



OPTIMAL HEDGE RATIO AND HEDGING EFFECTIVENESS OF  
TURKISH STOCK INDEX FUTURES

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OPTIMAL HEDGE RATIO AND HEDGING EFFECTIVENESS OF  
TURKISH STOCK INDEX FUTURES

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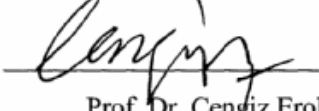
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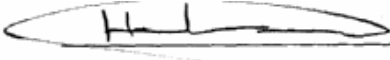
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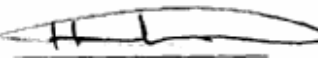
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## ABSTRACT

### OPTIMAL HEDGE RATIO AND HEDGING EFFECTIVENESS OF TURKISH STOCK INDEX FUTURES

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The objective of this thesis is to estimate optimal hedge ratio for ISE-30 stock index futures by using several econometric models. The linear regression model, the bivariate vector autoregressive (VAR) model, the error correction model (ECM), the GARCH model and the multivariate GARCH (M-GARCH) model are conducted particularly in the study to calculate risk-minimizing hedge ratio. The appropriateness/superiority of the models' findings is evaluated under the hedging effectiveness criterion for each in-sample and out-of-sample data horizons. As a result, M-GARCH hedge ratio provides the highest variance (risk) reduction for all of the hedging periods along with both in-sample and out-of-sample data. However, there are no penetrating differences between the hedging performances of applied models. It is expected that the findings of the analysis will be beneficial for investors who wish to hedge price risk in Turkish stock market.

Keywords: Hedge Ratio, M-GARCH, Hedging Effectiveness

## ÖZET

### TÜRK HİSSE SENEDİ VADELİ İŞLEM SÖZLEŞMELERİNDE OPTİMAL KORUNMA ORANI VE KORUNMA ETKİNLİĞİ

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Bu tezin amacı, İMKB-30 endeks vadeli işlem sözleşmelerine ait optimal korunma oranının çeşitli ekonometrik modeller uygulanarak tespit edilmesidir. Optimal korunma oranının hesaplanmasında, Doğrusal Regresyon modeli, Yöney Kendiylebağlaşım (VAR) modeli, Hata Düzeltme modeli (ECM), GARCH modeli ve Çok Değişkenli GARCH (M-GARCH) modeli kullanılmıştır. Modeller tarafından tahminlenen korunma oranlarının, örneklem-içi ve örneklem-dışı veri setlerinde, karşılaştırılmasında korunma etkinliği kriteri baz alınmıştır. Buna göre, M-GARCH modeli tarafından tahminlenen korunma oranının hem örneklem-içi hem de örneklem-dışı veri setleri için en düşük değişirliği sağladığı gözlemlenmiştir. Diğer taraftan, modellerin korunma performansları arasında kayda değer farklılıklar bulunmamaktadır. Analizden elde edilen bulguların Türk hisse senedi piyasasındaki riskini en aza indirmek isteyen yatırımcılar için yararlı olması beklenmektedir.

Anahtar Kelimeler: Korunma oranı, M-GARCH, Korunma etkinliği

To my love Tamay

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I would first and foremost thank my supervisor Assoc. Prof. İ. Hakan Yetkiner for his encouragement, support, and guidance throughout the different phases of this paper. His comments have not only enhanced the contribution of this thesis but also improved my research skills.

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# Chapter 1

## Introduction

A futures contract is just a standardized agreement to buy or sell a commodity or financial instrument on a stipulated future date at a particular price agreed upon today by the buyer and seller. The demand for futures contracts has steadily expanded around the world over the last decade parallel to the drastic technological developments in the meantime. One of the major reasons behind this growth is exactly the hedging opportunity provided by futures markets to cope with the adverse effects of volatility in asset prices. The purpose of hedging is to minimize the risk of a portfolio for a given level of return. This is done by taking a position (sell or buy) in futures market that is opposite the one in spot market.<sup>1</sup> Consequently, a profit or loss in the spot position due to the variability in prices will be countered by taking a futures position.

Traditional hedging approach affirms that the absolute magnitudes of spot and futures positions should be same in order to offset price risk. However, the spot and futures prices are not perfectly correlated in reality. Hence, the magnitudes of the positions in spot and futures market would be different to follow a sound hedging strategy. Determining the proper amount of futures position that perfectly covers the spot exposure is crucial correspondingly. The optimal hedge ratio, which is described as the ratio of the size of portfolio taken in futures contracts to

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<sup>1</sup> For example, if an US exporter is expected to be paid Euro (long spot position), he needs to sell Euro or take short position in futures market. By this way, any reduction in the value of Euro against dollar will be compensated through the short position in futures market.

the size of the exposure in spot market under minimum risk (variance) constraint, is used to determine the proper amount of futures position that optimally covers the spot exposure. Nevertheless, the calculation of the optimal hedge ratio for any futures contracts is not a straightforward effort.

Even though there are plenty of suggested methods to compute optimal hedge ratio, there is still no agreement in the existing literature on which model is the best. The hedging effectiveness measure often decides the most favorable model for estimating the hedge ratio in the related studies. Hedging effectiveness can be defined as the percentage reduction in the variance of the spot portfolio by using futures contracts. Further, the performance of a hedging operation is disclosed via the hedging effectiveness criterion.

The econometric models that are used to estimate the hedge ratio can be grouped into two: constant models and dynamic models. Dynamic models, which are more sophisticated, take into account the heteroskedastic nature of financial time series, contrary to constant models. But they are suffered from the complex algorithms in the calculation process, which may sometimes be a disadvantage, compared to constant models. To sum up, each model has its own pros and cons in the estimation of the optimal hedge ratio.

In fact, the estimation of the optimal hedge ratio includes many dimensions different than the choice of empirical model, which can cause great variations in the hedging effectiveness. The choice of data frequency, the length of hedging period and whether the horizon is in-sample or out-of-sample all influence the

hedge ratio or hedging effectiveness results. Therefore, all these abovementioned issues are taken into account in this thesis.

Since the focus of this study is to explore newly launched Turkish futures contracts concerning optimal hedge ratio and hedging effectiveness, the ISE-30 stock index futures is chosen particularly for the empirical analyses. This futures contract is regarded as a benchmark for the Turkish Derivatives Exchange (TurkDEX) as it composes more than 90% of total trading volume in last three years. Moreover, the trading volume of the ISE-30 futures has been increasing steadily as it allows managing price risk in Turkish stock market.

The stock index futures contracts have emerged at various exchanges in the world since their first launching in 1982. However, the awareness of the index futures soared after the 1987 stock market crash. Especially institutional investors commonly prefer stock index futures to control the unsystematic risk of the stock portfolio they hold without changing the composition. These contracts are favored as an effective hedging tool because of their liquidity and relatively lower transaction costs.

This thesis aims to estimate the optimal hedge ratio for ISE-30 stock index futures contract by using a variety of empirical models. It further aims to contribute by comparing the superiority of constant or dynamic models for the estimation process, through associated assessments. The measure of hedging effectiveness will be handled particularly to compare the hedge ratio estimations of derived models within different hedging horizons for both in-sample and out-of-sample data.

The findings of this thesis will be important for two main reasons. First, hedge ratio estimations obtained from the analysis will be beneficial for the investors who wish to hedge price risk in Turkish stock market. Second, it will help to answer how effective is the ISE-30 futures contract as a hedging tool.

The remainder of the thesis is organized as follows. Section 2 extensively reviews the theoretical background and existing literature on the hedge ratio and hedging effectiveness. Section 3 describes the data set used in the study with brief information on the TurkDEX. Section 4 presents the empirical models conducted for the research. Section 5 portrays hedge ratio estimates of models and the related comparison of hedging effectiveness. Section 6 makes concluding remarks.

## **Chapter 2**

### **Literature Review**

A significant amount of empirical research in the hedging literature has focused on estimating the optimal hedge ratio and/or hedging effectiveness for a variety of futures contracts. In fact, the two concepts are directly related with each other and mostly considered in the same framework. To put it differently, the effectiveness of a futures hedging process can be enhanced simply through calculating and using the most accurate hedge ratio.

Theoretical hedging function of futures markets was early investigated by Working (1952) in a seminal study. He challenged the common view of classifying hedgers as pure risk-minimizers and stated that profit maximization is also the prior objective of hedgers as well as speculators. Furthermore, he emphasized that, spot and futures prices do not tend to move together entirely in practice and most hedging is done by expecting a divergence on these prices. Thus, holders of long positions in the spot market will only hedge if the basis (the difference between spot and futures prices) is expected to fall.

Johnson (1960) and Stein (1961) revealed the fundamentals of futures hedging by applying the mean-variance framework of Markowitz's (1952) portfolio theory. According to this framework, it is assumed that all investors have a common object of maximizing expected return for a given level of variance (risk) or alternatively minimizing variance (risk) at a given level of expected return. A



hedger can maximize his utility via minimizing the unconditional variance ( $\delta_p^2$ ) of the covered portfolio returns as follows:

$$\delta_p^2 = w_s \delta_{\Delta S}^2 + w_f \delta_{\Delta F}^2 - 2w_s w_f Cov(\Delta S, \Delta F) \quad (1.1)$$

where  $w_s$  and  $w_f$  represent the weights of spot and futures positions in the hedged portfolio, respectively.  $\Delta S$  and  $\Delta F$  demonstrate the price changes (or returns) in spot and futures market and  $Cov(\Delta S, \Delta F)$  denotes the covariance among these variables and  $\delta_{\Delta S}^2$  and  $\delta_{\Delta F}^2$  are the unconditional variance of price changes in spot and futures market. The proportion of futures contracts that should be held in the portfolio against existing spot exposure is defined as the Minimum Variance Hedge Ratio  $H^*$ .

$$H^* = \frac{w_f}{w_s} = \frac{Cov(\Delta S, \Delta F)}{\delta_{\Delta F}^2} \quad (1.2)$$

Following that, Ederington (1979) adopted Ordinary Least Squared (OLS) regression method to derive risk-minimizing hedge ratio. He showed that the optimal hedge ratio is just the slope coefficient of a regression equation where spot and futures price changes are defined as dependent and explanatory variables, respectively. The hedging effectiveness was also specified in the study by referring the percentage variance reduction between hedged and unhedged positions. Ederington (1979) employed the R-squared statistic of related regression as a hedging effectiveness measure in this manner. He stated that the OLS-based hedge ratio outperforms naïve one-to-one hedging strategy for

treasury-bill futures contracts by creating more reduction in the spot exposure variability.

Many researchers have used the OLS method regularly to estimate the hedge ratios for different futures contracts since Ederington's (1979) benchmark study. The hedge ratios estimated by this approach are classified as 'constant' in the literature as well, corresponding to the essential assumptions of the OLS regression model.<sup>2</sup> Hill and Schneeweis (1981) analyzed the hedge effectiveness of the major currency futures by implementing OLS model to derive hedge ratios. They demonstrated that the most significant reduction in the variance of the spot return (price-change) series is realized by hedging at the minimum risk hedge ratio estimated from the OLS regression. A comparison between the use of spot and futures price changes and/or price levels as proper time-series was also made in the paper. However, the price level regressions generally violate the OLS assumptions due to the high degree of autocorrelation detected in residuals, which leads statistically conflicting and inefficient hedge ratio estimates. Figlewski (1984) emphasized that the magnitude of the hedged stock position must be different than the underlying portfolio. The fluctuations of the basis that indicates the changes between spot and futures prices determine the risk and return components for index futures hedge. He examined empirically the Standard and Poor's (S&P) 500 futures in terms of hedging potential through pointing out the usefulness of OLS-based hedge ratios. He reported that minimum portfolio

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<sup>2</sup> The residuals have constant (homoscedastic) variance and the error terms are normally distributed.

variance (risk) is achieved with the hedge ratio of the OLS model for various holding periods.

Myers and Thompson (1989) developed a generalized approach for hedge ratio estimation by implementing OLS to a single multiple regression equation. This specification that takes heteroskedastic shocks into account differs from simple regression models. Nevertheless, it is denoted that the classical regression method provides reasonably accurate estimates even it is not complex as the general approach. Malliaris and Urrutia (1991) scrutinized the probable effects of the lengths of estimation periods and the hedging horizons on the hedging effectiveness for selected currency futures. They affirmed that the length of estimation period, assigned for the required computations of the OLS regression model, does not have any impact on hedging effectiveness. On the other hand, weekly (shorter) horizons were found to be more effective in hedging than monthly (longer) horizons. In a parallel research, Benet (1992) asserted that the hedging horizon should be minimized to reduce spot price risk successfully, since the frequent changes in market conditions deteriorate the stable estimates of the OLS model. He also documented the robustness of constant hedge ratios relative to time-varying (multiple) ratios, since the hedge ratio variability usually offsets the diversification benefits of the portfolio.

The reliability of OLS hedge ratios is noted in a recent study by Lien (2005). His findings illustrate that the hedge ratio gathered from the OLS regression would minimize the within sample unconditional variance and likely perform better than any strategy for out-of-sample comparison too if two sub-samples have equal

magnitudes. In addition, he claimed that the superiority of other methods on hedging effectiveness, relative to OLS regression model, is caused by either small sample sizes or structural changes between two sub-samples.

One of the limitations for the regression model is the serial correlation detected in residuals, which caused to yield biased hedge ratio estimates. To eliminate the serial correlation in residuals, Herbst *et al.* (1993) modeled the spot and futures returns through a bivariate vector autoregressive (VAR) framework. As consequence, they removed the serial correlation from the residuals and derived more realistic hedge ratios in the study.

It is clearly documented by numerous empirical researches that there is a close interaction between the spot and futures markets under the lead-lag pattern, which is taken into account by the bivariate VAR model from a different perspective. To name a few, Stoll and Whaley (1990), Chan (1992) and Brooks and Chong (2001) have all reported the feedback relations (mostly bidirectional) among the spot and futures prices, returns and volatilities for various markets. These findings provide reasonable support for applying a VAR framework to estimate hedge ratio, under the fact that trends in spot and future markets may affect the current price movement instantaneously.

Another structural viewpoint on hedge ratio estimation merely hinges upon the probable cointegration<sup>3</sup> relationship between the spot ( $S$ ) and futures ( $F$ ) prices, which is reinforced by cost-of-carry futures pricing model given in the following<sup>4</sup>:

$$F = S e^{(r+s-c)t} \quad (1.3)$$

where  $r$ ,  $s$  and  $c$  indicate risk-free interest rate, storage costs and convenience yields respectively that aroused from holding the futures contract until delivery date. The existence of such a correlation would not allow any prices to diverge extensively from the long-run equilibrium. Engle and Granger (1987) revealed that the cointegration among variables invalidates the findings of conventional regression method since the essential short-run dynamics are not taken into account through the OLS system. They suggested applying *error correction* mechanisms in the case of cointegration, which associate the change in one variable to past equilibrium errors and further past changes in other variables. A procedure for testing the null hypotheses of no cointegration against the alternative cointegration was also developed in their study.

The cointegration framework is early considered by Garbade and Silber (1983) over the estimation of optimal hedge ratio particularly. They specified a method that derived from the price elasticity of storable commodities (i.e. wheat, oats, gold) to define the interrelationships between cash market prices and futures

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<sup>3</sup> Cointegration is an econometric character of time-series parameters. If two or more series are themselves non-stationary (i.e. spot and future prices), but a linear combination of them is stationary, then the series are called as cointegrated. In our context, cointegration refers not to comovements in returns, but to comovements in raw asset prices. Cointegration process allows considering both short-run and long-run dynamics in the model.

<sup>4</sup> Cost-of-carry model is an arbitrage-free pricing model. Its main idea is that futures contract is so priced as to preclude arbitrage profit

prices. The degree of market integration is characterized by a simple function concerning the supply of arbitrage services for the short hedging horizons. Consequently, the optimal hedge ratio would be smaller if the cointegration relation is not counted in the process. Ghosh (1993) examined the major stock index futures in US with the aim of determining the most effective hedge ratio. Apart from simple regression, he employed error correction model (ECM) alternatively once the null hypothesis of non-cointegrating vector (rank=0) is rejected significantly. The empirical results in the paper demonstrate that the hedge ratios derived from traditional methods are underestimated because of the misspecification problem.<sup>5</sup> On the other hand, the hedging effectiveness is improved considerably by the estimates of error correction model which incorporates the long-run equilibrium relationship and the short-run dynamics. Furthermore, Lien and Luo (1993) confirmed the existence of cointegration between spot and futures prices for the major currency and stock index markets. They extended the well-known Garbade-Silver (GS) error correction model with the inclusion of lagged price terms to calculate the multi-period hedge ratios. It is concluded that, the hedge ratios obtained from error correction specification provide remarkable performance in hedging. Similar findings referring to the superiority of error correction model were also reported by Chou *et al.* (1996) and Kenourgios and Samitas (2008) for the Nikkei and S&P stock index futures, respectively.

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<sup>5</sup> By omitting the long run cointegration relationship from the estimation procedure, hedger would take a smaller than optimal futures position, which probably causes a relatively poor hedging performance.

Nonetheless, Lien (1996) highlighted that the misspecification issue might exist for the error correction representations as well by modeling a partial cointegration system instead of a complete one. In that case, the error correction model will not present the optimal solution in the calculation of the hedge ratios. In a recent study Moosa (2003) exhibited the negligible difference in hedging effectiveness with or without cointegration consideration. He pointed out that model selection (complexity) is not crucial as much as the correlation between the prices of the unhedged position and the hedging instrument.

Contrary to main assumptions of the OLS approach, most of the financial asset returns are not symmetrically distributed and do not have constant variance as confirmed in the literature. In this context, the variance of error terms tends to change through a deterministic function of time, so called as heteroskedastic. The stylized fact of volatility clustering is further observed characteristic of the financial time-series as a property of stochastic heteroskedasticity processes within the econometric perspective.<sup>6</sup> Therefore, the Generalized Autoregressive Conditional Heteroskedasticity process (GARCH) is developed by Bollerslev (1986) for considering idiosyncratic features of financial data in volatility modeling unlike conventional methods.

As many empirical investigations in the finance discipline, the GARCH based models (both univariate and multivariate specifications) are commonly conducted with the aim of estimating hedge ratios. This framework allows us to compute

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<sup>6</sup> As noted by Mandelbrot (1963) “large changes tend to be followed by large changes, of either sign or small changes tend to be followed by small changes”. While returns are themselves uncorrelated, absolute returns or their squares display a positive, significant, and slowly decaying autocorrelation function for ranging from a few minutes to several weeks.

*time-varying* (dynamic) hedge ratios since the conditional (past) variances and covariances are considered as the explanatory of current variance. Baillie and Myers (1991) adopted the bivariate GARCH model including an error correction term (diagonal VECM parameterization) to calculate optimal hedge ratios for each of six different commodity futures. They compared the conditional variances of portfolio returns under three hedging strategies: no hedging; hedging with constant hedge ratio estimated using regression methods; and hedging with a time-varying hedge ratio of bivariate GARCH model. Both the in-sample and out-sample performance evaluations indicate that time-varying hedge ratio of GARCH method generally provides the largest reduction in portfolio variances. After that, Kroner and Sultan (1993) found parallel results for the major currency futures by employing the constant correlation GARCH (1,1) specification. The cointegration issue was taken into account with the inclusion of error correction term to the benchmark model, which is required for currency markets. Further, Sephton (1998) advanced the existing GARCH methodology in the literature on hedge ratio estimation through developing another multivariate framework to allow temporal evolution in the processes. He stated that the hedge ratios calculated from multivariate GARCH model are optimal to reduce risk for the commodity futures traded in Canada.

Park and Switzer (1995) are the first who apply dynamic hedging to the stock index futures, S&P 500 and Toronto 35, in daily basis. It is concluded in the study that the bivariate GARCH model provides an improved hedging strategy in comparison to the OLS hedge and the OLS with cointegration between spot and futures prices. Additionally, Lypny and Powalla (1998) analyzed the hedging



effectiveness of the DAX futures. They showed that the application of a dynamic hedging strategy based on the GARCH (1,1) covariance parameterization reached significant progress in economic welfare over simple constant hedge and the error correction without the GARCH (1,1) structure. Allen and Yang (2004) employed a standard diagonal VEC M-GARCH (1,1) model under the constant correlation hypothesis. The dynamic hedge ratios estimated from conditional information set display high degree of non-stationarity through time, which is also consistent with the non-linear nature of the price distributions. He markedly expressed that the hedging effectiveness is preceded via the GARCH time-varying hedge ratios in terms of the minimum portfolio variance for longer hedging horizons particularly.

From a different perspective, Brooks *et al.* (2002) put emphasis on the asymmetry in GARCH modeling across the entire variance and covariance matrix since the OLS hedging is independent of news arriving to the market. Thus, the hedge ratios might be responsive to the fluctuations in prices resulting from information arrivals. Their findings related to the multivariate asymmetric GARCH model, which enables positive and negative price innovations to affect variance differently, confirm the significant developments in hedging performance. Moreover, Choudhry (2004) implied that the dynamic hedge ratios of various GARCH derivations outperform the constant hedge ratios obtained from different estimation procedures for many stock markets and their corresponding futures contracts. Despite the presence of excessive researches for developed markets on time-varying (GARCH) hedge ratio concept, emerging markets have not been detected sufficiently yet towards the same purpose. A few numbers of studies such as Choudhry (2003) for Hong-Kong and South Africa, Floros and Vougas

(2006) for Greece, and Bhaduri and Durai (2008) for India have emphasized the ability of GARCH framework to compute risk-minimizing hedge ratios in accordance.

Although the great number of studies in the literature asserts the superiority of the dynamic hedge ratios, there exist also considerable counterviews concerning the drawbacks of the GARCH estimation process. Myers (1991) revealed that the GARCH model perform only slightly better than constant hedge ratio estimation. Therefore, the linear regression approaches to optimal hedge ratio estimation may be a satisfactory approximation. Besides, Fackler and Mcnew (1994) underlined the disadvantage of GARCH formations due to their non-linear optimization algorithms. Holmes (1996) and Miffre (2004) also showed that the hedge ratios subject to estimations of the OLS regression method provide better hedging performance than other complex GARCH specifications. Through a descriptive work, Lien *et al.* (2002) examined the implementation and effectiveness of the time-varying hedge ratios attained from the constant correlation GARCH model, under the rolling-window out-of-sample forecasts. Their empirical comparisons demonstrated that GARCH estimations should not be performed for hedging purposes because of the computational complexity and costs. In a more recent study Lien (2007) outlined the fact that the time-varying (GARCH) hedge ratio minimizes the conditional variance of the hedged portfolio whereas the constant (OLS) hedge ratio minimizes the unconditional variance of the hedged portfolio. As the common hedging effectiveness measures care the proportional reduction in unconditional variance; the conventional hedge ratio dominates the other hedging strategies correspondingly.

Briefly, four fundamental methodologies have been conducted to estimate optimal hedge ratios in the literature. These are OLS regression method, VAR method, ECM method and GARCH (univariate or multivariate) method. However, it can be easily comprehended that there is no consensus about the best way to determine the hedge ratios using futures. Different studies have documented contradictory results in this regard. The majority of researches have decided the superiority of a method by comparing the estimates of stated models under the criterion of hedging effectiveness. A similar procedure is adopted in this study to determine optimal hedge ratio for the ISE-30 index futures, which based on comparing the hedge ratio estimates of various econometric models under hedging effectiveness criteria. Since there are very limited academic researches (only one) in the literature so far about the hedging effectiveness of Turkish futures market [Aksoy and Olgun (2009)], this thesis aims to make a concrete contribution to the hedging effectiveness literature.

## Chapter 3

### Data

#### 3.1 Institutional Features of TurkDEX

The TurkDEX is the first and unique derivatives market in Turkey, authorized by the Capital Markets Board (national regulatory agency). The opening bell for trading in the TurkDEX rang recently on 4<sup>th</sup> February 2005 with 34 members. As of May 2008, there are 86 registered members (brokerage houses and banks) in TurkDEX. Although the exchange is classified as a profit-seeking organization by the legislations, it has strategic and economic priorities from the point of establishing an efficient and successful risk management platform to sustain a robust economic structure. In this context, the principal objectives of the TurkDEX are sorted as: protection of investors, transparency in executions, optimal response to market demands and integration with international markets.<sup>7</sup>

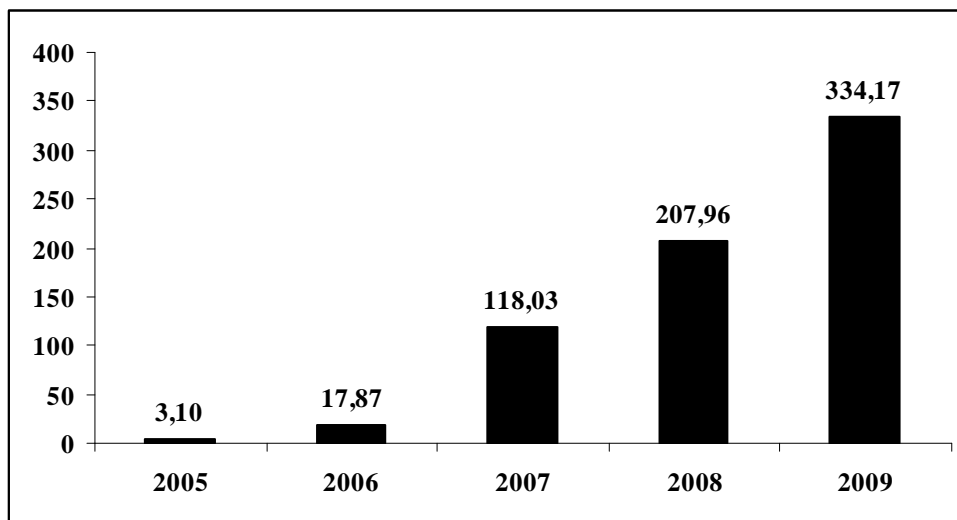
Only futures contracts from the main categories of financials (currency, interest rate and equity index) and certain commodities (gold, cotton, wheat) have been traded in TurkDEX so far. Futures prices are determined by executing “multiple price-continuous auctions” method. A single trading session called as “normal session” is adopted in TurkDEX, holding between 9.30 a.m. and 5.35 p.m. TurkDEX is tax-free for foreign and domestic investors since the trading was

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<sup>7</sup> See TurkDEX web site, <http://www.turkdex.org.tr/VOBPortalEng/DesktopDefault.aspx?tabid=101>

commenced in the exchange.<sup>8</sup> As TurkDEX utilizes a fully electronic trading system with remote access, there is no geographical restriction to invest.

The magnitude of trading volume in TurkDEX has been growing gradually in parallel with awareness of the futures contracts by Turkish investors. Figure 1 below evidently highlights the upward trend in total volume during the period of 2005-2009. As demonstrated in the figure that annual trading volume in the market soared more than 100 times by this time. Further, the daily average volume has reached 1.4 billion TL in 2009, which covers 55%-65% of daily trading volume in Istanbul Stock Exchange.



**Figure-1: Annual Trading Volume in TurkDEX (bill. TL)**

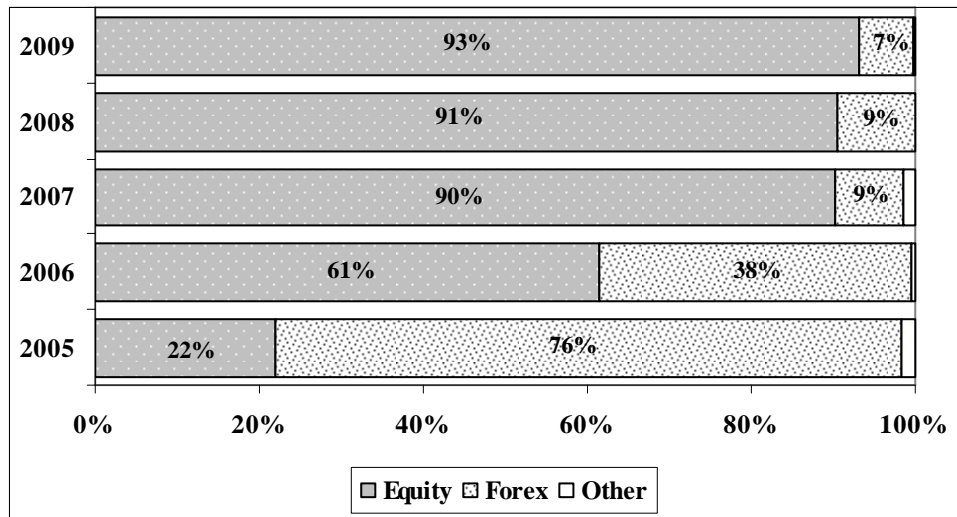
Source: TurkDEX

Nevertheless, the breakdown of total trading volume in TurkDEX points out the presence of a heterogeneous structure on the basis of individual futures contracts. Correspondingly, it can be affirmed that ISE-30 equity index futures has

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<sup>8</sup> This regime is going to continue for domestic investors until the end of 2008. However, there is no an announced deadline for foreign investors yet. The proportion of foreign investors in the market is approximately 25%.

dominated the market since the beginning of 2006. This contract composed 91.22% and 93.16% of market trading volume by itself in 2008 and 2009 respectively (see Figure-2).



**Figure-2: Trading Volume Breakdown in TurkDEX**  
Source: TurkDEX

As the ISE-30 index futures contract is comprehended as a good proxy to represent the whole market, it would be reasonable to use this contract for investigating the hedging effectiveness of TurkDEX. The ISE-30 futures are quoted by dividing the underlying spot index (ISE-30) by 10 simply. Six delivery months- February, April, June, August, October and December- are specified but only three of them are opened to trade at the same time considering expiration of previous contracts.<sup>9</sup> The ISE-30 futures contracts are cash-settled as other futures in TurkDEX. Daily price limit is +/-10%; position limits are 5000 contracts (absolute) and 10% of total open positions (proportional).

<sup>9</sup> For instance, when February contract is expired August contract is activated.

### 3.2 Data Description

The data set employed in the study comprise daily spot and futures prices of the ISE-30 stock index covering the period from 2 May 2005 to 30 April 2009 by 1006 observations.<sup>10</sup> The spot and futures data were collected from the official web sources of the Istanbul Stock Exchange (ISE) and the TurkDEX. The ISE-30 stock index contains the shares of 30 largely capitalized firms from various sectors quoted on Turkish stock market, which is accounting for approximately 60% of total market value. To eliminate the adverse effects of thin trading near expiration, the contracts in settlement month are rolled over to next two-month contracts whenever the daily open interest (as proxy of trading volume) of next two-month contract exceeds the daily open interest of the contract in settlement month.<sup>11</sup> Moreover, the last 276 observations (from 21 March 2008 to 30 April 2009) in the data set are not included to estimation process in order to evaluate out-of-sample forecasting abilities of the empirical models in the thesis. Figure 3 provides an insight for the basis between spot and futures prices of the ISE-30 index.

Daily returns for ISE-30 index (spot and futures) are calculated by the following formulas;

$$R_{s,t} = \log\left(\frac{S_t}{S_{t-1}}\right) \quad (2.1)$$

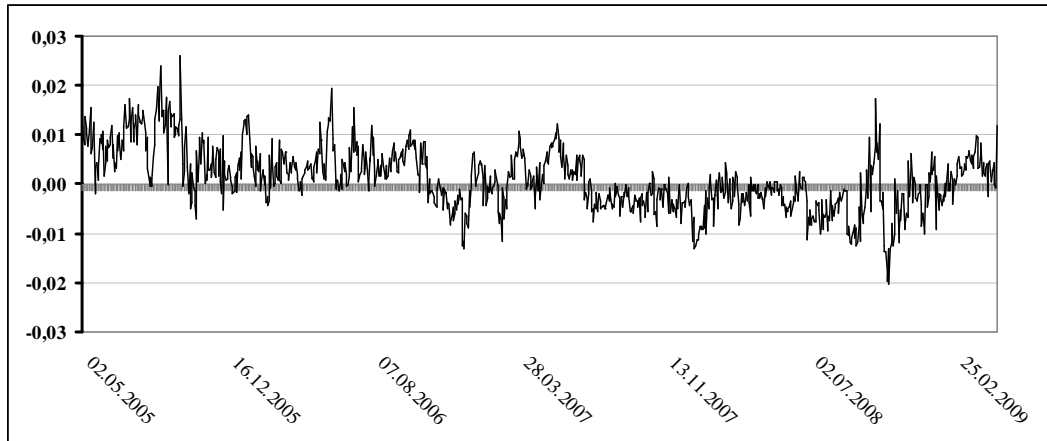
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<sup>10</sup> As the ISE-30 index future contracts were not heavily traded during the first months of the exchange, the dataset is started from 02 May 2005 instead of the launching date, 4 February 2005.

<sup>11</sup> For instance, the open interest of the February 2005 contract exceeded the December 2005 contract firstly at the rollover date of 29.12.2005 (one day before December 2005 contract expiration). Hence, the settlement prices of February 2005 contract are started to be used from 29.12.2005 by ignoring remained settlement prices in the December 2005 contract.

$$R_{f,t} = \log\left(\frac{F_t}{F_{t-1}}\right) \quad (2.2)$$

where  $R_s$  and  $R_f$  represent daily spot and futures returns respectively. Closing values of ISE-30 index are shown by  $S_t$  for spot and  $F_t$  for futures, on the corresponding day  $t$ .



**Figure-3: Basis between ISE-30 Index Spot and Futures Prices (log.)**  
Source: ISE and TurkDEX

### 3.2 Summary Statistics

Several descriptive statistics are calculated for the purpose of recognizing fundamental statistical properties of the dataset prior to empirical analysis. Table 1 shows summary statistics related to univariate spot and futures daily return series for the selected period.

**Table-1: Summary Statistics**

	Mean	Stand. Dev.	Variance	Skewness	Kurtosis	JB Statistics
$R_s$	2.57E-04	0.0221	0.0005	-0.0073	4.5564	65.14*
$R_f$	2.64E-04	0.0226	0.0005	-0.0458	4.6659	65.39*

Note: \* denotes 1% significance level.



As firstly seen in the table, both samples exhibit positive average daily return (mean) values over three year period, though futures returns are slightly greater than the spot returns. The volatility measures, standard deviation and variance, are highly identical for spot and futures returns as presumed. In addition, both series are negatively skewed by pointing out an asymmetrical left-tailed distribution. The excess kurtosis estimates denote that daily return distributions have fat tails (leptokurtic) relative to normal distribution. On the other hand, asymmetric distribution of spot and futures returns are verified through significant Jarque-Bera statistics as well rejecting the null hypothesis of normality<sup>12</sup>.

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<sup>12</sup> Jarque-Bera (JB) statistic is employed to test the null hypothesis of normality along with an asymptotic chi-square distribution. JB values tend to be closer to 1, or less than 1 for symmetric normal distributions.

## Chapter 4

### Methodology

As stated previously, there is nothing like “the best method” in the literature to estimate optimal hedge ratio for the selected futures contracts. Each suggested model has its own strengths and weaknesses. For instance, the primitive models, such as linear regression, are simple and easy to implement; but they do not fully cover the features (e.g. heteroskedasticity) of the financial data. On the other hand, the sophisticated models, as multivariate GARCH, may lead to calculate more realistic dynamic hedge ratios. Nevertheless, they have numerous constraints which make estimation process so difficult and complicated.

Moreover, the predictions of the econometric models do not engender identical hedging performances for each derivative market particularly.<sup>13</sup> Therefore, most of the researches concerning optimal hedge ratio estimation have been conducted by employing various econometric models together and interpreting their results in terms of hedging effectiveness. The appropriateness of an empirical model and the optimal hedge ratio for a specific futures contract is determined through comparing the findings of these models respectively. This common application will also be adopted in this thesis. Accordingly, Linear Regression Model (Model I), Bivariate Vector Autoregressive Model (Model II), Error Correction Model (Model III), GARCH Model (Model IV) and Multivariate GARCH Model (Model

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<sup>13</sup> Whilst the hedge ratio calculated by simple regression method is characterized as optimal by Choudhry (2003) for Germany and United Kingdom, the hedging performance of the same model is found as the worst for Australia and Hong Kong.

V) are chosen for the econometric comparisons to estimate optimal hedge ratio for the ISE-30 index futures contract. However, it is expected that multivariate GARCH (M-GARCH) hedge ratio will provide a superior hedge performance due to the statistical strength of the model.

#### **4.1 Linear Regression Model**

Linear regression is the most widely applied econometric technique, which aims to explain the statistical relationship between two or more variables by fitting a linear equation to observed data. In other words, it attempts to estimate the expected value of an interest variable (dependent variable), using some explanatory variables (independent variables) along with a linear function. To fit a regression line for the observed data, the sum of squares belonging to vertical deviations (residuals) from each data point is minimized, which is called as ordinary least-squares (OLS) method. The classical assumptions of the linear regression model can be stated as normally distributed and uncorrelated residuals that have an unconditional constant variance (homoscedasticity).

Since the interaction between spot and futures prices composes the key principal of hedging, the linear regression model is employed commonly to calculate hedge ratio through relating the spot price changes (dependent) with the changes of futures prices (independent). Furthermore, it is characterized as the simplest and quickest model in the literature. Equation (3.1) developed by Ederington (1979) is used to estimate hedge ratio specifically;

$$R_{s,t} = c + \beta R_{f,t} + \varepsilon_t \quad (3.1)$$

where the slope coefficient  $\beta$  represents risk-minimizing hedge ratio.  $R_{s,t}$  and  $R_{f,t}$  indicate actual daily spot and futures returns (price changes) respectively. The constant term in the model is shown by  $c$  and residuals (error terms) are symbolized by  $\varepsilon_t$ .

The validity of the linear regression model is highly dependent on satisfying its conventional assumptions. For this reason, the consistency of these assumptions should be monitored through some diagnosis tests. Jarque-Bera test for normality, Breusch-Godfrey test for autocorrelation and ARCH-LM test for homoscedasticity will be used in this thesis and will be discussed exclusively in the empirical results of linear regression model in the next chapter.

## 4.2 Bivariate Vector Autoregressive (VAR) Model

Vector autoregression is basically an econometric approach designed to capture interdependencies among multiple time-series. It can be perceived as a generalized extension of the univariate autoregressive model (AR) in practice. The VAR specifications are found to be functional in reflecting the dynamic features of the financial time-series and often generate superior forecasts than univariate models. In addition, estimates of VAR models are not rigid because of the conditional forecasts on potential lead-lag structure of specified variables.<sup>14</sup> There are two essential considerations for the variables of structural VAR model from

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<sup>14</sup> Theoretically, all the variables used in the VAR process have to be at the same order of integration [I(0) or I(1)]. Otherwise, the required transformations should be provided in order to prevent biased estimations.

econometric perspective: (i) error terms are not correlated, and more importantly (ii) variables can have contemporaneous interactions with other variables and their own lags.<sup>15</sup>

The VAR modeling has comprehensible advantages (especially on linear regression model) over estimation of a robust hedge ratio. First, it can eliminate serial correlation (autocorrelation) in residuals, which usually causes biased results predicted by conventional methods. Second, the VAR procedure helps to examine the simultaneous interactions between spot and futures returns by underlining the fact that fluctuations in both markets may influence the current price mechanisms.

The bivariate version of the standard VAR model is employed in the thesis particularly, consistent with the existed hedge ratio literature. It is just a multiple regressions process by two equations that contains two variables (spot and futures returns). For the purpose of calculating a risk-minimizing hedge ratio, the following bivariate VAR specification is applied particularly [Kroner and Sultan (1993)].

$$R_{s,t} = c_s + \sum_{i=1}^k \beta_{s,i} R_{s,t-i} + \sum_{i=1}^k \lambda_{s,i} R_{f,t-i} + \varepsilon_{st} \quad (3.2)$$

$$R_{f,t} = c_f + \sum_{i=1}^k \beta_{f,i} R_{s,t-i} + \sum_{i=1}^k \lambda_{f,i} R_{f,t-i} + \varepsilon_{ft} \quad (3.3)$$

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<sup>15</sup> See, Lütkepohl (2008).

Equations (3.2) and (3.3) demonstrate explanatory parameters on daily spot and futures returns, respectively. Meanwhile, daily spot and futures returns as main variables in both equations are characterized by  $R_{s,t}$  and  $R_{f,t}$  respectively.  $\beta_{s,i}$ ,  $\beta_{f,i}$ ,  $\lambda_{s,i}$  and  $\lambda_{f,i}$  are positive VAR parameters; and  $c_s$  and  $c_f$  indicate constant terms in the equations. Further, identically distributed independent vectors are shown by  $\varepsilon_{s,t}$  and  $\varepsilon_{f,t}$  in the model. The optimal hedge ratio ( $h$ ) is derived from the bivariate VAR model by:

$$h = \frac{\delta_{sf}}{\delta_f^2} \quad (3.4)$$

where  $\delta_f^2$  is the variance of  $\varepsilon_{f,t}$  calculated from equation (3.3) and  $\delta_{sf}$  is the covariance between  $\varepsilon_{s,t}$  and  $\varepsilon_{f,t}$ .

One critical point for the success of model is deciding the optimal lag length (shown by  $k$ ) to remove autocorrelation in residuals permanently. To this end, Akaike's Information Criterion (AIC) and Schwarz's Information Criterion (SIC) are employed to determine the proper lag length before running the model. Moreover, the autocorrelation structure of residuals is checked through the Lagrange-Multiplier test.

### 4.3 Error Correction Model (ECM)

Error correction mechanisms primarily allow considering long-run cointegration relationship between non-stationary (level) forms of variables in estimation process along with short-run impact of changes for stationary (differenced) forms.

If the presence of such a cointegration relationship is ignored, it would be probable to attain spurious results due to excluding the effect of previous period's error (adjustment) term for the long-run equilibrium. All the variables in the ECM must be stationary, I (0), and cointegrated as pre-requisites of the model. The estimation procedure of ECM consists of two steps. The first step is computing the error correction vector between non-stationary [I(1)] variables. Next, the pivot model for stationary variables is built by including appropriate error correction terms derived from the first step.

There is a long-run relationship between spot and futures prices under theoretical framework [equation (1.3)]. This structure clearly signals the presence of a cointegration relation among the variables, which is not taken into account by the previous hedge ratio models. However, numerous studies claim that such a correlation for spot and futures prices (at level) may cause some misspecifications on hedge ratio estimations and produce biased results apparently.

With the aim of incorporating both long-run and short-run dynamics simultaneously to the forecasting process, the error correction model is employed as the third method to determine the optimal hedge ratio. The univariate ECM model and its cointegrating vector are defined as follows, respectively:

$$R_{s,t} = c + \beta R_{f,t} + \sum_{i=1}^k \eta_i R_{f,t-i} + \sum_{m=1}^n \theta_m R_{s,t-m} + \lambda EC_{t-1} + \varepsilon_t \quad (3.5)$$

$$EC_{t-1} = S_{t-1} - (a + bF_{t-1}) \quad (3.6)$$

In equations (3.5) and (3.6),  $R_{s,t}$  and  $R_{f,t}$  indicate daily spot and futures returns [I(1)], where previous day's non-stationary spot and futures prices (logged) are shown by  $S_{t-1}$  and  $F_{t-1}$  in cointegration equation (3.6). In equation (3.5),  $EC_{t-1}$  represents lag-one error correction term and the optimal hedge ratio is demonstrated by  $\beta$ . The constant and error term (residual) of the model are characterized by  $c$  and  $\varepsilon_t$  as other models. Further, the absolute value of  $\lambda$  parameter provides an interpretation about the speed of adjustment in the long-run relationship. Optimal lag-lengths of spot ( $n$ ) and futures ( $k$ ) returns might be different for the ECM unlike the bivariate VAR model. We followed “general to specific” approach of Hendry (1995) to verify suitable lag-lengths. The pre-conditions of ECM given above are controlled through various unit-root tests and Johansen (1988) cointegration test before setting the model. Nevertheless, the same diagnosis tests previously applied to linear regression model are also implemented to ECM, as it is just an extension of the simple regression model.

#### **4.4 GARCH Model**

The heteroskedastic nature of the financial time-series has been confirmed by numerous empirical studies over the last decade. In fact, the variances of error terms for these series are not constant over time, as assumed in conventional econometric perspective, and the current variances are considered as conditional upon the past values. As a consequence of reflecting the characteristics of the data reasonably, the GARCH model developed by Bollerslev (1986) is classified as superior for modeling financial asset returns in particular. The process permits the variance change over time through a long-term memory contemporaneously. To



put it differently, it has a key advantage of attaching heteroskedasticity into the estimation procedure and capturing the tendency for volatility clustering in financial series. According to the GARCH framework, there are two main parameters affecting current variance: past values of error terms (ARCH effect) and the conditional variances generated by information arrivals to the market.

For a standard GARCH (p, q) model, p and q represent the lag length of conditional variances and past values of error terms, respectively. Generally log-likelihood ratio test is conducted in order to determine the optimal lag length for the GARCH model. As a rule of thumb, the GARCH (1,1) specification generally provides the best fit and forecast accuracy for the financial data within other alternatives.

Consistent with the prior empirical works, we expect that the spot and futures return series used in this thesis will follow a heteroskedastic pattern. The asymmetric structures of these series have already been exhibited in descriptive statistics. Correspondingly, the GARCH (p, q) model is constructed below [equations (3.7) and (3.8)] on estimation of the optimal hedge ratio as the fourth model. The most important contribution of this model would be calculating a hedge ratio that considers heteroskedastic nature of the data.

$$R_{s,t} = c + \beta R_{f,t} + \varepsilon_t \quad ; \varepsilon_t^2 | \varphi_{t-1} \equiv N(0, \delta_t^2) \quad (3.7)$$

$$h_t = \lambda_0 + \lambda_1 \varepsilon_{t-q}^2 + \lambda_2 h_{t-p} \quad (3.8)$$

Equation (3.7) is the mean equation of the model, where  $R_{s,t}$  and  $R_{f,t}$  represent daily spot and futures returns.  $\beta$  characterizes the optimal hedge ratio and the constant term is shown by  $c$ . Further,  $\varepsilon_t$  is random error term while  $\varphi_{t-1}$  coefficient indicates available past information that has an effect on error term. In volatility equation (3.8), the coefficients of  $\lambda_1$  and  $\lambda_2$  are the ARCH and GARCH effects respectively on the conditional volatility  $h_t$ . The lag-lengths,  $p$  and  $q$ , for the model are chosen by log-likelihood ratio test as stated before.

#### **4.5 Multivariate GARCH (M-GARCH) Model**

Multivariate GARCH framework can be simplified just as an advanced version of univariate GARCH pattern, which aim to provide a better forecast of asset pricing, portfolio selection and hedging. Technically, it focuses on predicting the sequential dependence in the cross order moments of time-series by employing information set through complete variance-covariance matrix of errors. In fact, the distinctive feature of M-GARCH model, compared to univariate specifications, is using historical information from various markets together instead of concentrating in only one market. Pagan (1984) affirms that the generated regressor problem detected for univariate models is eliminated by the generated regressor problem detected for univariate models is eliminated by M-GARCH extension since all parameters are estimated jointly. Moreover, it is asserted by Engle and Kroner (1995) that variance and covariance elements in M-GARCH model depend upon the information set in a vector ARMA manner.

As the spot and futures series are closely interrelated, it would be more precise to conduct a multivariate model for computing the optimal hedge ratio. Thus, M-GARCH specification of Kroner and Sultan (1993) is utilized lastly in the thesis with the anticipation of deriving the best hedge ratio plausibly. As a combination of the bivariate VAR and the error correction model, equations (3.9) and (3.10) present conditional mean equations of the M-GARCH model. Hereby,  $R_{s,t}$  and  $R_{f,t}$  demonstrate daily spot and futures returns and  $EC_{t-1}$  is the error correction term from Equation (3.6). The coefficients  $c_s$  and  $c_f$  display constant terms. Further, the residuals of the equations are shown by  $\varepsilon_{s,t}$  and  $\varepsilon_{f,t}$ .

$$R_{s,t} = c_s + \sum_{i=1}^k \beta_{s,i} R_{s,t-i} + \sum_{i=1}^k \lambda_{s,i} R_{f,t-i} + \gamma EC_{t-1} + \varepsilon_{st} \quad (3.9)$$

$$R_{f,t} = c_f + \sum_{i=1}^k \beta_{f,i} R_{s,t-i} + \sum_{i=1}^k \lambda_{f,i} R_{f,t-i} + \gamma EC_{t-1} + \varepsilon_{ft} \quad (3.10)$$

Conditional volatility specifications of the M-GARCH (p, q) model are expressed through the following equation (and matrices) in this manner.

$$\begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + A_i \begin{bmatrix} \varepsilon_{s,t-p}^2 \\ \varepsilon_{s,t-p}, \varepsilon_{f,t-p} \\ \varepsilon_{f,t-p}^2 \end{bmatrix} + B_i \begin{bmatrix} h_{ss,t-q} \\ h_{sf,t-q} \\ h_{ff,t-q} \end{bmatrix} \quad (3.11)$$

In equation (3.11),  $h_{ss}$  and  $h_{ff}$  exhibit conditional variance of the error terms ( $\varepsilon_{s,t}$ ,  $\varepsilon_{f,t}$ ) from the mean equations of the model. Besides, the conditional covariance among daily spot and futures returns is represented by  $h_{sf}$ . It is evidently seen that there are 21 parameters to estimate in the model. In order to ease complications,

Bollerslev *et al.* (1988) suggested a restricted parameterization technique, called as diagonal VEC, which assumes  $A_i$  and  $B_i$  matrices in equation (3.11) are diagonal. It is also assumed that the conditional variances (spot and futures) are only related with the past values of its lagged squared residuals. Therefore, diagonal elements deducted to estimate conditional variances and covariance of the M-GARCH (p, q) model are found as:

$$h_{ss,t} = c_{ss} + \alpha_{ss} \varepsilon_{s,t-p}^2 + \beta_{ss} h_{ss,t-q} \quad (3.12)$$

$$h_{sf,t} = c_{sf} + \alpha_{sf} \varepsilon_{s,t-p} \varepsilon_{f,t-p} + \beta_{sf} h_{sf,t-q} \quad (3.13)$$

$$h_{ff,t} = c_{ff} + \alpha_{ff} \varepsilon_{f,t-p}^2 + \beta_{ff} h_{ff,t-q} \quad (3.14)$$

Subsequent to the estimation of parameters above, time-varying daily hedge ratios ( $H_t$ ) are calculated by:

$$H_t = \frac{h_{sf,t}}{h_{ff,t}} \quad (3.15)$$

However, it may not be so practical to change the position in futures market day by day for a hedger due to the transaction costs and initial margins. Hence, the arithmetic mean value of time-varying hedge ratios will be taken as unique M-GARCH hedge ratio for that reason.<sup>16</sup>

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<sup>16</sup> There are numerous studies using this process. To name a few, see Myers (1991), Kroner and Sultan (1993), and Brooks, Henry and Persaud (2002).

## Chapter 5

### Empirical Results

#### 5.1 Testing for Unit Root

The presence of unit-root for the time-series data is checked through various procedures as a pre-condition. Granger and Newbold (1974) indicated that using non-stationary variables in the OLS process leads potentially spurious regression results due to the time-variant variance feature of the unit-root series.<sup>17</sup> In this case, the persistence of shocks will be infinite and permanent by invalidating the assumptions of asymptotic OLS analysis. Hence, the non-stationary data incorporated to conventional regression model may cause higher  $R^2$  values and t-ratios for even completely unrelated variables.

Even though there are numerous methods proposed in the literature for investigating a unit-root in time-series, the most widely conducted two procedures are Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP). We also employed these two procedures in this study to test the stationarity condition of the series.<sup>18</sup>

The ADF test is developed by Dickey and Fuller (1981) which assumes error term ( $\varepsilon_t$ ) is white noise and not autocorrelated. However, it is reported by many related studies [e.g. Phillips (1987)] that the error term ( $\varepsilon_t$ ) is unlikely to be white noise

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<sup>17</sup> Contrary to stationary series, the non-stationary time series do not have a constant variance. The variance depends on time and it has a long memory, which approaches to infinity as time goes. Thus, it is usually not clear-cut to estimate parameters over past and future intervals of time for these series.

<sup>18</sup> Leybourne and Newbold (1999) stated that evaluating the results of both tests jointly would be more precise.

in practice. Therefore, Dickey and Fuller (1981) suggested an ‘augmented’ version that adds  $p$  lags of the dependent variable ( $\Delta Y$ ) to the standard autoregressive time-series model by the following:

$$\Delta Y_t = \gamma Y_{t-1} + \sum_{i=1}^p \alpha_i \Delta Y_{t-i} + \varepsilon_t \quad (5.1)$$

The test-statistics that is compared with the critical values computed by Dickey and Fuller (1981) under the null hypothesis  $H_0: \gamma = 0$  (there is unit-root) is calculated as:

$$DF_t = \frac{\tilde{\gamma}}{SE(\tilde{\gamma})} \quad (5.2)$$

where  $SE$  stands for standard error. If the test statistics is greater than the critical value then the null hypothesis of  $\gamma = 0$  is rejected and the absence of unit root for  $Y_t$  is verified. Dickey and Fuller (1981) derived also two more expansions of their core process [equation (5.2)] to monitor random walk against a stationary autoregressive process. These are called “intercept” and “trend” procedures, which are also checked separately in this study.

On the other hand, several limitations are present for the ADF test. First, the adequacy of the test declines clearly as the lag-length,  $p$  increases. Second, it assumes that residuals are uncorrelated and have constant variance; but this is generally not true for the financial time-series. By considering the shortcomings of the ADF test, Phillips and Perron (1988) suggested an alternative approach on unit-root testing that allows taking into consideration heteroskedasticity and

autocorrelated residuals. A non-parametric correction is used to account for the probable serial correlation. The hypotheses and the decision making process for the PP test are identical with the ADF test. The test-statistics that will be compared with the relevant critical values is computed via the formula below:

$$Z(\tau_\mu) = \left( \frac{S_u}{S_\pi} \right) \tau_\mu - 0.5 \left( \frac{S_\pi^2}{S_u^2} \right) \left[ S_\pi \left\{ T^2 \sum_{i=2}^T (y_{t-1} - \bar{y})^2 \right\}^2 \right]^{-1} \quad (5.3)$$

$S_\pi$  and  $S_u$  are consistent estimators developed by Phillips and Perron (1988), corresponding to dependent variable  $y$ .  $T$  is the number of observations and  $\tau_\mu$  is the t-statistics applied in testing the null hypothesis of unit root.

Table 2 demonstrates the results of ADF and PP tests for the spot and futures return series. Moreover, the logarithmic values of spot and futures indexes at levels are examined as well since their unit root structure is important to reveal the type of cointegration relationship between the variables [equation (3.6)]. The optimal lag-length that removes autocorrelation among residuals is determined according to Akaike Information Criteria (AIC). It is denoted in the table that both of the return series are stationary at 1% significance level whilst the logarithmic spot and future index series (at level) are likely to contain unit-root, hence the series are non-stationary.

**Table-2: Unit Root Tests**

Variables	Lag-Length (p)	Critical Values	ADF Statistics	PP Statistics
S <sub>t</sub>	1	-3.430	-2.508	-2.543
F <sub>t</sub>	1	-3.430	-2.492	-2.464
R <sub>s</sub>	2	-3.430	-16.142*	-28.438*
R <sub>f</sub>	1	-3.430	-20.321*	-29.381*

**Note:** S<sub>t</sub> and F<sub>t</sub> represent logarithmic spot and future values for ISE-30 index (level) respectively. R<sub>s</sub> and R<sub>f</sub> demonstrate daily spot and future returns in this manner. Tests are applied according to “intercept” and “trend” procedures as well; as the results do not change we have not presented in the table.

\* 1% significance level.

As the variables are integrated of order one [I(1)], we need to analyze the cointegration relationship. The next subsection does this.

## 5.2 Testing for Cointegration

Once the presence of I(1) process is detected for the spot and futures price (level) series, it is now possible to analyze the cointegration relationship between the mentioned non-stationary variables. In this context, Johansen (1988) cointegration test is used particularly to observe probable long-run interactions among the spot and future markets. This methodology has precise advantages over alternative testing procedures such as the Engle and Granger (1987) method. It primarily lets all test parameters to react as endogenous variables during the estimations. The basic VAR mechanism applied in Johansen method is as follows.

$$\Delta Y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p+1} \Gamma_i \Delta y_{t-1} + \varepsilon_t \quad (5.4)$$

$y_t$  is an  $n \times 1$  vector of variables that are I(1) process,  $\mu$  is vector of constants and  $t = 1, 2, \dots, T$  is the number of observations. The lagged terms capturing the long-run



dynamics is represented by  $y_{t-1}$ . However, the rank of  $\Pi$  plays a key role in the process for estimating the number of cointegrating vectors. If the rank of  $\Pi$  is  $r$  and  $r < n$  then there exists  $n \times r$  cointegrating vector among the variables. Johansen (1988) suggested two different likelihood tests to monitor the reduced rank of the  $\Pi$  matrix, which are called as “the trace test” and “the maximum eigenvalue test”. The following equations basically exhibit related computations for these statistics respectively that will be compared with critical values found by Johansen (1988).

$$J_t = -T \sum_{i=r+1}^n \ln(1 - \tilde{\lambda}_i) \quad (5.5)$$

$$J_m = -T \ln(1 - \tilde{\lambda}_i) \quad (5.6)$$

$\lambda$  represents the largest canonical correlation and  $T$  is the sample size. Whilst the trace statistics is used to test the null hypothesis of  $r$  cointegrating vector against the alternative hypothesis of  $n$  vectors; the maximum eigenvalue statistics tests the presence of  $r$  cointegrating vectors (null) against  $r+1$  (alternative). If the calculated statistics is greater than the critical value, the null hypothesis would be rejected as usual.

Table 3 provides Johansen cointegration test results. Widely applied two-step process is followed to test the rank of cointegrating vector between logarithmic spot and future prices. According to this framework, firstly the lack of cointegrating vectors,  $H_0: r = 0$  is tested firstly against the hypothesis that there is at least one vector,  $H_1: r = 1$ . Next, the null hypothesis of maximum one vector

$H_0: r \leq 1$  is tested against the alternative hypothesis of two cointegrating vectors  $H_1: r = 2$ . The lag-length of VAR model is calculated as 3 by the AIC. 1% and 5% Johansen critical points are also presented in the table.

**Table-3: Johansen Cointegration Test**

	$H_0: r = 0$ and $H_1: r = 1$			$H_0: r \leq 1$ and $H_1: r = 2$		
	Statistics Value	Critical Values		Statistics Value	Critical Values	
		1%	5%		1%	5%
Trace	42.59*	24.60	19.96	6.40	12.97	9.42
Eigenvalue	36.18*	24.60	15.67	6.40	12.97	9.24

**Note:** The VAR model employed in process is specified with trend and constant terms.

\* exhibit the rejection of  $H_0$  at 1% significance.

As shown in the table, the null hypothesis that there is no cointegration relationship among spot and futures prices ( $H_0: r = 0$ ) is rejected significantly against the alternative hypothesis that there is one cointegrating vector ( $H_1: r = 1$ ). Nonetheless, the null hypothesis stating that the number of cointegrating vectors is not greater than one ( $H_0: r \leq 1$ ) cannot be rejected by either trace or eigenvalue statistics. Therefore, the presence of a cointegration relationship between level variables is approved with rank of one ( $r = 1$ ). In that case, the cointegrating vector among logged spot and futures prices is found by using OLS regression method as follows:

$$S_t = 0.4389 + 0.9599F_t \quad (5.7)$$

This vector will be used to derive error correction term [equation (3.6)] among spot and futures prices, which is a parameter of the ECM and M-GARCH models specified above.

### 5.3 Hedge Ratio Estimations of Specified Models

The hedge ratios computed by proposed empirical models for the ISE-30 index futures are reported in this section with related diagnostics respectively.

#### 5.3.1 Estimates of Linear Regression Model

The OLS results obtained by running the regression equation (3.1) are shown in Table 4 (Panel A). Accordingly, the optimal hedge ratio that is just the coefficient of daily futures returns ( $R_{f,t}$ ) is calculated as 0.8938. Statistical meanings of estimated coefficients are checked through standard errors, t-statistics and related p-values as well.<sup>19</sup> The coefficient representing the hedge ratio is found as significant (1%) in this regard, contrary to constant parameter ( $c$ ) of the model.

**Table-4: Results of Linear Regression Model**

<b>Panel A – Parameter Estimates</b>				
<b>Variable</b>	<b>Coefficient</b>	<b>S. Error</b>	<b>t-statistics</b>	<b>p-value</b>
c	1.7e-5	0.0003	0.06	0.954
$R_{f,t}$	0.8938	0.0159	56.35	0.000*
<b>Panel B-Diagnostic Tests</b>				
	<b>Shapiro-Wilk</b>	<b>Breusch-Godfrey</b>	<b>ARCH-LM</b>	<b>White's</b>
<b>Test Statistic</b>	12.23*	82.63*	48.81*	20.35*

**Note:** The null hypotheses for diagnostic tests can be conveyed as: “normally distributed residuals” for Shapiro-Wilk, “no serial correlation in residuals” for Breusch-Godfrey, “no ARCH effects in residuals” for ARCH-LM and “constant variance” for White’s test.

\* indicates 1% significance level.

As a matter of fact, the robustness of the linear regression model strongly depends on validity of the OLS assumptions. Hence, the required specification tests are applied to the estimates of regression model. Panel B (in Table 4) denotes the

<sup>19</sup> This statistical evaluation process will be followed similarly for the findings of other empirical models in the dissertation.

results of the essential diagnostics in particular. The assumptions of normality and no serial correlation are rejected via the Shapiro-Wilk and Breusch-Godfrey tests respectively. In addition, the results of the ARCH-LM and White's tests state that the spot and future return series have time-varying variance unlike the OLS assumptions. Consequently, it can be argued that there exist potential biases in the estimations of the regression model.

### 5.3.2 Estimates of Bivariate VAR Model (II)

The bivariate VAR model [equations (3.2) and (3.3)] is employed secondly in the thesis to calculate optimal hedge ratio, which aims removing the adverse effects of serial correlation detected by the regression method. The appropriate lag-order for the VAR model is determined as 2 according to AIC. Table 5 displays the estimated coefficients from the bivariate VAR (2) model. Nevertheless, the LM statistics (up to lag 4) are also provided in the table to ensure the condition of no serial correlation.

**Table-5: Results of Bivariate VAR Model**

<b>Variable</b>	<b>Coefficient</b>	<b>S. Error</b>	<b>z-statistics</b>	<b>p-value</b>
<b><u>Spot Eq.</u></b>				
$c_s$	0.0006	0.0007	0.82	0.411
$R_{s,t-1}$	-0.0457	0.0889	-1.98	0.048**
$R_{s,t-2}$	-0.0283	0.0922	-0.31	0.759
$R_{f,t-1}$	0.0582	0.0920	0.63	0.527
$R_{f,t-2}$	0.0375	0.0917	0.41	0.682
<b><u>Futures Eq.</u></b>				
$c_f$	0.0006	0.0007	0.67	0.503
$R_{s,t-1}$	0.4030	0.0913	4.41	0.000*
$R_{s,t-2}$	0.2107	0.0917	2.30	0.022*
$R_{f,t-1}$	-0.4029	0.0912	-4.41	0.000*
$R_{f,t-2}$	-0.1705	0.0889	-1.99	0.045**
<b>LM -Test</b>	<b>Lag 1</b>	<b>Lag 2</b>	<b>Lag 3</b>	<b>Lag 4</b>
	5.62 (0.22)	4.47 (0.35)	7.42 (0.11)	1.35 (0.85)

**Note:** The null hypothesis for LM-test is “no autocorrelation at lag order k”. Numbers in brackets represent corresponding p-values. \* and \*\* indicate the significance levels of 1% and 5% respectively.

After predicting the variance-covariance matrix of residual series for the spot ( $\varepsilon_{s,t}$ ) and futures ( $\varepsilon_{f,t}$ ) equations, the hedge ratio is calculated as 0.9322 [equation (3.4)] by the VAR method.

### 5.3.3 Estimates of Error Correction Model (III)

Since the presence of a long-run cointegration relationship is confirmed between spot and futures prices, the Error Correction Model (ECM) ought to be employed alternatively for calculating the optimal hedge ratio. Accordingly, the ultimate model [equation (3.5)] is derived by adding one-period lagged error correction term to the simple regression equation [equation (3.1)] with specified lag orders of spot and futures returns. The time-series regarding the error correction term is created by referring the cointegration vector computed previously [equation (5.9)]. Before running the model, the proper lag-length for spot and future returns is determined as 2 by applying the Hendry's (1995) "general to specific" approach. The outcomes from the error correction model are noted in Table 6 below.

**Table-6: Results of Error Correction Model**

<b>Panel A – Parameter Estimates</b>				
<b>Variable</b>	<b>Coefficient</b>	<b>S. Error</b>	<b>t-statistics</b>	<b>p-value</b>
c	1.5e-05	0.0002	-0.06	0.956
$R_{f,t}$	0.9271	0.0132	62.79	0.000*
$R_{s,t-1}$	-0.3107	0.0485	-6.41	0.000*
$R_{s,t-2}$	-0.1592	0.0441	-3.61	0.000*
$R_{f,t-1}$	0.3348	0.0489	6.85	0.000*
$R_{f,t-2}$	0.1445	0.0446	3.24	0.001*
$EC_{t-1}$	-0.1731	0.0316	-5.49	0.000*
<b>Panel B – Diagnostic Tests</b>				
	<b>Shapiro- Wilk</b>	<b>Breusch- Godfrey</b>	<b>ARCH-LM</b>	<b>White's</b>
<b>Test Statistic</b>	12.39*	0.62	25.98*	4.88**

\* and \*\* indicate the significance levels of 1% and 5% respectively.

Nearly all estimated parameters are statistically significant, as demonstrated in the table (Panel A). This finding remarkably gives us an idea about the fitness of error correction model. The hedge ratio that is just the coefficient of current future returns ( $R_{f,t}$ ) is found as  $0.9271$ . Furthermore, the sign of error correction term ( $EC_{t-1}$ ) clarifies the fact that the direction of adjustment is from spot to future market unsurprisingly. The speed of adjustment is determined as approximately 6 days in this manner.

Panel B in the table reports the results of essential diagnostic tests, which are applied for linear regression model as well. The diagnostic tests show that the residuals are abnormally distributed and do not have constant variance (heteroskedastic). On the other hand, the absence of serial correlation cannot be rejected significantly for given lag orders.

#### **5.3.4 Estimates of GARCH Model (IV)**

The heteroskedastic nature of the return series has encouraged us to set a GARCH model in estimation of the optimal hedge ratio. Within many specifications, the GARCH ( $p, q$ ) model [equations (3.7) and (3.8)] is chosen due to its accuracy on many financial time series. The lag-orders of conditional variance,  $p$ , and past-squared residuals,  $q$  are defined as 1 by the log-likelihood procedure.<sup>20</sup> Table 7 summarizes the coefficient findings of GARCH (1, 1) model. To support the robustness of the model, the Ljung-Box-Q (LB-Q) and ARCH-LM test statistics are presented in the table as additional diagnostics.

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<sup>20</sup> After estimating several GARCH ( $p, q$ ) specifications for  $p=1, 2$  and  $q=1, 2$  it is decided that the GARCH (1, 1) model is the most appropriate model according to log-likelihood statistics.

**Table-7: Results of GARCH (1,1) Model**

<b>Panel A – Parameter Estimates</b>				
<b>Parameters</b>	<b>Coefficient</b>	<b>S. Error</b>	<b>z-statistics</b>	<b>p-value</b>
c	2.1e-04	2.3e-05	0.08	0.929
$\beta$	0.9093	0.0098	93.51	0.000*
$\lambda_0$	2.4e-06	6.89e-07	3.56	0.004*
$\lambda_1$	0.0871	0.0157	5.57	0.000*
$\lambda_2$	0.8796	0.0201	42.80	0.000*

<b>Panel B – Diagnostic Tests</b>			
	<b>Test-statistics</b>	<b>lag-length</b>	<b>p-value</b>
<b>ARCH-LM</b>	0.81	2	0.469
<b>Ljung-Box-Q</b>	18.35	20	0.313

**Note:** The null hypothesis for Ljung-Box test is “no serial correlation for given lag order”. The lag-lengths for diagnostic tests are determined by using AIC.  
\* and \*\* indicate the significance levels of 1% and 5% respectively.

The GARCH (1, 1) model yielded that (see Panel A in Table 7) the optimal hedge ratio, represented by  $\beta$ , is *0.9093* and significant.<sup>21</sup> More importantly, the hedge ratio of the GARCH (1,1) model has the lowest standard error compared to results of previous methods so far.  $\lambda_1$  and  $\lambda_2$  parameters appear to be highly significant by correcting the conditional volatility over the spot and futures returns. The condition of  $\lambda_1 + \lambda_2$  close to unity can be interpreted as past volatility information suppresses the outsized market shocks in forecasting current volatility.<sup>22</sup> To put differently, these shocks decay with time. The insignificant LB-Q and ARCH-LM statistics at selected lags (Panel B) confirm the adequacy of the model.

### 5.3.5 Estimates of M-GARCH Model

Although the bivariate VAR model and the error correction model (ECM) have taken into account the short-run and long-run interactions between spot and futures prices respectively, the ARCH effects in residuals could not be captured

<sup>21</sup> Estimations are made under General Error Distribution (GED) assumption as it is more effective for non-normal distributions.

<sup>22</sup> The persistency in volatility is captured by  $\lambda_1 + \lambda_2$  for GARCH (p, q) model.

by these models. At this point, the most crucial feature of the M-GARCH model is to consider the bivariate cointegration relationship between spot and futures prices from the *heteroskedastic* pattern jointly. It basically combines the bivariate VAR and the error correction models through mean equations [equations (3.9) and (3.10)] and the GARCH framework as in conditional variance-covariance equations [equations (3.12), (3.13) and (3.14)]. Time-varying hedge ratios from the M-GARCH model are therefore expected to give better results in terms of hedging effectiveness.

Table 8 indicates the estimation results from the M-GARCH (1, 1) model.<sup>23</sup> However, the coefficient findings of mean equations are not presented in the table since the main drive for calculating time-varying hedge ratios is to obtain conditional variance and covariances. To estimate the coefficients in the model, the Marquardt algorithm under t-distribution is used specifically.

**Table-8: Results of M-GARCH (1,1) Model**

<b>Panel A – Parameter Estimates</b>				
<b>Parameters</b>	<b>Coefficient</b>	<b>S. Error</b>	<b>z-statistics</b>	<b>p-value</b>
$c_{ss}$	7.8e-06	3.1e-06	2.58	0.001*
$c_{ff}$	6.1e-06	2.2e-06	2.75	0.006*
$c_{sf}$	5.9e-06	2.1e-06	2.75	0.006*
$\alpha_{ss}$	0.0558	0.0130	4.29	0.000*
$\alpha_{ff}$	0.0511	0.0114	4.45	0.000*
$\alpha_{sf}$	0.0525	0.0115	4.55	0.000*
$\beta_{ss}$	0.9251	0.0159	58.06	0.000*
$\beta_{ff}$	0.9330	0.0139	66.70	0.000*
$\beta_{sf}$	0.9301	0.0138	67.39	0.000*

<b>Panel B – Diagnostic Tests</b>				
	<b>Residuals</b>	<b>Test-statistics</b>	<b>lag-length</b>	<b>p-value</b>
<b>Ljung-Box-Q</b>	Spot	23.74	20	0.254
	Futures	25.32	20	0.190

**Note:** The lag-lengths for Ljung-Box-Q test are determined by using AIC.

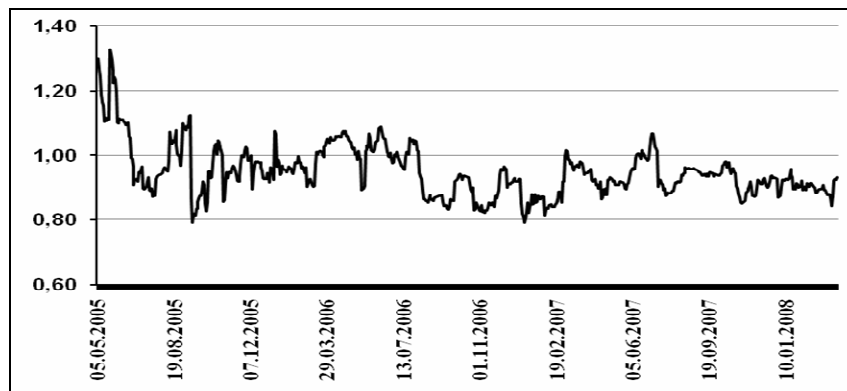
\* indicates the significance level of 1%.

<sup>23</sup> Once again the lag-lengths of p=1 and q=1 for the GARCH model provides the best combination corresponding to log-likelihood functions.



As stated in the table (Panel A), all coefficients of the conditional variance and covariance equations are statistically significant and positive, which satisfies the model constraint at first sight. Further, the GARCH parameters' sum ( $c_{ss} + \alpha_{ss} + \beta_{ss}$ ) and ( $c_{ff} + \alpha_{ff} + \beta_{ff}$ ) is close to unity for each variance equation. In other words, the persistence in volatility is high for the dataset. The sign of the covariance parameters ( $\alpha_{sf}, \beta_{sf}$ ) also corrects the positive interaction between the two prices. Panel B in the table provides Ljung-Box-Q statistics of standardized residuals for the spot and futures returns [equation (3.9) and (3.10)]. According to the results of Ljung-Box-Q test, there is no autocorrelation in residuals of both specifications, which confirms that the M-GARCH model is capable of estimating the dynamics in the second moments of spot and futures returns.

Figure 4 below plotting the time-varying hedge ratios calculated by the standard M-GARCH (1, 1) model [equation (3.15)] shows that it ranges from a minimum of 0.79 to a maximum of 1.32.



**Figure-4: Time-varying hedge ratios**

The mean value for the time-varying hedge ratio series is computed as *0.9490*. In order to compare the empirical models applied in the study robustly, the sample mean of the dynamic hedge ratios is taken as a benchmark.

#### 5.4 Hedging Effectiveness Comparison

The performance of the hedge ratios based on five different empirical models is compared in this section from the perspective of the hedging effectiveness. Ederington (1979) defines the hedging effectiveness as the relative percentage reduction in the unhedged portfolio variance after the hedging transaction. Therefore, we need to construct unhedged and hedged portfolios virtually first. While the unhedged portfolio just contains the ISE-30 spot index, the hedged portfolios are composed from the ISE-30 spot index and the ISE-30 futures index at different weights (using different hedge ratios) together. Then, the returns of these portfolios for different hedging horizons (i.e. 5, 10, 15 and 20 days) are computed by the following equations.

$$r_{u,t} = (S_t - S_{t-p}) \quad (5.8)$$

$$r_{h,t} = (S_t - S_{t-p}) - h^* (F_t - F_{t-p}) \quad (5.9)$$

$r_{u,t}$  and  $r_{h,t}$  represent the daily returns of unhedged and hedged portfolios respectively, where  $S_t$  and  $F_t$  indicate logged spot and futures prices.  $p$  equals 5, 10, 15 and 20 for different hedging horizons. The hedge ratio determining the weight of the futures position is shown by  $h^*$ . Nonetheless, the variances of the

created portfolio return series are calculated through the equations (5.10) and (5.11) below.

$$\text{Var}(U) = \text{Var}(r_u) \quad (5.10)$$

$$\text{Var}(H) = \text{Var}(r_h) \quad (5.11)$$

The variances of the unhedged and hedged portfolios are demonstrated by  $\text{Var}(U)$  and  $\text{Var}(H)$  respectively. Consequently, the measure of the hedging effectiveness ( $HE$ ) that is used to evaluate the performances of the estimated hedge ratios is calculated as follows [Ederington, (1979)]:

$$HE = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)} \quad (5.12)$$

As the main purpose of the hedging is to diminish or abolish the price risk (variance) of the underlying asset, the comparison of the effectiveness of hedge ratios via various techniques is made under this criterion essentially. There are few other studies [Howard and D'Antonio, (1984)] which assert that the return after hedging is also important and should be considered for the effectiveness of hedging. For that reason, the average daily returns of the constructed portfolios are also calculated and analyzed as a supportive tool for evaluating the effectiveness of hedging via different models.

Both in-sample and out-of-sample data are applied in the model assessment process. Whilst we can monitor statistical robustness of the empirical models by

analyzing in-sample data; out-of-sample data consider the forecast accuracy of the proposed models.<sup>24</sup>

Table 9 and 10 depict the effectiveness results of the hedge ratios of the methodologies used in the study for in-sample and out-of-sample data, respectively. The mean returns and the variances of the hedged and unhedged portfolios are presented as well in the tables with the measure of the hedging effectiveness, *HE*.

The results indicate that even if all hedge ratio estimates reduce the variance of the unhedged portfolio, time-varying hedge ratio of the M-GARCH model outperforms the findings of competing methods in majority of cases for both in-sample and out-of-sample data in terms of HE measure. This implication is evidently consistent with Baillie and Myers (1991), Kroner and Sultan (1993), Choudhry (2003) and Yang and Allen (2004). Hence, we can assert that the M-GARCH model is the most appropriate model (as expected) to estimate the risk-minimizing hedge ratios for ISE-30 index. The optimal hedge ratio for ISE-30 index contracts is determined as *0.9490* in this manner as an arithmetic mean of the daily hedge ratios from the multivariate model. Thus, if a hedger takes a reverse position in futures market which covers approximately 95% of his spot exposure, he will attain the most effective hedging. In addition, the significance of the hedging effectiveness increases parallel to the length of the hedging period through in-sample data but not for out-of-sample data.

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<sup>24</sup>However, Baillie and Myers (1989) and Park and Switzer (1994) reveal that the out-of-sample data provide more reliable results for the comparison of hedging effectiveness.

**Table-9: In-sample Hedging Effectiveness**

<b>Model</b>	<b>Hedge Ratio</b>	<b>Mean Return</b>	<b>Variance</b>	<b>HE (%)</b>
<b><u>5-day</u></b>				
OLS	0.8938	6.78e-05	1.29e-04	94.26
VAR	0.9322	-6.56e-05	1.21e-04	94.65
ECM	0.9271	-4.79e-05	1.23e-04	94.61
GARCH	0.9093	3.68e-05	1.25e-04	94.44
M-GARCH	0.9490	-1.23e-04	1.12e-04	<b>95.03</b>
Unhedged	0.0000	3.17e-03	2.24e-03	-
<b><u>10-day</u></b>				
OLS	0.8938	3.06e-04	1.95e-04	95.13
VAR	0.9322	2.23e-05	1.81e-04	95.47
ECM	0.9271	5.99e-05	1.83e-04	95.44
GARCH	0.9093	2.87e-04	1.89e-04	95.29
M-GARCH	0.9490	-1.01e-04	1.47e-04	<b>96.34</b>
Unhedged	0.0000	6.91e-03	4.02e-03	-
<b><u>15-day</u></b>				
OLS	0.8938	5.88e-04	2.71e-04	95.40
VAR	0.9322	1.49e-04	2.38e-04	<b>95.96</b>
ECM	0.9271	2.07e-04	2.42e-04	95.90
GARCH	0.9093	4.64e-04	2.54e-04	95.71
M-GARCH	0.9490	-4.32e-04	2.08e-04	95.79
Unhedged	0.0000	1.08e-02	5.90e-03	-
<b><u>20-day</u></b>				
OLS	0.8938	5.26e-04	2.59e-04	96.92
VAR	0.9322	-4.27e-05	2.18e-04	97.40
ECM	0.9271	3.29e-05	2.22e-04	97.36
GARCH	0.9093	3.79e-04	2.36e-04	97.21
M-GARCH	0.9490	-2.92e-04	1.97e-04	<b>97.66</b>
Unhedged	0.0000	1.37e-02	8.43e-03	-

**Note:** In-sample period covers 730 observations from 02.05.2005 to 20.03.2008.

**Table-10: Out-of-sample Hedging Effectiveness**

<b>Model</b>	<b>Hedge Ratio</b>	<b>Mean Return</b>	<b>Variance</b>	<b>HE (%)</b>
<b><u>5-day</u></b>				
OLS	0.8938	-2.25e-03	5.44e-05	93.12
VAR	0.9322	-2.24e-03	5.38e-05	93.21
ECM	0.9271	-2.24e-03	5.38e-05	93.22
GARCH	0.9093	-2.26e-03	5.39e-05	93.22
M-GARCH	0.9490	-2.19e-03	5.14e-05	<b>93.51</b>
Unhedged	0.0000	-2.26e-03	7.93e-04	-
<b><u>10-day</u></b>				
OLS	0.8938	-2.24e-03	5.61e-05	90.75
VAR	0.9322	-2.31e-03	5.80e-05	90.43
ECM	0.9271	-2.30e-03	5.76e-05	90.49
GARCH	0.9093	-2.28e-03	5.59e-05	<b>90.77</b>
M-GARCH	0.9490	-2.33e-03	5.94e-05	90.18
Unhedged	0.0000	-7.06e-04	6.06e-04	-
<b><u>15-day</u></b>				
OLS	0.8938	-1.82e-03	7.12e-05	93.28
VAR	0.9322	-1.80e-03	7.17e-05	93.23
ECM	0.9271	-1.80e-03	7.14e-05	93.26
GARCH	0.9093	-1.79e-03	7.09e-05	93.31
M-GARCH	0.9490	-1.77e-03	6.95e-05	<b>93.38</b>
Unhedged	0.0000	-2.25e-03	1.05e-04	-
<b><u>20-day</u></b>				
OLS	0.8938	-4.54e-03	3.60e-05	95.05
VAR	0.9322	-4.65e-03	3.44e-05	95.27
ECM	0.9271	-4.64e-03	3.45e-05	95.26
GARCH	0.9093	-4.55e-03	3.51e-05	95.19
M-GARCH	0.9490	-4.41e-03	3.24e-05	<b>95.54</b>
Unhedged	0.0000	-2.10e-03	7.28e-04	-

**Note:** Out-sample period covers 276 observations from 21.03.2008 to 30.04.2009.

When the returns of the portfolios are taken into account, there is a slight difference between in-sample and out-of-sample datasets. Whilst, the highest returns are provided by the regression method for in-sample data, the M-GARCH hedge ratio performs the best in three of four cases for out-of-sample data with negative returns particularly. Since the out-of-sample data is especially used to

evaluate the applicability of the empirical findings, it can be claimed that the best hedging period is 20 days or 4-weeks for the ISE-30 futures contracts as the lowest variance is realized in this period by the time-varying hedge ratio of the M-GARCH model. Therefore, hedgers should reorganize their portfolios every twenty days actively.

## **Chapter 6**

### **Conclusion**

This study examines the optimal hedge ratio for the ISE-30 futures contracts, traded in the TurkDEX, by running several competitive econometric models. Accordingly, the linear regression model, the bivariate vector autoregressive (VAR) model, the error correction model (ECM), the standard GARCH model and the multivariate GARCH (M-GARCH) model are conducted to estimate the risk-minimizing hedge ratio. The appropriateness/superiority of the models is investigated under the hedging effectiveness criterion for each in-sample and out-of-sample data horizons.

The empirical results demonstrate that the hedge ratio estimated by the M-GARCH model provides the highest variance (risk) reduction for majority of the hedging periods along with both in-sample and out-of-sample data. This finding is consistent with the general expectation in the thesis since the M-GARCH model has a more complex structure that takes into account the interactions among spot and future prices jointly from the heteroskedastic wisdom. On the other hand, there are no penetrating differences between the findings of empirical models in terms of hedging effectiveness. The hedging effectiveness also improves in longer hedging periods for just in-sample data.

Moreover, the return performance of the hedged portfolios that constructed by utilizing the estimated hedge ratios is also checked as a minor comparison tool for



the models. Consequently, the highest returns are provided by the linear regression model for the in-sample data and the M-GARCH model for the out-of-sample data.

It is expected that the findings of the thesis would be useful practically for the institutional investors, who want to hedge their exposure in Turkish stock market, through resolving the magnitude of the futures position. We can also assert that the ISE-30 index futures contract succeed to reduce price variability in the spot market more than 95%. Therefore, the TurkDEX is able to perform its hedging function perfectly even it does not have a long history as developed futures markets. This fact might help to increase the awareness and reputation of the TurkDEX in near future. Nonetheless, this study will enhance the poor academic literature about the newly established Turkish futures market through providing a recent discussion point on the optimal hedge ratio and the hedging effectiveness.

One important proposal for a possible further research is changing the frequency of the data for the empirical analyses. To concentrate on high-frequency data might provide more realistic results as it captures all of the dynamics tick by tick between the spot and future markets particularly.

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