abstract

We examine the acoustic and magnetic energy generation and propagation in β Hydri (G2 IV). The underlying motivation for this work is based on the solar, stellar, and galactic relevance of β Hyi (a star in the Southern hemisphere), which is readily understood as a prime example and proxy of the future Sun – thus allowing assessments and analyses of the secular decay of solar activity. Regarding the magnetic energy generation, we consider longitudinal flux tube waves. We also assess acoustic waves. For the acoustic wave energy flux, the difference between the results obtained for β Hyi and the Sun is significantly smaller than typically attained for main-sequence stars, which is largely due to the gravity-dependence of the acoustic energy generation. Furthermore, we study the height-dependent behavior of the magnetic energy flux for different magnetic filling factors corresponding to different flux tube spreadings. Finally, we comment on possible directions of future research.

Keywords Stars: activity · Stars: individual: β Hydri · Stars: magnetic fields · Stars: chromospheres · Sun: evolution · Turbulence

1 Introduction

Solar-type stars, while situated on the main-sequence, are known to exhibit fundamental changes over long timescales, i.e., millions and billions of years. The changes pertain both the stellar interior as well as the atmospheres and winds, entailing large sets of observational consequences. In this study, we focus on β Hydri (β Hyi; HD 2151, HR 98, HIP 2021), a star in the Southern Hemisphere that is part of the constellation Hydrus. Previous modelling by Brandão et al. (2011) has reconfirmed β Hyi's spectral type as G2 IV, among other spectral properties.

Dravins et al. (1993a,b,c) noted various aspects of interests about β Hyi, which are (1) intriguing facts about its photosphere such as relatively large (a factor of $\simeq 5$) convective cells (granules) as revealed by 3-D numerical simulations, (2) reduced photospheric pressure leading to higher granular velocities, (3) decreased chromospheric activity compared to the Sun (but showing intricate time variability), (4) fainter transition region and coronal emission relative to the Sun, and (5) stellar wind conditions shaped by post–main-sequence thermodynamics. Based on further work, β Hyi's age has been identified as about 6.5 Gyr (Dravins et al. 1998; Brandão et al. 2011).

Another aspect of β Hyi is the existence of solar-type oscillations (e.g., Bedding et al. 2001; Carrier et al. 2001; Bedding et al. 2007; Fernandes and Monteiro 2003; Di Mauro et al. 2003; Doğan et al. 2010), which allowed to further establish β Hyi's evolutionary status and to place additional constraints on β Hyi's stellar parameters. The asteroseismic analysis of those data allowed to identify both differences and similarities to the solar spectrum. In addition, those detailed studies of β Hyi served as an important application and test-bed of existing stellar evolution and structure codes.

Stars akin to β Hyi are characterized by reduced outer atmospheric heating as well as a noticeably decreased level of chromospheric emission, with the latter confirmed by observations (e.g., Dravins et al. 1993b; Buccino and Mauas 2008). Clearly, when stars evolve away from the main-sequence approaching the subgiant stage, the magnitude of magnetic activity is considerably reduced as a consequence of the restructuring of the stellar interior, associated with the

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Table 1 Stellar parameters

Parameter	Value	Reference
M ($M_\odot$)	1.08 ± 0.03	Brandão et al. (2011)
R ($R_\odot$)	1.809 ± 0.015	Brandão et al. (2011)
$T_{\rm eff}$ (K)	5872 ± 44	North et al. (2007)
L ($L_\odot$)	3.494 ± 0.087	Brandão et al. (2011)
log $g_\star$ (cgs)	4.02 ± 0.04	da Silva et al. (2006)
[Fe/H] (dex)	-0.10 ± 0.07	Bruntt et al. (2010)
Age (Gyr)	6.40 ± 0.56	Brandão et al. (2011)

Note: All symbols have their usual meaning

Table 2 Determination of $T_{\rm eff}$

Parameter	Value	Reference
$T_{\rm eff}$ (K)	5872 ± 44	North et al. (2007)
...	5774 ± 60	Di Benedetto (1998)
...	5800 ± 100	Dravins et al. (1993a)

Table 3 Determination of the stellar age

Parameter	Value	Reference
Age (Gyr)	6.40 ± 0.56	Brandão et al. (2011)
...	6.75 ± 0.35	Fernandes and Monteiro (2003)
...	$\sim$6.7	Dravins et al. (1998)
...	9.5 ± 0.8	Dravins et al. (1993a)

redistribution of the angular momentum, and the loss of angular momentum due to the stellar wind (e.g., Schrijver and Pols 1993; Charbonneau et al. 1997; Johnstone et al. 2015); see also Metcalfe et al. (2022) for updated analyses. Preliminary results for β Hyi based on theoretical two-component chromosphere models (acoustic and magnetic) have been given by Cuntz and Fawzy (2022), which also include some comparisons with observations.

In this work, we explore aspects of acoustic and magnetic energy generation in β Hyi with a focus on longitudinal flux-tube waves (LTWs) and to a lesser degree on acoustic waves (ACWs). In Sect. 2, we convey our theoretical approach, including a discussion of the stellar parameters and the employed flux-tube models. Our results and discussion are given in Sect. 3. In Sect. 4, we present our summary and conclusions.

2 Theoretical approach

2.1 Stellar parameters

Studies of acoustic and magnetic energy generation require the usage of various stellar parameters. In case of β Hyi information is given in Table 1. As pointed out in this study as well as previous studies, the initial acoustic and magnetic wave energy depend on the effective temperature $T_{\rm eff}$, surface gravity log $g_\star$, and the metallicity. The key stellar parameters of β Hyi read as follows: $M_\star = 1.08\ M_\odot$, $R_\star = 1.81\ R_\odot$, $T_{\rm eff} = 5872$ K, $L_\star = 3.49\ L_\odot$, log $g_\star = 4.02$ (cgs), and [Fe/H] $= -0.10$ (dex) (da Silva et al. 2006; North et al. 2007; Bruntt et al. 2010; Brandão et al. 2011).

Dravins et al. (1993a) thoroughly discussed the status of β Hyi as a nearby subgiant and a valid proxy for the future Sun, exhibiting greater granular velocities (about a factor of $\simeq$1.5 to 2) noting that an almost identical surface energy flux must be carried by a lower photospheric gas density. Moreover, the scale of the photospheric granulation is expected to be larger, and a considerably smaller number of granules will simultaneously fit on its stellar surface; an outcome that is relevant for β Hyi's presumed magnetic surface structure.

Furthermore, consensus has been achieved about β Hyi's effective temperature (see Table 2). The age of β Hyi has been given as about 6.5 Gyr (Dravins et al. 1998; Brandão et al. 2011); see Table 3. Brandão et al. (2011) applied detailed asteroseismic modelling, stellar surface structure assessments, and uncertainty analysis to arrive at that value. Previous studies in part also based on β Hyi's solar-like oscillations have been given by Carrier et al. (2001) and Fernandes and Monteiro (2003). The latter authors also focused on a determination of β Hyi's mass, providing further support of the interpretation that this star very closely represents the future Sun.

2.2 Theory of acoustic and magnetic energy generation

Akin to our previous work, the amount of energy generated by ACWs or LTWs in the stellar convective zone uses the concept of the mixing-length theory, including adjustments made based on previous 3-D (magneto-)hydrodynamic models of stellar convection; see Trampedach et al. (1997) and Stein et al. (2009a,b). Previous results in that regard have been given by Musielak et al. (1989, 1994, 1995), Ulmschneider et al. (1996), and Ulmschneider et al. (2001).

For example, the work of Ulmschneider et al. (2001) examines LTW wave energy fluxes carried along vertical magnetic flux tubes embedded in the atmospheres of late-type stars, including solar-type stars, subgiants and giants. In their work, the main physical process responsible for the generation of these waves is the nonlinear time-dependent response of the flux tubes to continuous and impulsive external turbulent pressure fluctuations; the latter are represented by an extended Kolmogorov spatial and modified Gaussian temporal energy spectrum. Underlying theoreti-

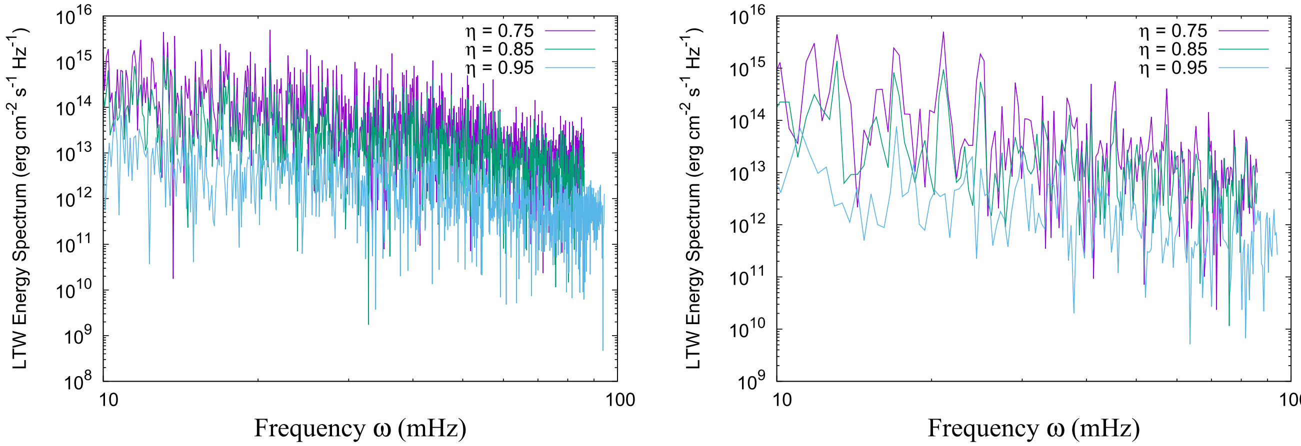

Fig. 1 LTW energy spectra for β Hyi assuming $\alpha_{\mathrm{ML}} = 1.8$ and different values of η while depicting the not-smoothed version (left panel) and the smoothed version of the data (right panel)

cal studies about stellar convection include work by Steffen et al. (1989), Cattaneo et al. (1991), and Nordlund et al. (1997).

In this study, we assume a mixing-length parameter of $\alpha_{\mathrm{ML}} = 1.8$ as advocated by Stein et al. (2009a,b). Our main magnetic models are based on $\eta = 0.85$ (see definition below). Figure 1 conveys the LTW energy spectra for β Hyi; see Table 1 for information on the stellar parameters. In fact, we present both the not-smoothed version and the smoothed version of the data, with the latter being utilized for the acoustic and magnetic energy generation; see Sect. 3.1.

2.3 Flux tubes models

Another important step of our approach is the construction of unperturbed and unheated initial magnetic flux-tube models; they are characterized by outwardly decreasing temperatures. This step requires specifying four physical parameters, which are: the stellar effective temperature T_{eff}, the surface gravity log $g_\star$, the magnetic field strength at the base of the flux tube $B_0(z = 0 \text{ km})$, and the magnetic filling factor f. Another parameter (with limited influence on our results) is the stellar metallicity, which for β Hyi is very close to the solar value (Bruntt et al. 2010). For more detailed discussions about the selected approach to call attention to the work by Ulmschneider et al. (2001) and references therein. For the current computations, we consider the thin flux-tube approximation (e.g., Roberts and Webb 1979; Spruit 1981; Ferriz-Mas et al. 1989; Hasan et al. 2003), which is readily applicable to most chromospheric layers of late-type stars (e.g., Stenflo 1978; Solanki 1993; Yelles Chaouche et al. 2009).

The height dependence of the tube geometry is physically governed by the initial photospheric magnetic filling factor and by the conservation of the horizontal magnetic pressure with the external gas pressure, noting that the guiding equation is given as: $B_{\mathrm{eq}}^2/8\pi + p_i = p_e$ with p_i and p_e being the internal and external gas pressures at a given height. The maximum tube opening is determined from the following equation: $f = r_{\mathrm{bottom}}^2/r_{\mathrm{top}}^2$ where r_{bottom} and r_{top} are the bottom and top radii of the flux tube at the stellar surface and at the maximum opening height, respectively. Akin to previous solar models (e.g., Fawzy et al. 1998), the bottom radius of the magnetic flux tube is assumed as about half the local pressure scale height. For early studies of β Hyi's photospheric structure see Dravins et al. (1993a).

For the determination of the magnetic field strength at height $z = 0$ km, we first compute the maximum allowed field of a void tube assuming the inner gas pressure $p_i = 0$ dyne cm^{-2}; this values also known as the equipartition field strength given by $B_{\mathrm{eq}} = \sqrt{p_e/8\pi}$ taken at 0 km, the photospheric reference height. According to the initial atmospheric model, the resulting equipartition magnetic field strength is given as $B_{\mathrm{eq}} = 1505$ G. A comparison to the solar case, as well as in alignment to previous magneto-acoustic model simulations for other stars (e.g., Fawzy and Cuntz 2018, 2021; Cuntz et al. 2021), we choose a surface magnetic field strength given as 0.85 B_{eq}. Nevertheless, as part of our parameter study of magnetic energy generation based on longitudinal flux-tube waves, we consider a larger set of parameters $\eta = B/B_{\mathrm{eq}}$, given as 0.75, 0.85, and 0.95.

In the current study, we consider three different surface magnetic filling factors, namely, 1%, 5%, and 10%, resulting in a bottom radius of about $r_{\mathrm{bottom}} = 196$ km and three top opening radii, given as $r_{\mathrm{top}} = 1750$ km, 781 km, and 552 km, respectively; see Fig. 2. Note that our work is most relevant for the low and middle part of β Hyi's chromosphere. Future studies aimed at β Hyi's high chromosphere and wind would need to consider also the impact of wave coupling effects and wave mode conversions; see, e.g., Srivastava et al. (2021) and references therein.

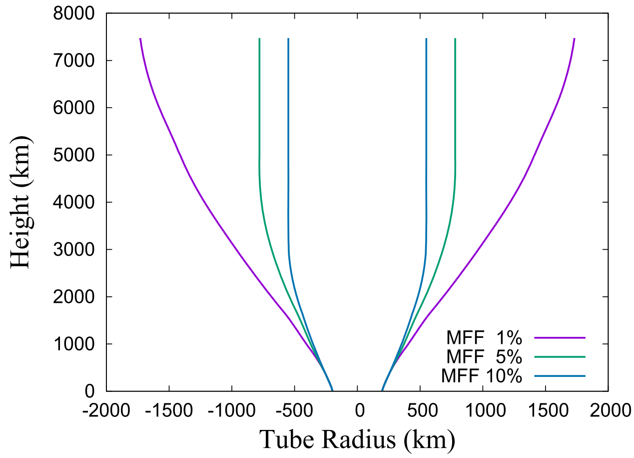

Fig. 2 Flux tube models for β Hyi regarding different magnetic filling factors

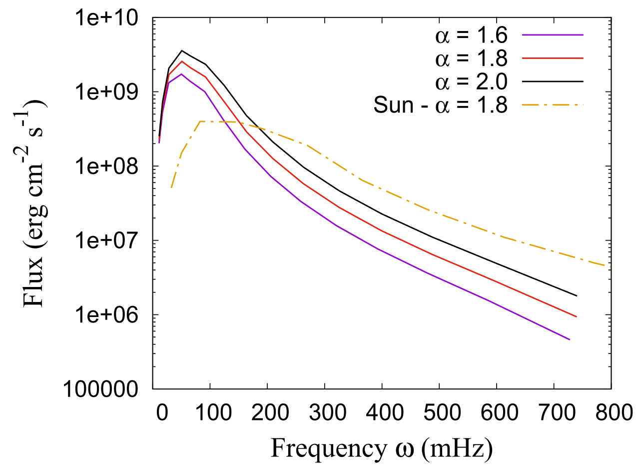

Fig. 3 Acoustic wave energy fluxes for β Hyi with mixing-length parameters $\alpha_{\mathrm{ML}} = 1.6$, 1.8, and 2.0 and the Sun assuming $\alpha_{\mathrm{ML}} = 1.8$

3 Results and discussion

3.1 Acoustic and magnetic energy generation

The main component of this study concerns the acoustic and magnetic energy generation, with a focus on LTWs, for β Hyi as facilitated at photospheric levels. As discussed in Sect. 2.2, we make use of previous works in the literature (e.g., Ulmschneider et al. 2001). Regarding LTWs, these kinds of models consider the nonlinear time-dependent response to external pressure fluctuations acting on the flux tubes embedded in the stellar atmosphere. Following detailed 3-D simulations for the Sun, Stein et al. (2009a,b) indicated that $\alpha_{\mathrm{ML}} = 1.8$ might be most realistic. Since it is still unsettled how this kind of result would translate to stars of lower surface gravity, calculations assuming a larger set of α_{ML} are desirable.

Figure 3 depicts acoustic wave energy fluxes for mixing-length parameters between 1.6, 1.8, and 2.0. For comparison, we also give the result for the solar case with $\alpha_{\mathrm{ML}} = 1.8$; see Fawzy and Cuntz (2011) and references therein for information on the relevant model parameters. Furthermore, Fig. 4 depicts a comparison between the generated flux for LTWs for β Hyi with mixing-length parameters $\alpha_{\mathrm{ML}} = 1.6$, 1.8, and 2.0 and for $\eta = 0.85$. Moreover, a comparison between the results for β Hyi pertaining to LTW fluxes for $\alpha_{\mathrm{ML}} = 1.8$ and for $\eta = 0.75$, 0.85, and 0.95 is conveyed in Fig. 5. The numerical values of the energy flux generation for ACWs and LTWs are given in Table 4 and 5, respectively.

It is found that for ACWs, the amount of generated energy is notably higher (about a factor of 2.3) in β Hyi compared to the Sun. This behavior closely related to the greater photospheric granular velocity of β Hyi due to its lower surface gravity; see Dravins et al. (1993a). However, for LTWs, the amount of generated energy is somewhat lower (about a

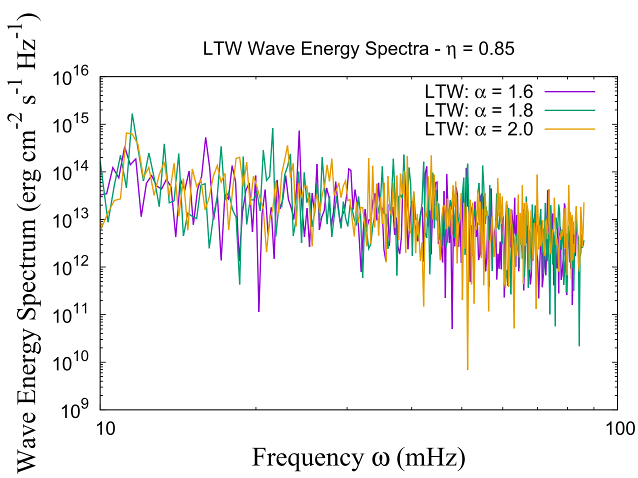

Fig. 4 A comparison between the generated flux for LTWs for β Hyi with mixing-length parameters $\alpha_{\mathrm{ML}} = 1.6$, 1.8, and 2.0 and for $\eta = 0.85$

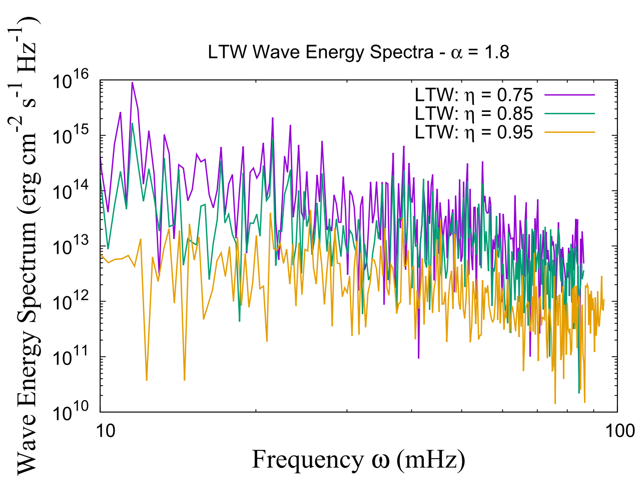

Fig. 5 A comparison between the generated flux for LTWs for β Hyi with mixing-length parameter $\alpha_{\mathrm{ML}} = 1.8$ and for $\eta = 0.75$, 0.85, and 0.95

Table 4 Acoustic energy generation

α_{ML}	Wave energy flux[a]	
	β Hyi	Sun
1.6	1.62e+8	6.94e+7
1.8	2.51e+8	1.05e+8
2.0	3.65e+8	1.58e+8

[a]Unit: $\mathrm{erg\,cm^{-2}\,s^{-1}}$

Table 5 LTW energy generation

α_{ML}	η	Wave energy flux[a]	
		β Hyi	Sun
1.6	0.75	3.65e+8	4.54e+8
...	0.85	2.19e+8	2.55e+8
...	0.95	6.47e+7	7.27e+7
1.8	0.75	4.89e+8	5.52e+8
...	0.85	2.74e+8	3.15e+8
...	0.95	7.22e+7	9.41e+7
2.0	0.75	5.79e+8	6.84e+8
...	0.85	2.88e+8	3.99e+8
...	0.95	8.92e+7	1.13e+8

[a]Unit: $\mathrm{erg\,cm^{-2}\,s^{-1}}$

factor of 1.1 to 1.3) in β Hyi compared to the Sun. Hence, the difference between magnetic and acoustic energy generation is relatively small for β Hyi, contrary to solar-type stars, including the Sun. Additionally, it is found that lower values of $\eta = B/B_{eq}$ lead to higher amounts of magnetic energy generation, a finding akin to previous results for other stars, especially main-sequence stars. In fact, those results indicate that the magnetic energy flux for tubes $\eta = 0.75$ can be about one order of magnitude higher compared to models of 0.95. This can be explained by the fact that the increase in η increases the stiffness of the magnetic flux tube, which in turn decreases the efficiency of the LTW wave generations.

Figure 6 depicts the two primary input wave energy spectra β Hyi obtained in this study. We show the case of LTWs with $\alpha_{ML} = 1.8$ and $\eta = 0.85$ and the corresponding case of ACWs. It is found that the difference in flux for LTWs and ACWs is much smaller than readily obtained for main-sequence stars; in fact, the wave energy flux for ACWs in evolved stars is relatively high owing to the gravity-dependence of the acoustic energy generation (e.g., Ulmschneider et al. 1996). Specifically, for $\alpha_{ML} = 1.8$ and $\eta = 0.85$ the ratio for the wave energy fluxes between β Hyi and the Sun reads 2.3 for ACWs and 0.87 for LTWs. From an astrophysical perspective, the relatively high ACW energy flux in β Hyi is due to the relatively high convective flow speeds in subgiants relative to main-sequence stars with comparable stellar parameters.

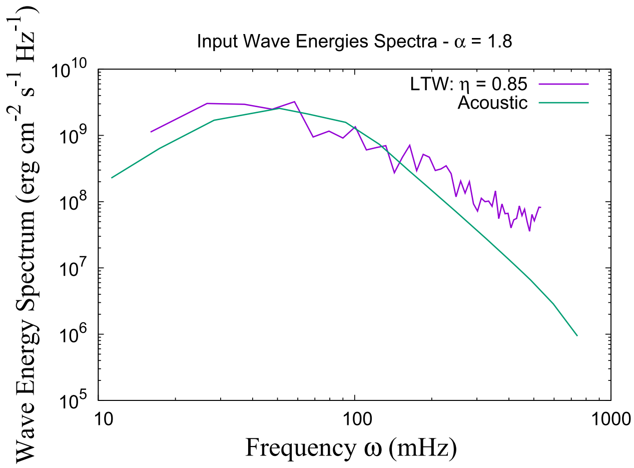

Fig. 6 Depiction of input wave energy spectra β Hyi. We show the case of longitudinal flux-tube waves with $\alpha_{ML} = 1.8$ and $\eta = 0.85$ and the corresponding case of acoustic waves

3.2 Comments on wave heating models and flux tube spreading

Figure 7 gives an example of a magnetic heating models for β Hyi. We convey a snapshot of a longitudinal flux tube model with MFF = 1% at an elapsed time of 3364 s. Various quantities are depicted, including a strong shock at height 2700 km with a shock strength measured as $M_{sh} = 6.3$. Time-dependent ionization has been considered. It is a monochromatic wave model with a wave period of 400 s, a value significantly higher than in models previously given for the Sun in responds to β Hyi's reduced surface gravity. A total of 14 shocks have been inserted. Additional time-dependent results for β Hyi's magnetically heated outer atmosphere have previously been given by Cuntz and Fawzy (2022). These models also considered the computation of Ca II fluxes, including tentative comparisons with observational constraints.

In Fig. 8, we examine the height-dependent behavior of the mechanical energy flux for LTWs regarding β Hyi. We consider both monochromatic and spectral waves while considering models based on MFF of 1%, 5%, and 10%, respectively. The initial wave energy fluxes are given as $2.74 \cdot 10^8\ \mathrm{erg\,cm^{-2}\,s^{-1}}$; this value corresponds to our main model given as $\alpha_{ML} = 1.8$ and $\eta = 0.85$. It is found that a higher MFF entails a somewhat smaller decrease of the LTW energy flux as a function of height, especially in the middle chromosphere, mostly associated with the difference in tube spreadings. However, at large heights other effects are relevant as well, especially in narrow tubes, including effects associated with strong shocks, which initiate both quasi-adiabatic cooling and a reduction of the time-averaged densities; see, e.g., Fawzy et al. (2012) for previous results.

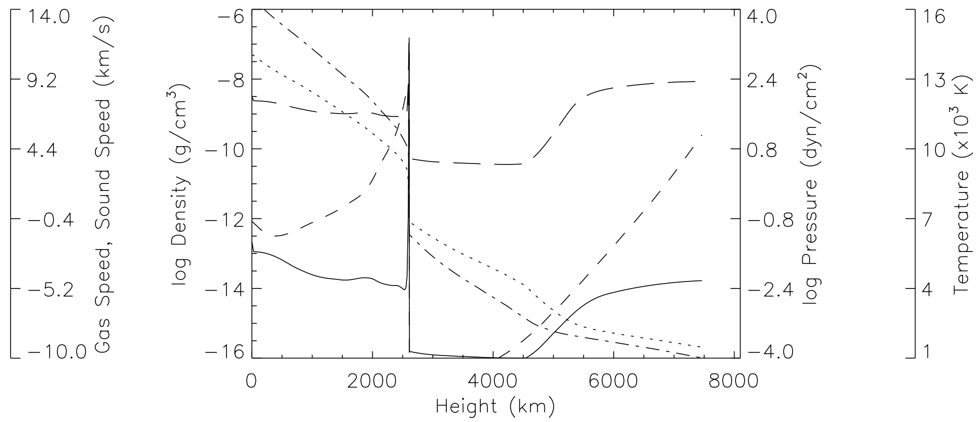

Fig. 7 Snapshot of a longitudinal flux tube model with MFF = 1% at an elapsed time of 3364 s. The following quantities are shown: temperature (solid line), gas density (short-dashed line), gas speed (midway-dashed line), sound speed (long-dashed line), and gas pressure (dashed-dotted line)

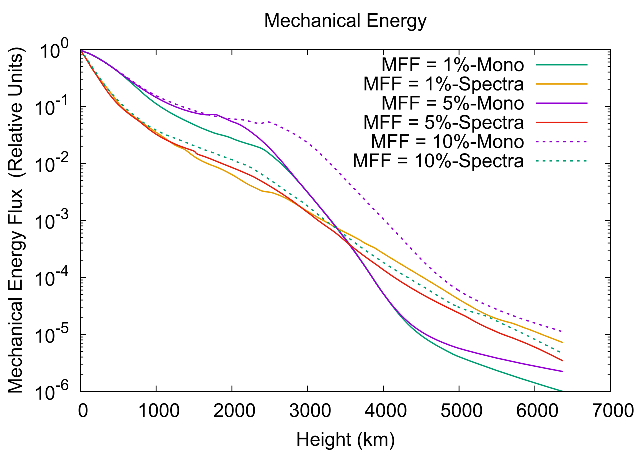

Fig. 8 Height-dependent behavior of the mechanical energy flux in models of longitudinal flux-tube waves (relative units) for β Hyi. We consider both monochromatic waves and wave energy spectra assuming magnetic filling factors of 1%, 5%, and 10%, respectively. The initial wave energy fluxes are given as $2.74 \cdot 10^8$ erg cm^{-2} s^{-1}

3.3 The big picture

It is of obvious interest to compare both the acoustic and magnetic heating in β Hyi to the solar case. Previous work by Bohn (1984) determined that the amount of the generated acoustic energy obeys the relation $F_M^{AC} \propto T_{eff}^{9.75} g_\star^{-0.5}$ with T_{eff} and $g_\star$ as stellar effective temperature and surface gravity, respectively. Based on $T_{eff} = 5872 \pm 44$ K for β Hyi (North et al. 2007), the generated acoustic energy is increased between 10% and 25% depending on the adopted value of T_{eff}. However, based on β Hyi's surface gravity, F_M^{AC} is increased by a factor of about 2.5 (see Table 4), as expected for evolved stars. Concerning the LTW models, the difference in the wave energy generation between β Hyi and the Sun is notable as well. For the different values of α_{ML} and η (see Table 5), the initial wave energy in β Hyi relative to the Sun is increased by a factor of 1.1 to 1.3, depending on the model.

For evolved stars, exhibiting very low chromospheric activity, also referred to as chromospheric basal flux stars, previous observational work by, e.g., Schrijver (1987) and Rutten et al. (1991) indicates that their emergent emission in Ca II and Mg II is close to the values obtained for low-activity main-sequence stars; for more information see Schrijver (1995). This outcome has also been reproduced by detailed theoretical heating models as given by, e.g., Ulmschneider (1989), Buchholz et al. (1998), Cuntz et al. (1994), and Fawzy et al. (2002).

This behavior is due to the fact that evolved stars possess more extended radiative zones. In those stars, the wave energy flux decreases much more rapidly as a function of height compared to main-sequence stars; hence, the amount of energy available in the Ca II and Mg II formation regions is largely insensitive to the amount of initial wave energy and (by implication) to the stellar surface gravity. Stars akin to β Hyi are characterized by moderate amounts of magnetic activity; hence, those atmospheric structures are more complex. Consequently, more detailed models are required, ideally encompassing different kinds of magnetic heating modes and magnetic field configurations.

Another approach has been pursued by Pérez Martínez et al. (2011). They considered a set of cool G, K, and M giants and supergiants (with evolutionary stages significantly beyond that of β Hyi) and inspected their Mg II $h + k$ emissions. They found that the results (when considering the various statistical uncertainties) agree well with the assumption of nonmagnetic energy dissipation (supposedly provided by acoustic waves) in the turbulent chromospheres. This is a stark motivation to further investigate the relationship between (magneto-)acoustic heating and chromospheric emission for β Hyi, an intermediate case between stars of considerable magnetic activity and chromospheric basal flux stars.

4 Summary and conclusions

The aim of this work has been the examination of the generation and propagation of acoustic and magnetic energy regarding β Hyi. This star is considered a benchmark G2 IV star situated in the solar neighborhood. Early studies about β Hyi (Dravins et al. 1993a,b,c) focused on stellar evolutionary aspects, photospheric structure, chromospheric activity and variability as well as its transition region, corona, and stellar wind. In the meantime, β Hyi's age has been determined as approximately 6.5 Gyr (e.g., Brandão et al. 2011), thus confirming its status as a proxy of the future Sun, thus establishing β Hyi's relevance for studies of stellar evolution and circumstellar habitability (e.g., Sackmann et al. 1993; Di Mauro et al. 2003; Ribas et al. 2005, and related work).

In this study, we focus on calculations of the initial magnetic and acoustic energy generation with a concentration on longitudinal flux-tubes waves. Previous theoretical work in that context has been given by Musielak et al. (1989, 1994, 1995) and Ulmschneider et al. (2001). Specifically, we consider models with MFF of 1%, 5%, and 10% to explore the height-dependent behavior of the magnetic wave energy flux. As previously identified for models of other stars, including the Sun (e.g. Fawzy et al. 1998), as smaller valuer MFF generally implies reduced spreading of the magnetic flux tubes at both photospheric and chromospheric heights.

Concerning ACW and LTW photospheric energy generation, we found the following results:

(1) Both for ACWs and LTWs, a larger mixing-length parameter α_{ML} entails a higher amount of energy generation in agreement with previous results (e.g., Ulmschneider et al. 1996, and related work).
(2) Regarding ACWs, the amount of generated energy is notably higher (about a factor of 2.3) in β Hyi compared to the Sun. This behavior is due to the greater photospheric granular velocity of β Hyi due to its lower surface gravity; see Dravins et al. (1993a).
(3) Regarding LTWs, the amount of generated energy is somewhat lower (about a factor of 1.1 to 1.3) in β Hyi, a behavior associated with its different thermodynamic and magnetic field conditions. Consequently, the difference between magnetic and acoustic energy generation is relatively small for β Hyi, contrary to solar-type main-sequence stars.
(4) Lower values of $\eta = B/B_{\mathrm{eq}}$ result in higher amounts of magnetic energy generation, a finding akin to previous results for other stars, including main-sequence stars; e.g., Ulmschneider et al. (2001). In these stars the flux can be about one order of magnitude higher in tubes with $\eta = 0.75$ compared to 0.95.

The fact that both the acoustic and magnetic energy generation in β Hyi is reduced compared to the Sun is a consequence of differences regarding their thermodynamic and magnetic properties at photospheric heights. In case of β Hyi a reduced surface density as well as a higher photospheric granular velocity (convective speed) occurs. Moreover, a higher MFF entails a somewhat smaller decrease of the LTW energy flux as a function of height, especially in the middle chromosphere, due to the MFF-dependent tube spreadings. At large heights, those differences are less pronounced because the relatively high energy fluxes in narrow tubes initiate strong shocks, which notably affect the thermal structure of the tubes due to quasi-adiabatic cooling; see, e.g., Fawzy et al. (2012) and Carlsson and Stein (1992, 1995) for previous results pertaining to magnetic and acoustic waves.

The differences in the magnetic energy generation at photospheric heights go hand-in-hand with the decrease of stellar activity for stars like β Hyi relative to the Sun. Observationally, it has been found that there is a notably reduced Ca II and Mg II emission in the line core fluxes, as well as evidence for cyclic chromospheric activity; see, e.g., Dravins et al. (1993b) and Buccino and Mauas (2008). Previous studies about β Hyi's upper atmosphere, notably the stellar corona and wind, have been given by, e.g., Dravins et al. (1993c), Güdel et al. (1998), and Pizzolato et al. (2000). Based on those analyses, it is expected that the modes of energy propagation and dissipation taken into account in this study, although nonmagnetic processes are expected to dominate in evolved stars, are insufficient to account for the vast variety of dynamic features in those regions, implying the presence of other processes. For moderately evolved stars, there is also evidence pointing to the significance of 3-D structures, associated with large-scale turbulence, as pointed out by Judge and Cuntz (1993) and others.

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Author contributions D.E.F. pursued the calculations and prepared the figures. M.C. has been the main contributor to the text.

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Data Availability The data that support the findings of this study are available from D.E.F. upon reasonable request.

Declarations

Informed consent All authors reviewed the manuscript and agreed to its content.

Competing interests The authors declare no competing interests.

References

Bedding, T.R., Butler, R.P., Kjeldsen, H., et al.: Astrophys. J. Lett. **549**, L105 (2001)

Bedding, T.R., Kjeldsen, H., Arentoft, T., et al.: Astrophys. J. **663**, 1315 (2007)
Bohn, H.U.: Astron. Astrophys. **136**, 338 (1984)
Brandão, I.M., Doğan, G., Christensen-Dalsgaard, J., et al.: Astron. Astrophys. **527**, A37 (2011)
Bruntt, H., Bedding, T.R., Quirion, P.-O., et al.: Mon. Not. R. Astron. Soc. **405**, 1907 (2010)
Buccino, A.P., Mauas, P.J.D.: Astron. Astrophys. **483**, 903 (2008)
Buchholz, B., Ulmschneider, P., Cuntz, M.: Astrophys. J. **494**, 700 (1998)
Carlsson, M., Stein, R.F.: Astrophys. J. Lett. **397**, L59 (1992)
Carlsson, M., Stein, R.F.: Astrophys. J. Lett. **440**, L29 (1995)
Carrier, F., Bouchy, F., Kienzle, F., et al.: Astron. Astrophys. **378**, 142 (2001)
Cattaneo, F., Brummell, N.H., Toomre, J., Malagoli, A., Hulburt, N.E.: Astrophys. J. **370**, 282 (1991)
Charbonneau, P., Schrijver, C.J., MacGregor, K.B.: In: Jokipii, J.R., Sonett, C.P., Giampapa, M.S. (eds.) Cosmic Winds and the Heliosphere. Space Science Series, p. 677. University of Arizona Press, Tucson (1997)
Cuntz, M., Fawzy, D.E.: Res. Notes AAS **6**, 147 (2022)
Cuntz, M., Rammacher, W., Ulmschneider, P.: Astrophys. J. **432**, 690 (1994)
Cuntz, M., Schröder, K.-P., Fawzy, D.E., Ridden-Harper, A.R.: Mon. Not. R. Astron. Soc. **505**, 274 (2021)
da Silva, L., Girardi, L., Pasquini, L., et al.: Astron. Astrophys. **458**, 609 (2006)
Di Benedetto, G.P.: Astron. Astrophys. **339**, 858 (1998)
Di Mauro, M.P., Christensen-Dalsgaard, J., Paternò, L.: Astrophys. Space Sci. **284**, 229 (2003)
Doğan, G., Brandão, I.M., Bedding, T.R., et al.: Astrophys. Space Sci. **328**, 101 (2010)
Dravins, D., Lindegren, L., Nordlund, Å., VandenBerg, D.A.: Astrophys. J. **403**, 385 (1993a)
Dravins, D., Linde, P., Fredga, K., Gahm, G.F.: Astrophys. J. **403**, 396 (1993b)
Dravins, D., Linde, P., Ayres, T.R., et al.: Astrophys. J. **403**, 412 (1993c)
Dravins, D., Lindegren, L., VandenBerg, D.A.: Astron. Astrophys. **330**, 1077 (1998)
Fawzy, D.E., Cuntz, M.: Astron. Astrophys. **526**, A91 (2011)
Fawzy, D.E., Cuntz, M.: Astrophys. Space Sci. **363**, 152 (2018)
Fawzy, D.E., Cuntz, M.: Mon. Not. R. Astron. Soc. **502**, 5075 (2021)
Fawzy, D.E., Ulmschneider, P., Cuntz, M.: Astron. Astrophys. **336**, 1029 (1998)
Fawzy, D., Ulmschneider, P., Stępień, K., Musielak, Z.E., Rammacher, W.: Astron. Astrophys. **386**, 983 (2002)
Fawzy, D.E., Cuntz, M., Rammacher, W.: Mon. Not. R. Astron. Soc. **426**, 1916 (2012)
Fernandes, J., Monteiro, M.J.P.F.G.: Astron. Astrophys. **399**, 243 (2003)
Ferriz-Mas, A., Schüssler, M., Anton, V.: Astron. Astrophys. **210**, 425 (1989)
Güdel, M., Guinan, E.F., Skinner, S.L.: In: Donahue, R.A., Bookbinder, J.A. (eds.) Cool Stars, Stellar Systems, and the Sun 10. Astr. Soc. Pac. Conf. Ser., vol. 154, p. 1041. ASP, San Francisco (1998)
Hasan, S.S., Kalkofen, W., van Ballegooijen, A.A., Ulmschneider, P.: Astrophys. J. **585**, 1138 (2003)
Johnstone, C.P., Güdel, M., Brott, I., Lüftinger, T.: Astron. Astrophys. **577**, A28 (2015)
Judge, P.G., Cuntz, M.: Astrophys. J. **409**, 776 (1993)
Metcalfe, T.S., Finley, A.J., Kochukhov, O., et al.: Astrophys. J. Lett. **933**, L17 (2022)
Musielak, Z.E., Rosner, R., Ulmschneider, P.: Astrophys. J. **337**, 470 (1989)
Musielak, Z.E., Rosner, R., Stein, R.F., Ulmschneider, P.: Astrophys. J. **423**, 474 (1994)
Musielak, Z.E., Rosner, R., Gail, H.P., Ulmschneider, P.: Astrophys. J. **448**, 865 (1995)
Nordlund, Å., Spruit, H.C., Ludwig, H.-G., Trampedach, R.: Astron. Astrophys. **328**, 229 (1997)
North, J.R., Davis, J., Bedding, T.R., et al.: Mon. Not. R. Astron. Soc. **380**, L80 (2007)
Pérez Martínez, M.I., Schröder, K.-P., Cuntz, M.: Mon. Not. R. Astron. Soc. **414**, 418 (2011)
Pizzolato, N., Maggio, A., Sciortino, S.: Astron. Astrophys. **361**, 614 (2000)
Ribas, I., Guinan, E.F., Güdel, M., Audard, M.: Astrophys. J. **622**, 680 (2005)
Roberts, B., Webb, A.R.: Sol. Phys. **64**, 77 (1979)
Rutten, R.G.M., Schrijver, C.J., Lemmens, A.F.P., Zwaan, C.: Astron. Astrophys. **252**, 203 (1991)
Sackmann, I.-J., Boothroyd, A.I., Kraemer, K.E.: Astrophys. J. **418**, 457 (1993)
Schrijver, C.J.: Astron. Astrophys. **172**, 111 (1987)
Schrijver, C.J.: Astron. Astrophys. Rev. **6**, 181 (1995)
Schrijver, C.J., Pols, O.R.: Astron. Astrophys. **278**, 51 (1993)
Solanki, S.K.: Space Sci. Rev. **63**, 1 (1993)
Spruit, H.C.: Astron. Astrophys. **102**, 129 (1981)
Srivastava, A.K., Ballester, J.L., Cally, P.S., et al.: J. Geophys. Res. Space Phys. **6**, e029097 (2021)
Steffen, M., Ludwig, H.-G., Krüß, A.: Astron. Astrophys. **213**, 371 (1989)
Stein, R.F., Nordlund, Å., Georgobiani, D., Benson, D., Schaffenberger, W.: In: Dikpati, M., Arentoft, T., González Hernández, I., Lindsey, C., Hill, F. (eds.) Solar–Stellar Dynamos as Revealed by Helio- and Asteroseismology. Astr. Soc. Pac. Conf. Ser., vol. 416, p. 421. ASP, San Francisco (2009a)
Stein, R.F., Georgobiani, D., Schaffenberger, W., Nordlund, Å., Benson, D.: In: Stempels, E. (ed.) Cool Stars, Stellar Systems, and the Sun 15. AIP Conf. Proc., vol. 1094, p. 764. Am. Inst. of Phys., New York (2009b)
Stenflo, J.O.: Rep. Prog. Phys. **41**, 865 (1978)
Trampedach, R., Christensen-Dalsgaard, J., Nordlund, Å., Stein, R.F.: In: Pijpers, F.P., Christensen-Dalsgaard, J., Rosenthal, C.S. (eds.) Solar Convection and Oscillations and Their Relationship, p. 73. Kluwer Academic, Dordrecht (1997)
Ulmschneider, P.: Astron. Astrophys. **222**, 171 (1989)
Ulmschneider, P., Theurer, J., Musielak, Z.E.: Astron. Astrophys. **315**, 212 (1996)
Ulmschneider, P., Musielak, Z.E., Fawzy, D.E.: Astron. Astrophys. **374**, 662 (2001)
Yelles Chaouche, L., Solanki, S.K., Schüssler, M.: Astron. Astrophys. **504**, 595 (2009)

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