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Research Note

Optimization with Genetic Algorithm (GA): Planar mechanism synthesis

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Abstract. Dimensional synthesis of mechanisms to trace given points is an important issue in mechanism and machine science. Having no exact solution makes this issue an optimization problem. This study offers an optimization approach to dimensional synthesis of planar mechanisms. Four-bar mechanisms having one Degree Of Freedom (DOF) are chosen as the configurations. The proposed method is implemented by establishing the objective functions with specified constraints and searching for the results by using an optimization algorithm. Genetic Algorithm (GA) in Optimization Toolbox-MATLAB® is selected as a solver. Different types of four-bar mechanisms like crank-rocker and double-crank including different target points are performed. Mechanisms are depicted by resulting parameters and a prepared MATLAB® script plays their animations. As a result, it is proved that the mechanisms whose dimensional properties are obtained by the GA solver have a good tracing capability for the desired paths. This study has the property of being a design guide. Its application is not limited to four-bar mechanism. Planar mechanisms with different configurations can be easily synthesized by using this technique.

1. Introduction

Dimensional synthesis is defined as the determination of kinematic dimensions of a mechanism to perform required motion (link lengths, offsets, etc.). Graphical and analytical methods are available. The application of optimization methods has gained popularity in recent years and different algorithms have been used. Problems are defined as motion generation, path generation, and function generation. Generally, a multidisciplinary design optimization procedure is applied to synthesize mechanisms. Some input data required by the designer are given in the algorithm. These are Degree Of Freedom (DOF), range of all geometric parameters, inputs to the mechanism, outputs from the mechanism, and the required kinematic characteristics of the mechanism [1].

Many simple machines use a four-bar linkage: a windshield wiper, a rock crusher, an oil well, a door closer. In addition, a high lift mechanism with 1 DOF can be given as an example. Four-bar mechanisms are practically important yet simple mechanisms. This study focuses on the four-bar linkage and presents brief relevant findings. Blackett (2001) investigated the optimal synthesis of planar five-bar linkages [1]. Cabrera et al. (2002) studied the optimal synthesis of mechanisms using Genetic Algorithm (GA) [2]. Zohoor and Nia (2005) suggested a GA based optimization for the synthesis of planar and spatial mechanisms [3]. Geletu (2007) studied optimization problems

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using MATLAB Optimization Toolbox [4]. Nariman-Zadeh et al. (2009) introduced hybrid multi-objective GA for pareto optimum synthesis of four-bar mechanisms. Two objective functions namely tracking error and transmission angle (μ) are minimized simultaneously [5]. Archaryya and Mandal (2009) performed a study on the synthesis of the four-bar mechanism using GA, Particle Swarm Optimization (PSO), and Differential Evolution (DE). A comparative study of three evolutionary algorithms was conducted [6]. Erdoğan (2011) had a thesis on the synthesis of planar mechanisms by applying evolutionary algorithms, GA [7]. Shete and Kulkarni (2015) studied the dimensional synthesis with GA upon analyzing three problems. The straight-line generation problem through six desired points without prescribed timing was explored [8]. Sleesongsom and Bureerat (2018) presented an optimum path generation for synthesizing a four-bar linkage. Seven metaheuristic techniques were applied and compared. Optimum parameters of a four-bar linkage were found [9]. Chaphet et al. (2019) studied the four-bar linkage as a high-lift mechanism. Teaching Learning-Based Optimization (TLBO) was applied in [10]. Pavlovic et al. (2019) studied the optimal synthesis of manipulator drive mechanisms in hydraulic excavators [11]. Sardashi et al. (2022) presented a methodology for path generation synthesis of the four-bar mechanism by introducing the Geometrical Similarity Error Function (GSEF) [12]. Kang et al. (2022) performed a comparative study on the optimum synthesis of path generation. Five metaheuristic optimization algorithms namely two swarm based (PSO, HPSO) and three evolutionary based (DE-gr, EPSDE, and L-EPSD) were applied to the four-bar mechanism [13].

The objective of this study is to employ an evolutionary method for the synthesis of four-bar mechanisms and to present a design guide on their application. The type of this mechanism can be changed easily with minor modifications to the functions. Path generation cases with and without prescribed timing are given in this paper. Minimization of structural error is performed by considering the size and geometric constraints including Grashof theorem and the transmission angle, for example. This study is organized as follows. Section 1 gives an introduction. Section 2 presents the definition and main principles of GA. Section 3 includes kinematics equations, objective, and constraint functions. Section 4 discusses the implementation of the four-bar mechanism on case studies. Finally, Section 5 concludes this paper.

2. Genetic Algorithm (GA)

GA was proposed by Holland in 1975 for solving different optimization-based problems. GA remains popular in solving complicated nonlinear problems in engineering. It was inspired by natural selection in the evolution of living organisms. The principles of genetics and natural selection are fundamentals of this technique. GA functions on the bases of analogy to chromosome encoding and natural selection [14]. Nobakhti (2010) performed a study on natural-based optimization [15]. Khan and Bajpai (2013) applied GA for different fields of mechanical engineering and other real-world problems characterized by their own advantages and limitations [16]. Bhoskar et al. (2015) presented a review of GA applications ranging from scheduling problems to manufacturing and material science [17]. Al-Smadi et al. (2022) investigated the sensitivity of four-bar coupler motion by analyzing position error employing GA for the four-bar mechanism. The hood mechanism chooses the Plymouth Satellite automobile as the case study [18]. In the course of optimization, an engineering issue is defined by considering the boundaries of the problem. This can be accepted as a constraint on the solution finding minima and maxima of functions. The problem must be formulated correctly.

3. Synthesis of mechanisms by optimization

Synthesis of a mechanism means the generation of solutions to any particular type of problem in the linkage design. It can be utilized as type synthesis, number synthesis, and dimensional synthesis [19]. Dimensional synthesis is the focus of this study to determine the lengths of links necessary to get the desired motions.

The mechanism to be synthesized is a four-bar mechanism given in Figure 1.

3.1. Closed loop equation of the mechanism

Initially, a loop equation must be written for the mechanism:

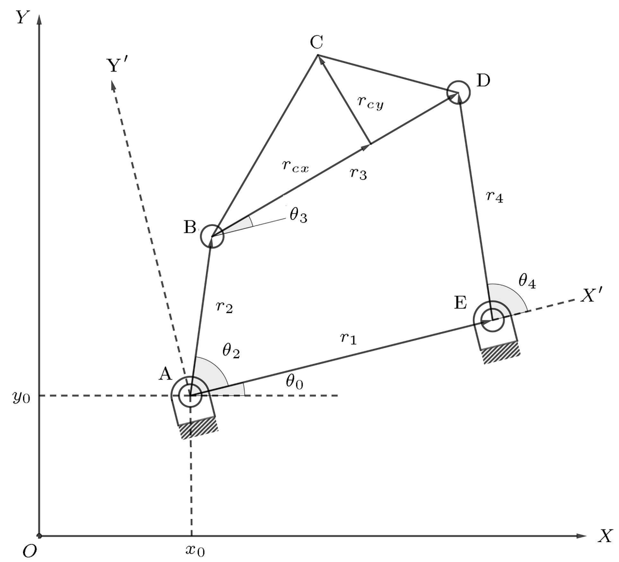

Figure 1. Four-bar mechanism.

$$\overrightarrow{r_2} + \overrightarrow{r_3} - \overrightarrow{r_4} - \overrightarrow{r_1} = 0. \tag{1}$$

Complex number notation is substituted and Eq. (1) is rewritten as follows:

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_4 e^{i\theta_4} - r_1 e^{i\theta_1} = 0. \tag{2}$$

Real and imaginary parts are separated as given in Eq. (3):

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0, \tag{3a}$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0. \tag{3b}$$

Angles θ_3 and θ_4 are solved for input angle θ_2 from Freudenstein's equation:

$$F_1 \cos\theta_4 - F_2 \cos\theta_2 + F_3 = \cos(\theta_2 - \theta_4), \tag{4a}$$

$$F_1 \cos\theta_3 + F_4 \cos\theta_2 + F_5 = \cos(\theta_2 - \theta_3), \tag{4b}$$

where:

$$F_1 = \frac{r_1}{r_2}, \quad F_2 = \frac{r_1}{r_4},$$

$$F_3 = \frac{{r_2}^2 - {r_3}^2 + {r_4}^2 + {r_1}^2}{2r_2r_4}, \quad F_4 = \frac{r_1}{r_3},$$

$$F_5 = \frac{{r_4}^2 - {r_1}^2 - {r_2}^2 - {r_3}^2}{2r_2r_3}.$$

The angular displacement of the third and fourth links is found, as given in Eq. (5):

$$\theta_{4_{1,2}} = 2\tan^{-1}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right), \tag{5a}$$

$$\theta_{3_{1,2}} = 2\tan^{-1}\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right), \tag{5b}$$

where:

$$A = \cos\theta_2 - F_1 - F_2 \cos\theta_2 + F_3, \qquad B = -2\sin\theta_2,$$

$$C = F_1 - (F_2 + 1)\cos\theta_2 + F_3,$$

$$D = \cos\theta_2 - F_1 + F_4 \cos\theta_4 + F_5,$$

$$E = -2\sin\theta_2, \qquad F = F_1 + (F_4 - 1)\cos\theta_2 + F_5.$$

Position of the coupler C in the reference frame $AX'Y'$ is:

$$C_{X_r} = r_2 \cos\theta_2 + r_{cx} \cos\theta_3 - r_{cy} \sin\theta_3, \tag{6a}$$

$$C_{Y_r} = r_2 \sin\theta_2 + r_{cx} \sin\theta_3 + r_{cy} \cos\theta_3, \tag{6b}$$

or written on world coordinate system OXY

$$\begin{bmatrix} C_X \\ C_Y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} C_{X_r} \\ C_{Y_r} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}. \tag{7}$$

3.2. Objective function

The objective function is to compute the position as the sum of the squares of the Euclidian distances between C_d^i and C^i. It is given in Eq. (8):

$$f_{obj} = \sum_{i=1}^{N} \left[\left(C_{xd}^i - C_x^i\right)^2 + \left(C_{yd}^i - C_y^i\right)^2 \right]. \tag{8}$$

C_d^i is a group of target points defined by the designer that should be traced by the coupler of the mechanism. C^i is a group of traced points obtained in Eq. (7). N denotes the number of points to be synthesized.

3.3. Constraint function

Two criteria are considered in defining the constraint function: Grashof condition and transmission angle.

According to Grashof theorem, a four-bar mechanism has at least one revolving link if:

$$s + l \leq p + q,$$

where s and l are the shortest and longest; p and q are the intermediate links of the mechanisms, respectively. Two options providing the above condition are the mechanisms called crank-rocker and double-crank, as shown in Figure 2(a) and (b):

- The mechanism is called crank-rocker if the shortest link is a side link. The shortest link is the crank in the mechanism;
- The mechanism is called double-crank if the shortest link is the frame.

Transmission angle is an important criterion for designing linkages. The force and the motion must be effectively transferred between linkages to ensure a smooth link movement with respect to each other. Transmission angle is denoted by μ, as illustrated in Figure 3.

It is clear that the optimum value of the transmission angle is 90°. In practice, it is determined that if the maximum deviation of the transmission angle from 90° exceeds 40° or 50°, the mechanism will be locked [20], as expressed in Eq. (9):

$$\mu = \cos^{-1}\left(\frac{r_3^2 - r_2^2 - r_1^2 + r_4^2 + 2r_1r_2\cos\theta_2}{2r_3r_4}\right). \tag{9}$$

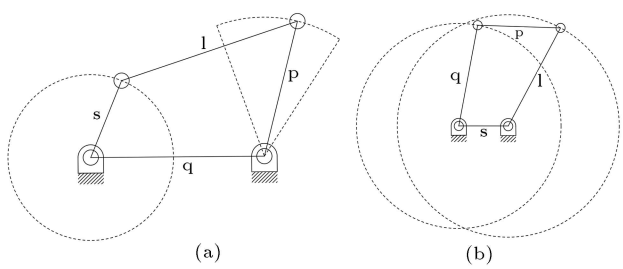

Figure 2. Crank-rocker and double-crank mechanisms.

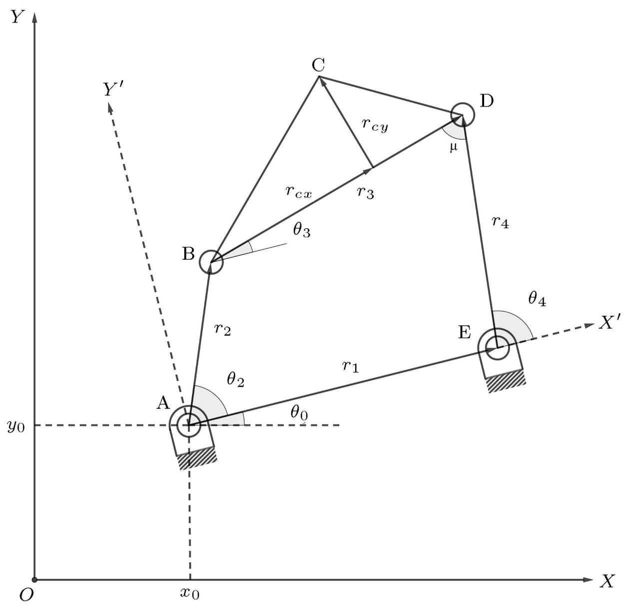

Figure 3. Transmission angle, μ on a four-bar mechanism.

4. Case studies

This section presents a group of results obtained when applying the functions derived in the previous section. Optimization Toolbox-MATLAB® providing a numeric computing environment is used in this section [21,22].

4.1. Case I-Path generation without prescribed timing

There are four coupler points required to determine an optimal solution. These points are designed to trace a vertical straight line by changing Y coordinate only. The problem is then defined by:

1. *The variables of the mechanism:*

$$V = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, x_0, y_0, \theta_0, \theta_2^i\right]$$

where:

$$i = 1, 2, \cdots, N \text{ and } N = 4.$$

2. *Target points are chosen as:*

$$\left[C_{xd}^i, C_{yd}^i\right] = \left[(20, 30), (20, 35), (20, 40), (20, 45)\right].$$

3. *Limits:*

$$r_1, r_2, r_3, r_4 \in [5, 60],$$

$$r_{cx}, r_{cy}, x_0, y_0 \in [-60, 60],$$

$$\theta_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4 \in [0, 2pi].$$

GA parameters are given in Table 1.

The crank-rocker mechanism variables obtained after synthesis are given below:

Table 1. Parameters of genetic algorithm.

Genetic algorithm	Values
Population size	100
Crossover possibility	0.9
Mutation possibility	Adaptive feasible
Selection type	Roulette wheel
Generation	1000

$$r_1 = 59.636, \quad r_2 = 24.941, \quad r_3 = 53.618,$$

$$r_4 = 37.089, \quad r_{cx} = -17.126, \quad r_{cy} = 8.173,$$

$$x_0 = 5.593, \quad y_0 = 9.28, \quad \theta_0 = 0.205, \quad \theta_2^1 = 0,$$

$$\theta_2^2 = 0.151, \quad \theta_2^3 = 0.339, \quad \theta_2^4 = 0.543, \quad f_{obj} = 1.306.$$

The coupler points tracing desired target points are given in Figure 4(a)–(d).

Desired and traced points in x and y directions and transmission angle obtained are given in Table 2.

The double-crank mechanism variables obtained after synthesis are given below:

$$r_1 = 10.501, \quad r_2 = 48.57, \quad r_3 = 43.381, \quad r_4 = 40.431,$$

$$r_{cx} = -1.189, \quad r_{cy} = 33.716, \quad x_0 = -9.287,$$

$$y_0 = 27.733, \quad \theta_0 = 0.655, \quad \theta_2^1 = 0.177, \quad \theta_2^1 = 0.177,$$

$$\theta_2^3 = 0.502, \quad \theta_2^4 = 0.649, \quad f_{obj} = 0.807.$$

The coupler points tracing the desired target points are given in Figure 5(a)–(d).

Desired and traced points in x and y directions and transmission angles are given in Table 3.

4.2. Case II-Path generation with prescribed timing

There are four coupler points required to determine an optimal solution. They are chosen to get an optimal solution for tracing an arc. The problem is then defined by:

1. *The variables of the mechanism:*

$$V = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, x_0, y_0, \theta_0\right].$$

2. *Target points are chosen as:*

$$\left[C_{xd}^i, C_{yd}^i\right] = [(0, 0), (1.9098, 5.8779), (6.9098, 9.5106), (13.09, 9.5106)],$$

$$\left[\theta_2^i\right] = \left[\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}\right].$$

3. *Limits:*

$$r_1, r_2, r_3, r_4 \in [5, 60], \quad r_{cx}, r_{cy}, x_0, y_0 \in [-60, 60].$$

Parameters of GA are given in Table 1.

The crank-rocker mechanism variables obtained after synthesis are given below:

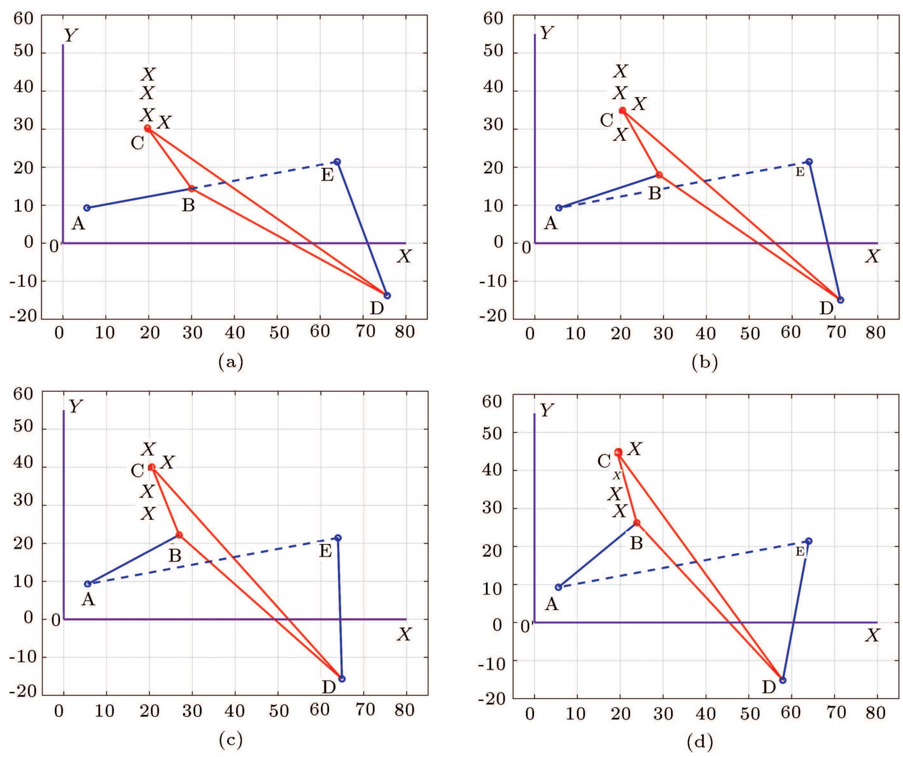

Figure 4. Tracing desired points with crank-rocker mechanism.

Table 2. Desired and traced points and transmission angles (μ) for crank-rocker mechanism.

X		***Y***		
Desired	**Traced**	**Desired**	**Traced**	**Transmission angle μ**
20	19.724	30	30.302	40°
20	20.467	35	34.937	40.75°
20	20.571	40	40.066	43.65°
20	19.31	45	44.66	48.81°

Table 3. Desired and traced points and transmission angles (μ) for double-crank mechanism.

X		***Y***		
Desired	**Traced**	**Desired**	**Traced**	**Transmission angle (μ)**
20	19.6	30	30.11	54.21°
20	20.45	35	34.99	55.08°
20	20.43	40	40.05	56.39°
20	19.53	45	44.89	57.98°

$$r_1 = 60, \quad r_2 = 20.267, \quad r_3 = 35.465,$$

$$r_4 = 59.71, \quad r_{cx} = 56.865, \quad r_{cy} = 3.414,$$

$$x_0 = -0.091, \quad y_0 = 44.654,$$

$$\theta_0 = 0, \quad f_{obj} = 2.15.$$

The coupler points tracing the desired target points are given in Figure 6 (a)–(d).

The desired and traced points in x and y directions and transmission angles are given in Table 4.

5. Conclusions

This study investigated the synthesis of planar mechanisms by a Genetic Algorithm (GA) solver and utilized the one-DOF four-bar mechanism as a simple example. The algorithm was developed only for well-

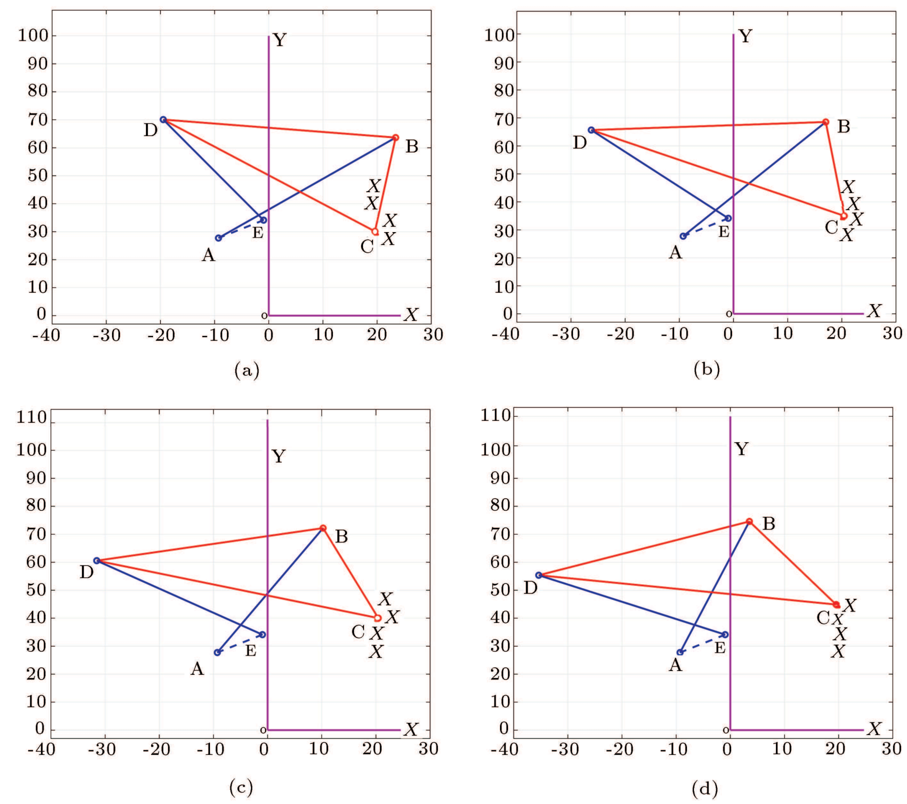

Figure 5. Tracing desired points with double-crank mechanism.

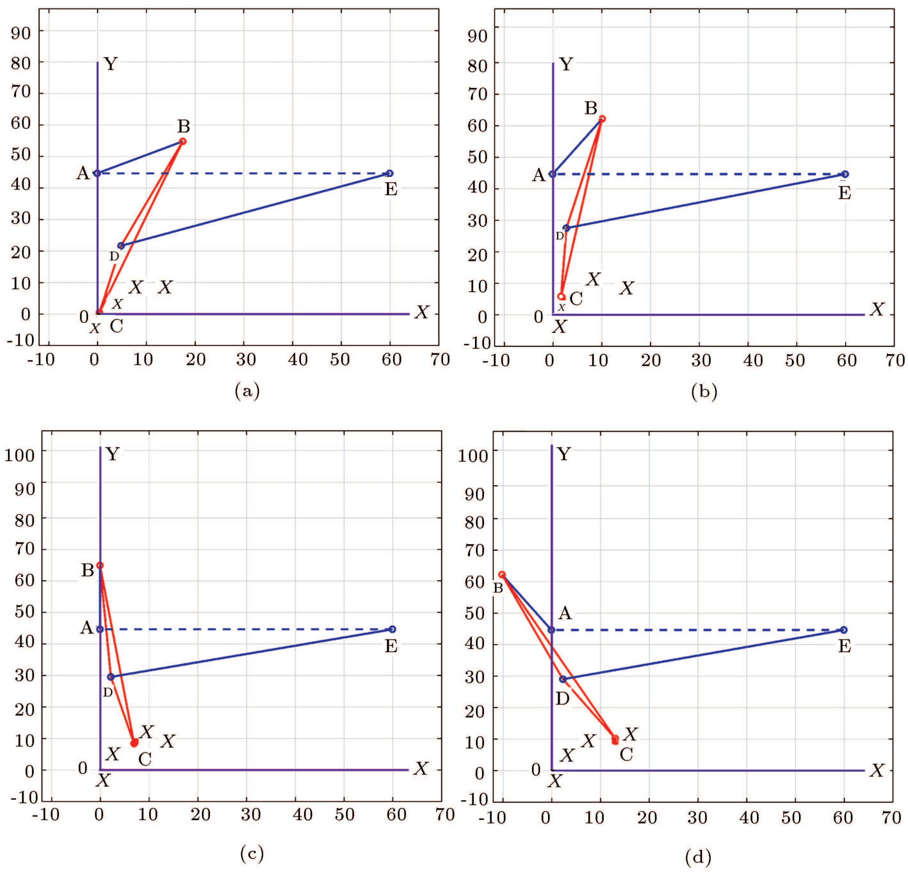

Figure 6. Tracing arc shaped path with crank-rocker mechanism.

Table 4. Desired and traced points and transmission angles (μ) for crank-rocker mechanism.

	X		***Y***		
$\boldsymbol{\theta_2}$	**Desired**	**Traced**	**Desired**	**Traced**	**Transmission angle ($\boldsymbol{\mu}$)**
$\pi/6$	0	0,357	0	0,40	46.44°
$\pi/3$	1,909	1,632	5,877	5,86	61.38°
$\pi/2$	6,909	6,904	9,51	8,39	78.94°
$2\pi/3$	13,09	13,02	9,51	10,19	95.47°

known Grashof-type four-bar mechanisms. Different case studies with crank-rocker and double-crank mechanisms having straight-line and arc-shaped target points were presented by using MATLAB® Optimization Toolbox. The study was enriched with case studies including path generations with and without prescribed timing. The position error between the desired and actual values of the coupler point of the mechanism was optimized using objective and constraint functions. This technique enjoyed basic and straightforward utility. The use of Genetic Algorithm (GA) in the dimensional synthesis of mechanisms is a good alternative solution to analytical and graphical solutions.

The proposed method gives users an option to increase and decrease the number of precision points easily without changing the mathematical expressions. Adapting this technique to other planar mechanisms is simply possible. This adaptation may include different objective functions except for position error minimization. The parameters of the options in the Optimization Toolbox can be studied to get better results.

Nomenclature

DOF	Degree Of Freedom
GA	Genetic Algorithm
C_d^i	Group of target point
C^i	Group of traced point
μ	Transmission angle

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Biographies

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