A CONTINUOUS-REVIEW INVENTORY MODEL WITH DISRUPTIONS AND REORDER POINT

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ABSTRACT

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We study a two-echelon continuous review inventory problem of a retailer and a supplier. It is an extension of economic order quantity model with lost sales and reorder point when both supplier and retailer are subject to random disruptions. It is assumed that supplier has two states which are available (ON) and unavailable (OFF), modeled with Markov chain. Whereas, if retailer is disrupted, all on-hand inventory is destroyed but afterwards retailer recovers immediately to serve the customers. All unsatisfied demand at retailer is assumed to be lost. In this study, the objective is to identify the optimal inventory policy for the retailer and investigate the importance of a non-zero reorder point for the retailer.

Utilizing Renewal Reward Theorem, expected total cost per unit time is derived. We analyze the sensitivity of the optimal values of expected cost per unit time, order-up-to level, and reorder level to different problem parameters. The problem parameters we investigated are unit holding, unit lost sales, fixed and variable ordering, supplier disruption and recovery rates, retailer recovery rate, and demand rate.

We use 1728 test cases in Sargut and Qi (2012) and another data set including 15360 test cases. In computational experiments, we also compare our solution with the optimal classical economic order quantity and we conclude that our model gives better or the same optimal average expected cost than EOQ. We conclude that adding a non-zero reorder point to the inventory policy is meaningful when supplier disruptions are more frequent than retailer disruptions. If the ratio of supplier disruption rate to recovery rate is greater than one, a positive reorder point can be optimal.

Keywords: Inventory control, disruption, economic order quantity, reorder point, lost sales.

ÖZ

KESİNTİLER VE YENİDEN SİPARİŞ NOKTASI İLE OLUŞTURULAN SÜREKLİ KONTROL KURALLI BİR ENVANTER PROBLEMİ

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Bu çalışmada, bir perakendecinin sürekli kontrol kurallı bir envanter problemini ele aldık. Bu problem, ekonomik sipariş miktarı probleminin üzerine kayıp satış ve yeniden sipariş verme noktasının eklenmiş halidir. Ayrıca, parekendeci ve toptancı rastlantısal kesintilere uğramaktadır. Toptancı, Markov zinciri esas alınarak, ulaşılabilir (açık) ve ulaşılamaz (kapalı) olarak iki farklı durumda modellenmiştir. Parekendeci, kesintiye uğraması halinde, elindeki tüm envanteri kaybetmektedir. Ancak parekendeci kesintiden sonra, hemen uygun duruma geçip, müşterilere servis verebilmektedir. Karşılanamayan siparişlerin tümü kaybolmaktadır. Bu çalışmada amacımız, parekendeci için optimal sipariş politikasını belirlemek ve pozitif yeniden sipariş verme noktasının önemini araştırmaktır.
Yenileme teorisi kullanılarak, birim süre için beklenen ortalama toplam maliyet elde edilmektedir. Bu maliyetle birlikte, sipariş verme noktası ve yeniden sipariş verme noktası da çeşitli parametrelerle analiz edilmektedir. İncelenen problem parametreleri, envanter maliyeti, birim kayıp satış maliyeti, sabit ve değişken sipariş verme maliyetleri, tedarikçi ve satıcı kesinti oranları, satıcı toparlanma süresi oranı ve talep oranıdır. Bu analizlerde, Sargut ve Qi (2012)'de verilen 1728 test kümesi ve 15360 örnek içeren başka bir test kümesi kullanılmıştır. Ayrıca işlemsel deneylerde, kendi modelimizle kayıp satışa izin veren ekonomik sipariş miktarı modelini karşılaştırdığımızda, çoğu örnekte kendi modelimizin, birim süre için beklenen toplam maliyetin ekonomik sipariş miktarı modeline göre daha düşük olduğunu gözlemledik. Sonuç olarak, sipariş politikasına pozitif yeniden sipariş verme noktası eklemek, toptancıda olan kesintilerin, perakendecide olan kesintiye göre daha sıklıkla olduğu durumlarda anlamlıdır. Eğer toptancının kesinti ve toparlanma süresi oranı bir den büyükse pozitif yeniden sipariş verme noktası optimaldir.

Anahtar Kelimeler:Envanter kontrol, kesinti, ekonomik sipariş miktarı, yeniden sipariş verme noktası, kayıp satış

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Chapter 1

Introduction

One of the oldest methods known in the area of inventory management is *classical Economic Order Quantity* (EOQ), which aims to find optimal order quantity that minimizes total ordering and holding costs. Assumption is that demand is deterministic and constant. There is no lead time, order is received immediately. It is well known that, the optimal order quantity (Q*) depends on demand rate, fixed cost for ordering and holding cost, which are λ, K and h, respectively. The optimal order quantity is given by

$$Q^* = \sqrt{\frac{2K\lambda}{h}} \tag{1.0.1}$$

In this model, there is only one party who is ordering from the supplier, and the party orders according to *zero inventory ordering policy* (ZIO), i.e., order is placed when inventory level reaches to zero. Inventory level by time for the Classical EOQ inventory model is shown in Figure 1.1. This triangle repeats itself up to infinity.

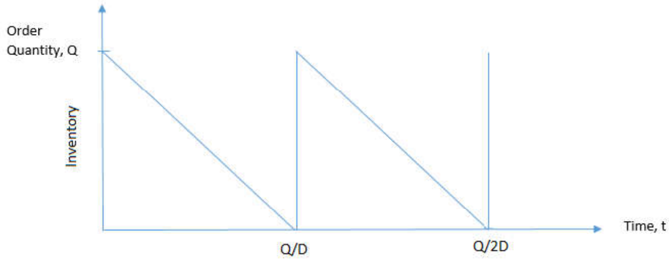

Figure 1.1: Classical EOQ Inventory Model

There are several different extensions of EOQ in the literature. EOQ can be extended by adding backorders (i.e, customer's orders cannot be filled immediately and they have to wait some time until inventory level is positive). Other extension is *Economic Production Quantity* (EPQ) where production company sends their goods while production is continuing and optimal production quantity is given by the following formula.

$$Q^* = \sqrt{\frac{2K\lambda}{h(1-\frac{\lambda}{P})}} \tag{1.0.2}$$

P represents the finite production rate. If the production rate is close to ∞, EPQ is equivalent to EOQ. Inventory level by time for EPQ model is shown at Figure 1.2 and T_p and T_d represents time spent at the production phase and sales and distribution phase, respectively.

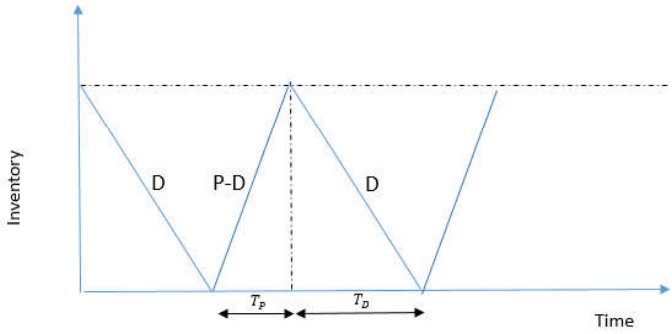

Figure 1.2: EPQ Model

Due to the assumptions of EOQ model, it is very unlikely to be used in practice. Therefore, to be more realistic, uncertainties should be taken into account such as both supplier and retailer are due to random disruptions. Supply disruptions are described as some events occur in random times where the supply chain will be affected as a result of these disruptions. The events may be natural disasters, terrorist attacks, machine breakdowns or other unpredictable situations which influence the supply chain in anyway. Both supplier and retailer can be influenced from the disruption at the same time or only one of them affected from the disruption. In two-echelon inventory model with disruptions and with ZIO policy, two cases can occur during the ordering process; first retailer gives an order when inventory level drops to zero and supplier can provide the demand, goods are sent immediately, in the second case, retailer again places an order when inventory level drops to zero but in this case the supplier is not available due to a disruption. Hence, in the second case, it leads to lost sales since goods cannot reach to the retailer from the supplier until supplier is recovered from its disruption. At this point, adding reorder point makes sense due to the supplier uncertainty and it gives an opportunity to decrease lost sales.

A disaster, may adversely affect the supply chains all around the world. For example, an Icelandic volcanic eruption realized in March 2010 which was one of the biggest disasters that affected air traffic ever, caused cancellation of flights during seven days due to the ash cloud throughout Europe. Some car manufacturers including BMW, Nissan and Audi had to stop their production because of the disruption at air freight (Graf and John, 2016). In February 2014, Japan was exposed to a catastrophic snowfall. The disaster that occurred in Japan, also influenced many other countries which were working in collaboration with Japan. Many manufacturers had to give a break to the production because of the electricity and transportation problems. Many flights carrying either passenger or freight were canceled. Due to the heavy snow weight on the roofs, the buildings were damaged. As a result, inventory, goods and manufacturer materials had to be destroyed (Mark and Faust, 2015). In such disasters, both retailer and supplier are subject to various interruptions. One of the disruptions that can be seen more frequently is power outages may be caused to destroy some perishable items. Moreover, accidents on the roads can also caused to supply chain disruption. To handle these disruptions, mitigation strategies can be used. The strategies are applied either before or after the disaster.

According to Snyder et al.(2015), mitigation strategies have four main categories. To mitigate the disruptions, they classify the strategies as inventory, flexibility and sourcing, facility location, and interaction with external stakeholders. First way to mitigate the supply disruptions is inventory control. To find the optimal order quantity and optimal reorder point help to reduce disruption effect. Second strategy is flexibility and sourcing. There are two types of sourcing: routine sourcing and contingent rerouting. In routine sourcing, retailer or producer supply their goods from more than one supplier at the same time regardless of there is a disruption or not. On the other hand in contingent rerouting, other suppliers which do not subject to disruption when the primary supplier is at OFF state, can provide the goods. Thus, under favor of to collaborate with multiple suppliers, disruptions are mitigated more easily. Tomlin(2009) states that "Demand switching is a tactic in which the firm provides incentives for a customer to purchase a different product if her preferred product is unavailable." Motivate the customers to take another product which is available on the inventory can be also used as a mitigation strategy. If the main product that customer wants to buy is not available at this time, this strategy is used. The other strategy is about the selection of facility location. It concerns about how to choose a facility location by taking into consideration the supply disruptions. Last mitigation strategy takes into account the relationship between the firm and external stakeholders of the firm.

In this study, we use inventory control to mitigate the supplier and retailer disruptions. We modeled supplier as a two-state continuous time Markov chain. On the other hand, retailer is assumed always available, if it is disrupted recovered immediately but all on-hand inventory is gone. We apply a modified (Q,R) policy. We need ordering quantity Q due to the setup cost. Besides, adding reorder non-negative point is reasonable because of supplier uncertainty. When supplier is not available a time which retailer should place an order, lead time occurs. Hence, reorder point helps to reduce expected average cost. Demand is assumed deterministic and constant. Planned backorders are not allowed. When inventory level is zero, all demand is assumed to be lost.

The rest of this thesis organized as follows. In Chapter 2, we present the related literature. Our model is demonstrated in Chapter 3. In Chapter 4, we show the solution method for the cost function. In Chapter 5, interpretations and results of the numerical experiments are presented. Lastly, we conclude the study and suggest future works in Chapter 6.

Chapter 2

Literature Review

We investigate the literature on inventory models with disruptions under two major categories; deterministic and stochastic demand.

2.1 Inventory Models with Disruptions under Deterministic Demand

Economic order quantity with disruptions (EOQD) is first introduced by Parlar and Berkin (1991). In this model, lead time is assumed as zero and shortages are not allowed. They construct an average cost function per unit time, using Reward Renewal Theorem. They assume ZIO and find the optimal order quantity. Bielecki and Kumar (1988) investigate unreliable manufacturing systems and point out that, ZIO is optimal in a range of parameters. Berk and Arreola-Risa (1994) correct Parlar and Berkin's cost function. Snyder(2014) show the convexity of the cost function and write the approximation model as closed form with considering ZIO assumption. Heimann and Waage (2007), construct a model with relaxing the ZIO assumption. They propose a closed-form approximate solution in their study. Parlar and Perry (1995) relax the ZIO assumption of Parlar and Berkin's model and add the reorder point as a decision variable. As it seen in Figure 2.1, Q unit of order is placed from the supplier when inventory level is at reorder point r or below r. Hence, inventory level increases to $R = Q + r$. If the retailer is out of inventory and cannot order due to the disruption, all unmet demand is backordered. They also define a decision variable T addition to ordering quantity Q and reorder point r, which decides how long retailer should wait to order when supplier is OFF.

Emer (2012), proposes an opportunity to give an order just before a supplier disruption. In this model only supplier is disrupted, so they try to find importance of this opportunity with comparing regular order-up to level. They conclude that, if the backorder cost is smaller than fixed and holding costs, the model where there is an opportunity to place an order does not place an order before the disruption.

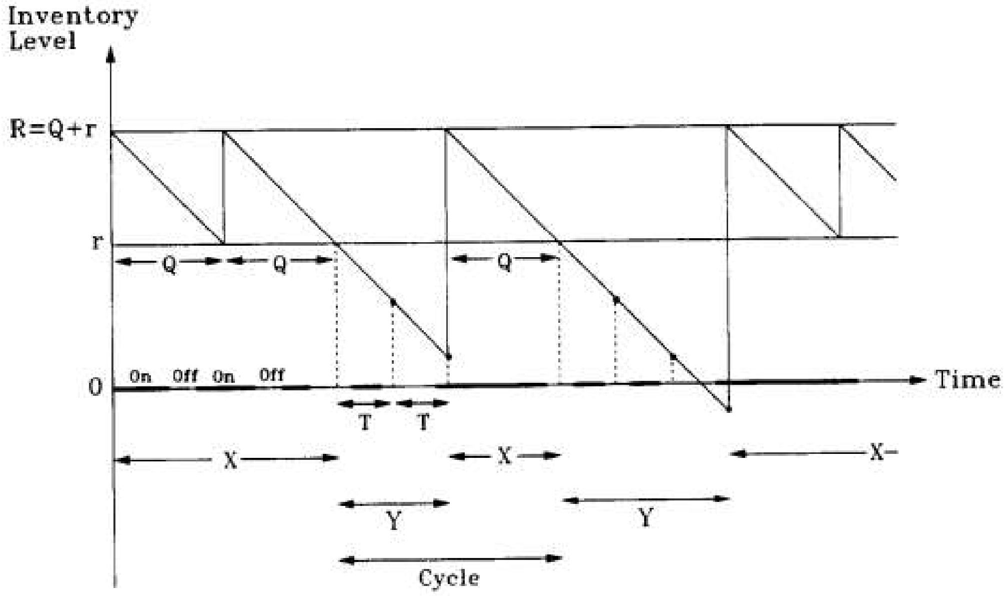

Figure 2.1: Inventory Policy of Parlar and Perry (1995)

Ramasesh et al.(1991) and Gürler and Parlar (1997) investigate the inventory model with two suppliers. Moreover, single-supplier, two-supplier and multiple-supplier versions are studied separately by Parlar and Perry (1996). They point out that if the number of suppliers is large enough, EOQD approximates the classical EOQ model. Furthermore, they conclude that, reorder point r can take negative values in addition to non-negative values when the backorder costs are finite (and small). Gürler and Parlar (1997) and Parlar and Perry (1996) determine the supplier costs as identical for all suppliers and assume suppliers' capacities as infinite. Different from them, Tomlin (2006) and Qi (2013) consider a problem where retailer can order from two suppliers, one of them is cheap but unreliable and the other one is reliable but expensive. Retailer can choose either the first supplier or the second supplier while trying to minimize total expected average cost per unit as seen in Figure 2.2. Besides, Tomlin (2006) assumes the suppliers capacities as finite. Chen et al. (2012) consider a periodic-review inventory model with two supplier where the main supplier is subject to random disruptions and the backup supplier is more costly and has an infinite capacity.

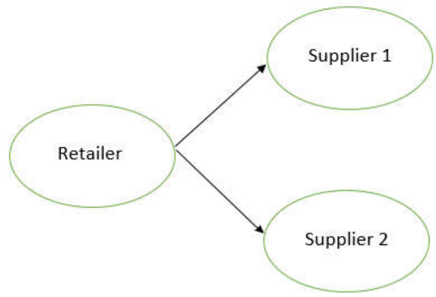

Figure 2.2: Inventory Model with two Suppliers

Unlike described works above, our model is similar to Qi et al.(2009) who examine two echelon supply disruptions (i.e. both supplier and retailer disruption). They assume that when retailer is disrupted, inventory level drops to zero. To find the expected cycle length at the retailer they divide it into two cases. At the first case, retailer is not disrupted until inventory level drops to zero and for the second case, retailer is subject to a disruption before inventory level drops to zero. They explain it basically in Figure 2.3. In the first and second cycles, first case is occurred. At the point A, when inventory level is zero supplier is available, on the other hand at point B, supplier is disrupted before inventory level reaches zero and it is still not available or it is at recovery period. Thus, the order cannot reach until the supplier is available. However, at the third and fourth cycles, retailer disrupted during Q/D. Therefore, all on-hand inventory is destroyed. Besides, at the forth cycle both retailer and supplier is not available at the same time when inventory level is zero. So, retailer has to wait until both itself and supplier are available for raise the inventory up to Q.

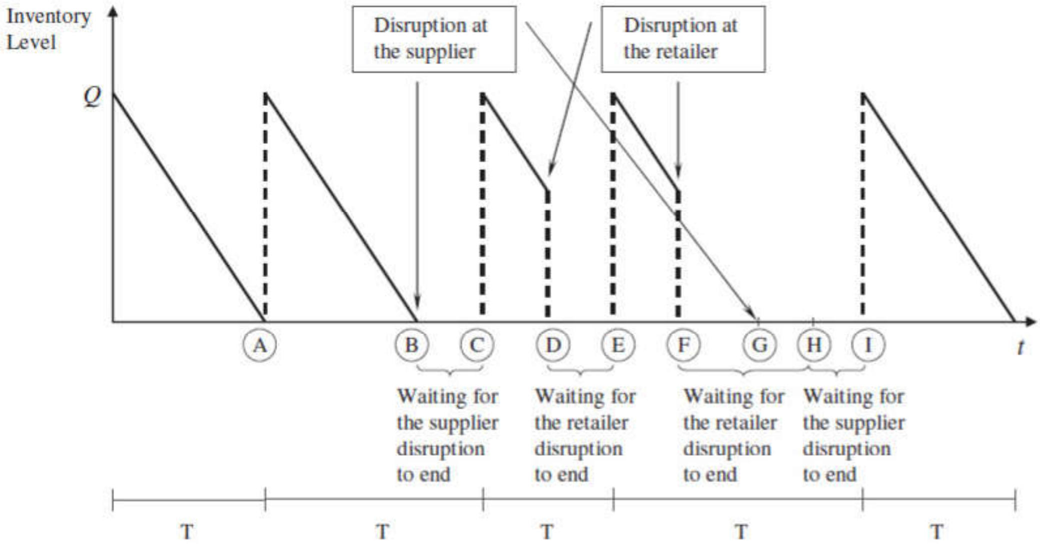

Figure 2.3: Inventory Policy at the Retailer with Random Cycle Length T of Qi et al.(2009)

They develop an expected inventory cost function and show that it is quasi-convex. As our model, when a retailer disruption occurs all on-hand inventory will be destroyed. However, they consider about the recovery time of the retailer which follows exponential distribution with rate β. Besides, they apply ZIO policy. Our problem extends their model by introducing reorder point as a decision variable in addition to Q.

Sargut and Qi (2012), also study an inventory model which includes both supplier and retailer disruptions. The main difference between the models is that when retailer faces a disruption, it becomes unavailable but keeps inventory on hand and concludes with lost sale only, whereas, in ours all on-hand inventory is destroyed.

At Figure 2.4, in the first retailer cycle, both retailer and supplier are disrupted. At point A, retailer is disrupted. Later on supplier is disrupted at point B and cannot recover when inventory level hits zero. Hence, retailer has to wait the supplier to be available for ordering. However, at point D, retailer disrupted again and at this time retailer cannot give an order until the point E where it becomes available again.

They obtained expected average cost per unit time. They analyzed the optimal order quantity and its sensitivity to problem parameters.

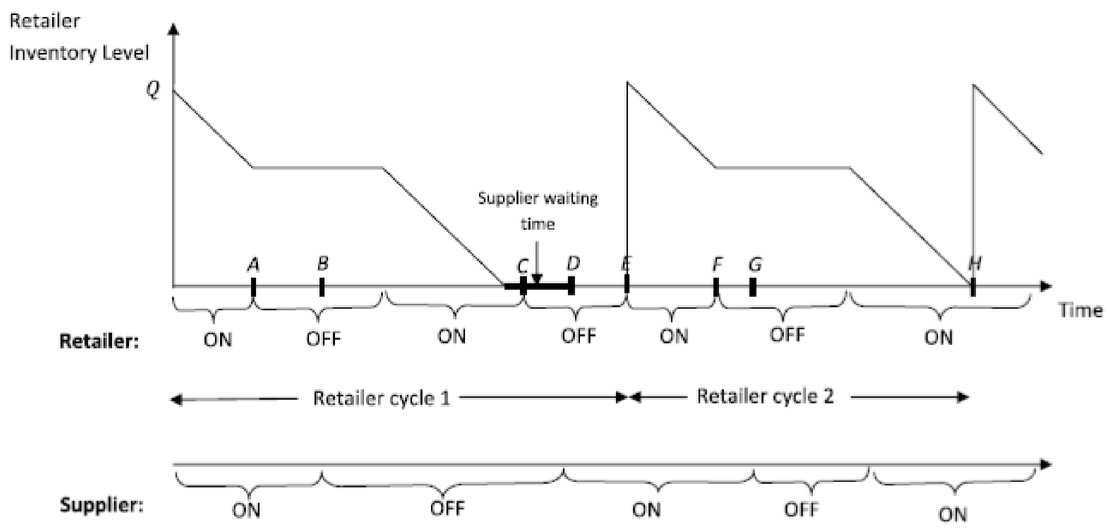

Figure 2.4: Inventory Policy at the Retailer with Random Cycle Length T of Sargut and Qi (2012)

2.2 Models with Disruption Under Stochastic Demand

In the literature, stochastic demand is studied as well. Bar-Lev et al. (1993) consider Brownian inventory system where supplier has exponential ON-OFF periods. They consider stochastic demand, which is different from Parlar and Berkin (1991). The cost function with stochastic demand and constant lead time is formulated by Gupta (1996). Supplier has ON-OFF periods and distributed exponentially.

Parlar (1997) studies continuous review inventory model with one supplier and one retailer with random demand. (Q,r) policy is used when inventory level drops to r, Q units are ordered to increase the inventory level to Q+r. Supplier is subject to random disruptions where ON periods follow k-state Erlang distribution whereas OFF periods are distributed with general distribution. Extended inventory model of Gupta (1996) is developed by Mohebbi (2003) where lead time follows Erlang (Ek) distribution and demand follows compound Poisson process. Mohebbi (2004) considers an inventory model with supplier disruption similar to Mohebbi (2003) and Gupta (1996) but he assumes lead time distribution is hyper-exponential. Moreover, supplier's ON periods follow general and OFF periods follow hyper-exponential distribution.

Schmitt et al. (2010) study on multi-period model with stochastic demand and supply with disruptions. For the base-stock level, a closed-form approximation is proposed.

In Table 2.1, we summarize the related literature, table includes seven different characteristics. As we mentioned before, retailer, supplier or both of them can be disrupted. s represents only supplier disruption and r represents only retailer disruption. *Both* means both of them can be disrupted. l and b indicates backordering and lost sale costs, respectively. When we show the features of demand, d, $s\&s$ and $c\&d$ represents deterministic; stationary and stochastic; and constant and deterministic, respectively. m represents multiple supplier. Lastly, C indicates supplier is capacitated and if the problem includes reorder point, it is also shown in Table 2.1.

	Disruption	Backordering/ lost sale	c	Demand	Lead time	# of supplier	Reorder Point
Parlar & Berkin(1991)	s	l		d		1	
Berk & Arreola-Risa (1994)	s	l		d		1	
Parlar & Perry (1995)	s	b		d		1	✓
Parlar & Perry (1996)	s	b		d		1, 2 ,m	✓
Tomlin (2006)	s	b	✓	s & s	✓	2	
Qi (2013)	s	Both		c & d		2	
Sargut & Qi (2012)	Both	l		c & d		1	
Snyder (2014)	s	l		c & d		1	
Qi et al. (2009)	Both	Both		c & d		1	
Our Study	Both	l		c & d		1	✓

Table 2.1: Summarize of the Related Literature

Chapter 3

Model

In this study, we consider a continuous review inventory policy when we have random disruptions at both retailer and the supplier. Retailer sells the products or consumer goods to customers and supply these from the supplier. Retailer orders the goods from the supplier according to its inventory level. Demand is constant and deterministic, unsatisfied demand is lost. Lead time consists of two parts; order lead time and transportation lead time. We assume transportation lead time is zero. Order lead time is defined as the time retailer waits for the supplier to be available. Order lead time is a random variable which depends on the distribution of supplier recovery time. The transportation lead time from supplier to retailer is assumed to be zero. We try to find an average total cost per unit time function for the retailer using renewal reward theorem referring to Ross(1970). We consider fixed and variable ordering costs, holding cost and lost sales cost. We assume transportation lead time is zero.

In our policy, Q and R are both decision variable where Q is order up to level and R is the reorder point. In classical (Q, R) policy, when inventory level drops to the reorder point, Q units of order is placed. When order arrives, then inventory position increases to $Q + R$. In our case we use modified (Q, R) policy where inventory level can be up to maximum level of Q. When modified (Q, R) policy is used, after the receipt of the order inventory level reaches to Q. We depict six different cases based on the states of the supplier and retailer, when the order is given.

Modified (Q,R) policy: When inventory level reaches to a level less than or equal to the reorder point, and retailer orders up to Q as soon as supplier is available and it arrives immediately. When we have excess demand we handle it by losing the sales. This cost corresponds to the revenue lost and loss of goodwill.

Qi et al.(2009) define four state continuous-time Markov which are 11, 01, 10, 00 and it can be seen in Figure 3.1. Since we assume $\beta = \infty$, we cannot model retailer and supplier states as four-state continuous Markov chain.

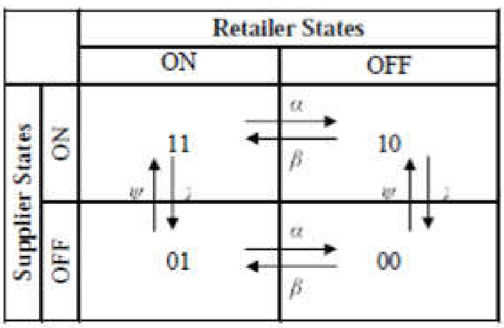

Figure 3.1: A four-state continuous-time Markov chain for the disruption and recovery states at the retailer and the supplier of Qi et al. (2009)

In our problem, supplier is modeled as a two-state continuous-time Markov chain. There are two states for the supplier: ON and OFF. If the supplier disrupted, the state of the supplier turns from ON to OFF. When the supplier is ON, the time until next disruption modeled distributed with exponential distribution with rate λ and when the supplier is disrupted the time until recovery is distributed with exponential distribution with rate ψ. We represent ON state by 0 and OFF state by 1 as we can see at Figure 3.2. We assume that supplier is at ON state at time zero, which is the beginning of the cycle.

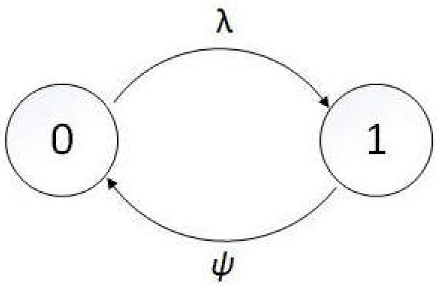

Figure 3.2: Supplier State Transition

Likewise, retailer is modeled as a two-state continuous-time Markov chain. When the retailer is ON, the time until next disruption modeled distributed with exponential distribution with rate α and when the retailer is disrupted the time until recovery is distributed with exponential distribution with rate β. In our model, we assume $\beta = \infty$. It means that retailer does not go through a recovery period, in other words, retailer never turns into OFF period. After retailer disruption, it recovers immediately. If retailer is disrupted, all ON-hand inventory is destroyed and inventory level drops to zero. When inventory level is less than or equal to R, retailer needs to order and the order quantity is equal to Q. The only effect of retailer's disruption is that inventory is completely destroyed. Retailer can place an order just after its disruption if the supplier is ON at that time.

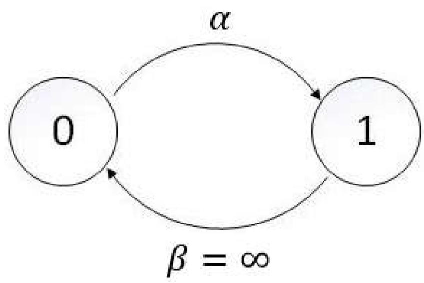

Figure 3.3: Retailer State Transition

A retailer cycle is defined as the time between two consecutive order arrivals. In Figure 3.4, four order cycles are depicted. In first cycle, both retailer and supplier are disrupted in different times.T_i: is the time for ith cycle. T_1 ends at point c. At point a, supplier is disrupted. After, at point b, retailer is disrupted and all inventory is destroyed. At point b, retailer could not give an order and waits until point c, where supplier is recovered. The order arrives instantaneously. In the second cycle, only supplier is disrupted (point d) and cannot recover until the point e. As an ordering policy, retailer should give an order when inventory level drops to reorder point, however in this cycle retailer have to wait until the supplier recovery and can give an order at point e. In the third cycle only retailer is disrupted at point g. Since the supplier is available, Q units of order is placed immediately (point g). In the forth cycle, neither the retailer nor the supplier is disrupted, retailer gives Q-R units of order when inventory level drops to reorder point at point f and order placed by the supplier instantly.

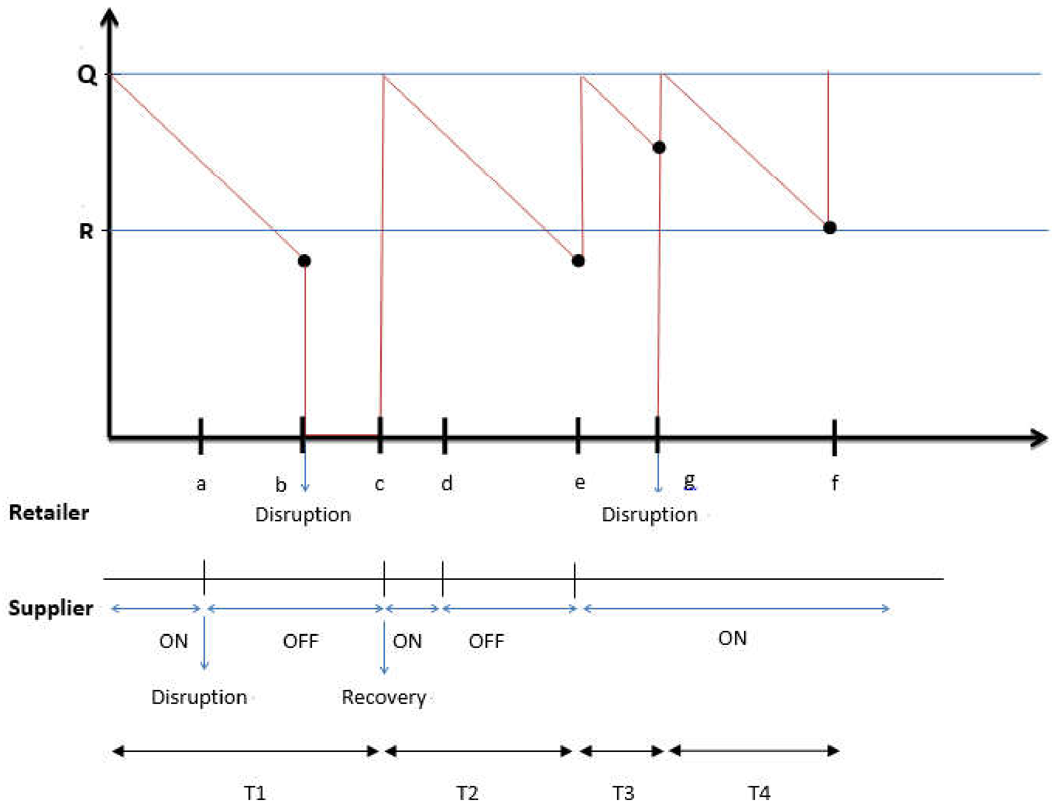

Figure 3.4: Retailer Cycle

We need the transition probabilities $P_{01}(t)$ and $P_{00}(t)$ which are given in Equations 3.0.1 and 3.0.2, respectively. Transition probabilities from a state to another state of a continuous-time Markov chain is introduced by Ross (1996). $P_{01}(t)$ is probability that supplier is OFF at time t, $P_{00}(t)$ is probability that supplier is ON at time t, assuming that supplier is on at time 0.

$$P_{01}(t) = \frac{\lambda}{\lambda + \psi}(1 - e^{-(\lambda+\psi)t}) \tag{3.0.1}$$

$$P_{00}(t) = 1 - (\frac{\lambda}{\lambda + \psi}(1 - e^{-(\lambda+\psi)t})) \tag{3.0.2}$$

Now, we will list the parameters of the model:

- λ: Supplier disruption rate, the expected number of supplier disruptions per unit time, time until disruption is distributed exponentially with rate λ
- ψ: Supplier recovery rate, the expected number of supplier recovery per unit time, time until recovery is distributed exponentially with rate ψ
- α: Retailer disruption rate, the expected number of retailer disruptions per unit time, time until disruption is distributed exponentially with rate α
- D: Demand rate per unit time
- F: Fixed cost per order
- a: Unit variable ordering cost
- h: Unit holding cost per unit per time
- s: Lost sales cost per unit

The Decision Variables of the modified (Q,R) policy are

- R: Reorder point
- Q: Order up to level

Retailer inventory will follow one of the six possible structures of retailer cycle. The inventory level by time is given in Figure 3.5.

Case A: Retailer disrupted before inventory level reaches reorder point and supplier is OFF at that time. Because of retailer disruption, inventory level hits zero and no order can be placed until supplier turns into ON state.

Case B: When inventory level reaches reorder point the supplier is OFF. Therefore, retailer cannot place an order. In Case B, retailer is disrupted before the supplier recovery so inventory level hits zero. No order can be placed until supplier turns into ON state.

Case C: Retailer disrupted before inventory level reaches reorder point and supplier is ON at that time. After disruption, inventory level reaches zero and retailer can place an order and level becomes Q immediately.

Case D: Neither the retailer nor the supplier will be disrupted before inventory level reaches reorder point. When the inventory level reaches the reorder point, an order is placed to increase the inventory level up to Q.

Case E: When inventory level reaches reorder point but the supplier is OFF at that time, retailer cannot place an order just in Case B. But in Case E, supplier recovers before retailer disruption. After the supplier recovery, retailer can placed an order to raise its inventory level up to Q.

Case F: Retailer is not disrupted before inventory level hits zero. When inventory level reaches reorder point, supplier is at OFF state and cannot turns into ON state until inventory level hits zero. After the supplier recovery, retailer can placed an order to raise inventory level up to Q.

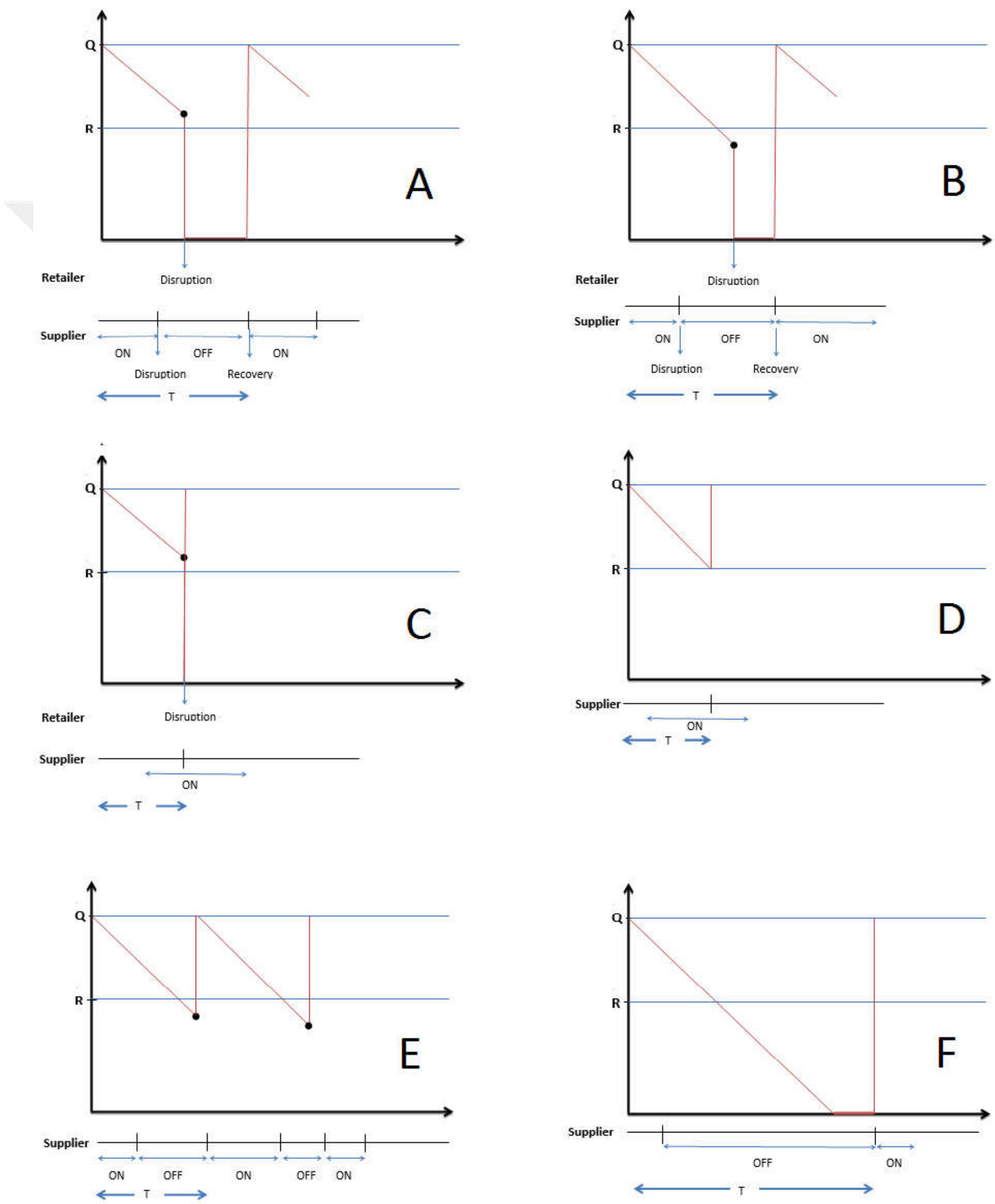

Figure 3.5: Cases

We also summarize the cases in Figure 3.6 verbally.

Retailer distrupted before R	Supplier is ON at time t when retailer distrupted C	Supplier is OFF at time t where retailer distrupted A	
Retailer is not disrupted in the cycle Supplier is OFF when inventory level = R		Recover after 0 F	
Retailer is disrupted after R Inventory level = R before it becomes 0	Supplier is ON when disrupted D	Supplier is OFF at R ON<dist. E	Supplier is OFF at R ON>dist. B

Figure 3.6: Cases

To apply reward renewal theorem, we calculate expected total cost per order cycle as $E(C)$ and expected cycle length as $E(T)$ and derive expected cost function per unit time. We need to consider all six cases while computing $E(T)$ and $E(C)$. The calculations for finding the functions are done with Maple.

3.1 Expected Retailer Cycle Length

To calculate expected retailer cycle length E[T], we define the following random variables:

- T_1 is the time of the first disruption for the retailer from the beginning of the cycle as it is seen at Figure 3.7. We assume that the retailer is at state ON at the beginning of the cycle, at time 0. The probability density function is denoted by $f_{T_1}(t) = \alpha e^{-\alpha t}$

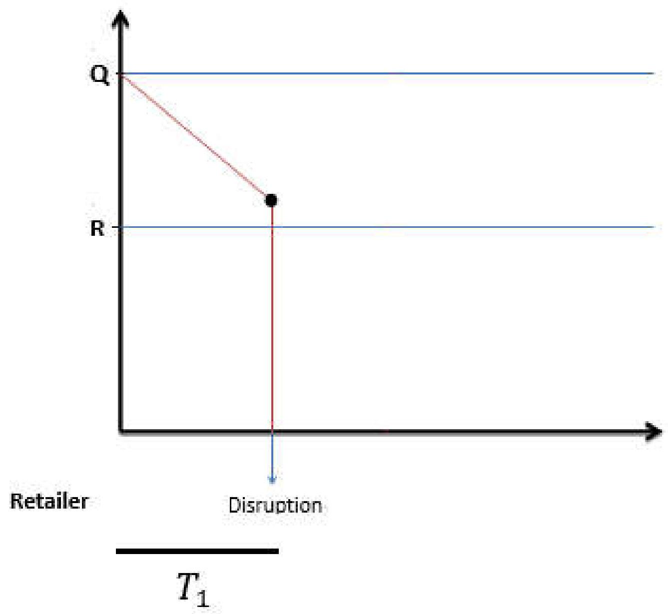

Figure 3.7: T_1

- T_2 is the length of the time, when inventory level is between Q and R in an order cycle. In the Figure 3.8, we depict two different retailer cycles where the retailer is disrupted before R in the first cycle and retailer is not disrupted in the second cycle.

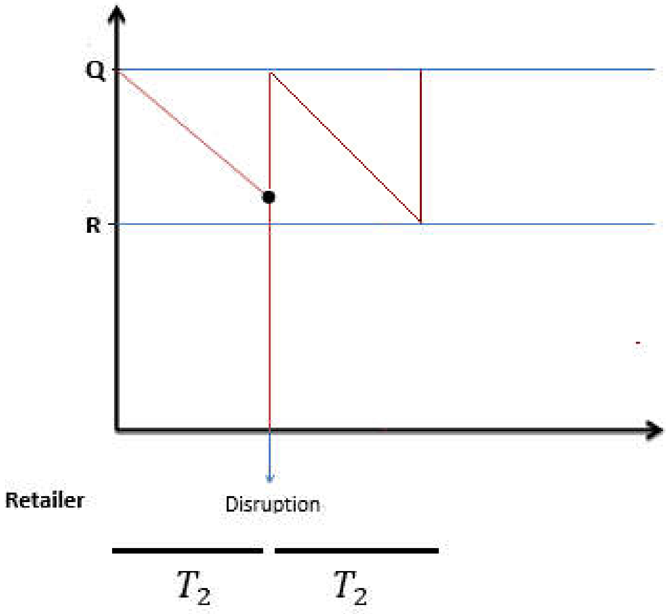

Figure 3.8: T_2

- T_3 is the time, inventory level is less than R, or the waiting time between the first time inventory level is less than or equal to R and the order arrival. We can see T_3 in Figure 3.9. The probability density function is denoted by $f_{T_3}(t) = \psi e^{-\psi t}$

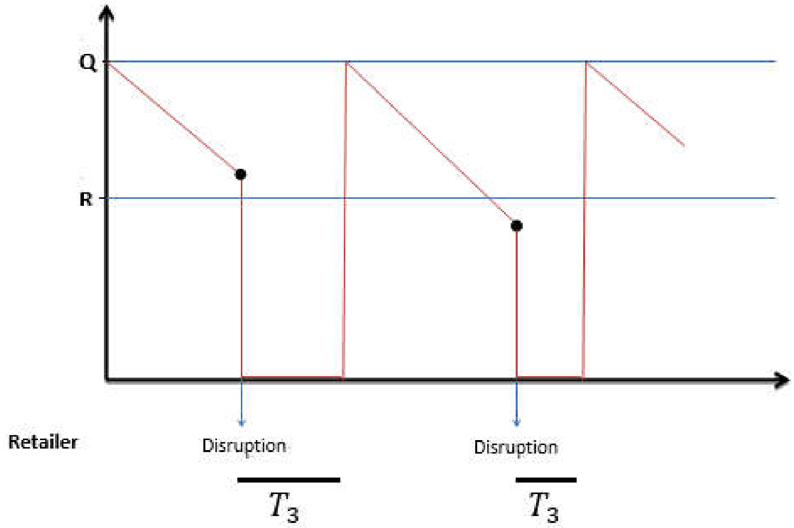

Figure 3.9: T_3

Expected cycle time is equal to $E(T_2 + T_3)$.

We condition the random variables on the time of retailer disruption. While finding T_2, we first find the expected time if a retailer disruption occurs before inventory level reaches reorder point. On the contrary, if there is no disruption at the retailer, the T_2 length will be $\frac{Q-R}{D}$. $E[T_3]$ is calculated to determine waiting time of supplier. $\frac{1}{\psi}$ is the expected waiting time. We multiply this time with its probability. There are two types of waiting times for supplier. First, when inventory level is between Q and R and if the retailer is disrupted when supplier is not available, there is a waiting time for supplier. Second, if the retailer is disrupted when inventory level is less than the reorder point and supplier is not available since $(Q - R)/D$, there is again an expected waiting time for supplier recovery. These two probabilities are used to calculate waiting time of supplier.

$$
\begin{aligned}
E[T] =& E[T_2] + E[T_3] \\
E[T_2] =& \int_0^\infty E[T_2|T_1 = t] f_{T_1}(t) dt \\
=& \int_0^{\frac{Q-R}{D}} t\alpha e^{-\alpha t} dt + \frac{Q-R}{D} \int_{\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt \\
E[T_3] =& \int_0^\infty E[T_3|T_1 = t] f_{T_1}(t) dt \\
=& \frac{1}{\psi} \left(\int_0^{\frac{Q-R}{D}} P_{01}(t) \alpha e^{-\alpha t} dt + P_{01}\left(\frac{Q-R}{D}\right) \int_{\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt \right)
\end{aligned}
$$

Now we can give our first theorem,

Expected retailer cycle length is:

Theorem 3.1.1

$$
E[T] = -\frac{\lambda}{(\alpha + \lambda + \psi)\,\psi} \left(\mathrm{e}^{-\frac{(Q-R)(\alpha+\lambda+\psi)}{D}} - 1 \right) - \frac{1}{\alpha} \left(\mathrm{e}^{-\frac{\alpha(Q-R)}{D}} - 1 \right)
$$

Proposition 3.1.1 *E[T] is an increasing and concave function of Q-R. It does not depend on Q and R, independently.*

Proof Let us define $Y = Q - R$ and α, λ, ψ and D are all positive constants,

$\frac{dE[T]}{d(Y)} = \frac{\lambda}{D\psi} \mathrm{e}^{-\frac{Y(\alpha+\lambda+\psi)}{D}} + \frac{1}{D} \mathrm{e}^{-\frac{\alpha Y}{D}}$

$\frac{d^2E[T]}{d^2(Y)} = -\frac{\lambda(\alpha+\lambda+\psi)}{D^2\psi} \mathrm{e}^{-\frac{Y(\alpha+\lambda+\psi)}{D}} - \frac{\alpha}{D^2} \mathrm{e}^{-\frac{\alpha Y}{D}}$

$\frac{dE[T]}{d(Y)} > 0$ and $\frac{d^2E[T]}{d^2(Y)} < 0$ for all Y where Y>0.

Hence, E[T] is a concave function of $Q - R$.

Proposition 3.1.2 *E[T] is a decreasing function of D.*

Proof α, λ, and ψ are all positive constants,

$$\frac{dE[T]}{d(D)} = -\frac{(Q-R)l}{D^2\psi}\mathrm{e}^{-\frac{(Q-R)(\alpha+\lambda+\psi)}{D}} - \frac{Q-R}{D^2}\mathrm{e}^{-\frac{\alpha(Q-R)}{D}} < 0$$

Since the slope grows when demand rate increases, time is spent faster as it also seen in Figure 3.10.

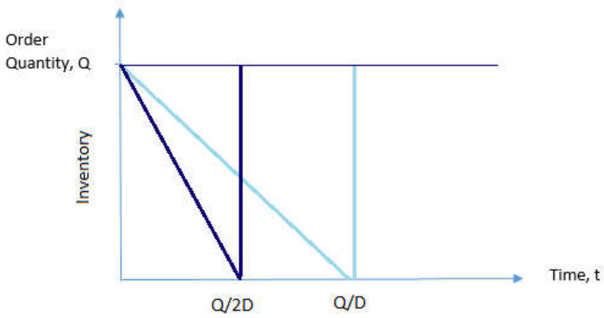

Figure 3.10: Effect of Demand on Retailer Cycle Time

As mentioned before, we compare our expected cycle length with Qi et al.(2009). Their expected retailer cycle length is:

$$E[T_Q] = \frac{\lambda w}{\lambda+\psi} \times (1 - \frac{\alpha\beta}{(\beta+\lambda+\psi)(\alpha+\lambda+\psi)})\left(1 - \mathrm{e}^{-\frac{(Q)(\alpha+\lambda+\psi)}{D}}\right) + (\frac{1}{\alpha} + \frac{1}{\beta})\left(1 - \mathrm{e}^{-\frac{\alpha(Q)}{D}}\right)$$

where w is the waiting time and equals to

$w = \frac{1}{\psi}(1 + \frac{\alpha(\lambda+\psi)}{(\alpha+\beta+\lambda+\psi)\beta})$

As we know, β is the retailer recovery rate. Since the retailer is recovered immediately in our model, we take β as infinity to compare two models. Also, they have no reorder point, thus, we accept reorder point as zero in our expected cycle length. When we set $\beta = \infty$ and reorder point to zero in our model, we see that these two functions are the same.

$\lim_{\beta\to\infty} E[T_Q] = \lim_{R\to 0} E[T]$

3.2 Expected Cost of a Retailer Cycle

The cost function consists of ordering cost, holding cost and shortage cost.

- Ordering cost is divided in two parts which are fixed and variable cost. Fixed cost does not depend the amount of goods ordered whereas, variable cost is incurred for every unit ordered. Ordering cost depends on the size of the order. Since the inventory is destroyed if retailer is disrupted in a retailer cycle, in this case, Q units of goods should be ordered. Otherwise, the amount of order is different.

- In any time, if there is any item in the inventory, holding cost should be calculated. The areas are calculated as expected holding cost.

- If retailer is out of inventory and supplier is not available for ordering, shortage cost is occurred.

We define new variables for calculating the total expected cost:

- C_2 is the cost during T_2
- C_3 is the cost during T_3
- t is the disruption time of retailer
- x is the recovery time of the supplier

$$
\begin{aligned}
E[C] =& E[C_2] + E[C_3] + F \\
E[C_2] =& \int_0^\infty E[C_2|T_1 = t] f_{T_1}(t) dt \\
=& h \int_0^{\frac{Q-R}{D}} (Qt - Dt^2/2)\alpha e^{-\alpha t} dt + h \frac{Q^2 - R^2}{2D} \int_{\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt + aQ \int_0^{\frac{Q-R}{D}} \alpha e^{-\alpha t} dt + \\
& P_{00}(\frac{Q-R}{D}) * a(Q-R) \int_{\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt
\end{aligned}
$$

We can verbally state $E[C_2]$ as below:

$E[C_2]$ = holding cost if retailer is disrupted when the inventory level is between Q and R+

holding cost if retailer is disrupted when the inventory level is less than R+

variable ordering cost if retailer is disrupted when the inventory level is between Q and R+

variable ordering cost if retailer is disrupted when the inventory level is less than R

$$E[C_3] = \int_0^\infty E[C_3|T_1 = t] f_{T_1}(t)dt$$

$$\begin{aligned}
E[C_3] =& P_{01}(\frac{Q-R}{D}) * h \int_{\frac{Q-R}{D}}^{\frac{Q}{D}} \frac{R^2 - (Q-Dt)^2}{2} \left(\int_{t-\frac{Q-R}{D}}^\infty \psi e^{-\psi x} dx \right) \alpha e^{-\alpha t} dt + \\
& P_{01}(\frac{Q-R}{D}) * h \int_0^{\frac{R}{D}} (Rx - Dx^2/2) \left(\int_{x+\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt \right) \psi e^{-\psi x} dx + \\
& P_{01}(\frac{Q-R}{D}) * sD \int_{\frac{Q-R}{D}}^{\frac{Q}{D}} \left(\int_{t-\frac{Q-R}{D}}^\infty (x - t + \frac{Q-R}{D}) \psi e^{-\psi x} dx \right) \alpha e^{-\alpha t} dt + \\
& \frac{sD}{\psi} \int_0^{\frac{Q-R}{D}} P_{01}(t) \alpha e^{-\alpha t} dt + \\
& sDP_{01}(\frac{Q-R}{D}) e^{-\alpha Q/D} * (1/\psi - \int_0^{R/D} x\psi e^{-\psi x} dx) + \\
& P_{01}(\frac{Q-R}{D}) * aQ \int_{\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt \int_{R/D}^\infty \psi e^{-\psi} dx \\
& P_{01}(\frac{Q-R}{D}) * a \int_0^{\frac{R}{D}} (Q - R + Dx) \left(\int_{x+\frac{Q-R}{D}}^\infty \alpha e^{-\alpha t} dt \right) \psi e^{-\psi x} dx
\end{aligned}$$

The second integral in the third term of C_3 is equal to

$$\begin{aligned}
&\int_{t-\frac{Q-R}{D}}^\infty (x - t + \frac{Q-R}{D}) \psi e^{-\psi x} dx = \\
&\int_{t-\frac{Q-R}{D}}^\infty x\psi e^{-\psi x} dx + \int_{t-\frac{Q-R}{D}}^\infty (\frac{Q-R}{D} - t) \psi e^{-\psi x} dx = \\
&1/\psi - \int_0^{t-\frac{Q-R}{D}} x\psi e^{-\psi x} dx + (\frac{Q-R}{D} - t) e^{-\psi (t - \frac{Q-R}{D})}
\end{aligned}$$

We can verbally state $E[C_3]$ as below:

$E[C_3]$ = holding cost if retailer disruption before supplier recovery+

holding cost if supplier recovery before retailer disruption+

lost sales cost if supplier recovers after disruption in the interval $(\frac{Q-R}{D}, Q/D)$+

lost sales cost if disruption happens before $\frac{Q-R}{D}$+

lost sales cost if retailer disruption happens after $\frac{Q}{D}$when supplier is OFF at $\frac{Q-R}{D}$+

variable ordering cost if retailer disruption before supplier recovery+

variable ordering cost if supplier recovery before retailer disruption

Then, expected cost of a retailer cycle is:

$$
\begin{aligned}
E[C] = {} & \frac{1}{\lambda+\psi}\left(\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)\left(-\frac{1}{2}\,\frac{h\alpha\left(-2De^{\frac{R(\alpha+\psi)}{D}}R\alpha-2De^{\frac{R(\alpha+\psi)}{D}}R\psi+2D^2e^{\frac{R(\alpha+\psi)}{D}}+R^2\alpha^2+2R^2\alpha\psi+R^2\psi^2-2D^2\right)e^{-\frac{Q\alpha+R\psi}{D}}}{\alpha^3+3\alpha^2\psi+3\alpha\psi^2+\psi^3}\right.\right. \\
& -\frac{1}{2}\,\frac{h\psi\left(-2De^{\frac{R(\alpha+\psi)}{D}}R\alpha-2De^{\frac{R(\alpha+\psi)}{D}}R\psi+2D^2e^{\frac{R(\alpha+\psi)}{D}}+R^2\alpha^2+2R^2\alpha\psi+R^2\psi^2-2D^2\right)e^{-\frac{Q\alpha+R\psi}{D}}}{D\left(\alpha^3+3\alpha^2\psi+3\alpha\psi^2+\psi^3\right)}+cQe^{-\frac{\alpha(Q-R)}{D}}e^{-\frac{\psi R}{D}} \\
& +\frac{c\psi\left(e^{\frac{R(\alpha+\psi)}{D}}Q\alpha+e^{\frac{R(\alpha+\psi)}{D}}Q\psi-e^{\frac{R(\alpha+\psi)}{D}}R\alpha-e^{\frac{R(\alpha+\psi)}{D}}R\psi+De^{\frac{R(\alpha+\psi)}{D}}-\alpha Q-Q\psi-D\right)e^{-\frac{Q\alpha+R\psi}{D}}}{\alpha^2+2\alpha\psi+\psi^2} \\
& \left.\left.+\frac{sD\alpha\left(e^{\frac{R(\alpha+\psi)}{D}}-1\right)e^{-\frac{Q\alpha+R\psi}{D}}}{(\alpha+\psi)\psi}\right)\right) \\
& +\frac{sD\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)e^{-\frac{\alpha Q}{D}}\left(\frac{1}{\psi}+\frac{e^{-\frac{\psi R}{D}}R\psi+De^{-\frac{\psi R}{D}}-D}{D\psi}\right)}{\lambda+\psi} \\
& +\frac{\lambda\left(e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\alpha+e^{\frac{\alpha(Q-R)}{D}}\lambda+e^{\frac{\alpha(Q-R)}{D}}\psi-\alpha-\lambda-\psi\right)e^{-\frac{\alpha(Q-R)}{D}}sD}{(\lambda+\psi)(\alpha+\lambda+\psi)\psi} \\
& -\frac{hQ\left(e^{-\frac{\alpha(Q-R)}{D}}Q\alpha-e^{-\frac{\alpha(Q-R)}{D}}R\alpha+De^{-\frac{\alpha(Q-R)}{D}}-D\right)}{\alpha D} \\
& +\frac{1}{2}\,\frac{h\left(e^{-\frac{\alpha(Q-R)}{D}}Q^2\alpha^2-2e^{-\frac{\alpha(Q-R)}{D}}QR\alpha^2+e^{-\frac{\alpha(Q-R)}{D}}R^2\alpha^2+2De^{-\frac{\alpha(Q-R)}{D}}Q\alpha-2De^{-\frac{\alpha(Q-R)}{D}}R\alpha+2D^2e^{-\frac{\alpha(Q-R)}{D}}-2D^2\right)}{\alpha^2 D} \\
& +\frac{1}{2}\,\frac{h\left(Q^2-R^2\right)e^{-\frac{\alpha(Q-R)}{D}}}{D}+cQ\left(1-e^{-\frac{\alpha(Q-R)}{D}}\right)+\left(1-\frac{\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)}{\lambda+\psi}\right)c(Q-R)e^{-\frac{\alpha(Q-R)}{D}}+F
\end{aligned}
$$

3.3 Expected Average Cost per Unit Time

Renewal reward theorem is used while calculating expected average cost per unit time $E[G(Q,R)]$. In this theorem, proportion of two different expected functions gives the average of a function. In our problem, $\frac{E[C]}{E[T]}$ gives expected average cost per unit time. Since a cycle repeats itself again when it passes another cycle, taking only one cycle is sufficient while calculating the expected function.

$$E[G(Q,R)] = \frac{E[C]}{E[T]}$$

$$
\begin{aligned}
E[G] = {} & \frac{1}{\lambda+\psi}\left(\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)\right. \\
& \left(-\frac{1}{2}\,\frac{h\alpha\left(-2De^{\frac{R(\alpha+\psi)}{D}}R\alpha-2De^{\frac{R(\alpha+\psi)}{D}}R\psi+2D^2e^{\frac{R(\alpha+\psi)}{D}}+R^2\alpha^2+2R^2\alpha\psi+R^2\psi^2-2D^2\right)e^{-\frac{Q\alpha+R\psi}{D}}}{\alpha^3+3\alpha^2\psi+3\alpha\psi^2+\psi^3}\right. \\
& -\frac{1}{2}\,\frac{h\psi\left(-2De^{\frac{R(\alpha+\psi)}{D}}R\alpha-2De^{\frac{R(\alpha+\psi)}{D}}R\psi+2D^2e^{\frac{R(\alpha+\psi)}{D}}+R^2\alpha^2+2R^2\alpha\psi+R^2\psi^2-2D^2\right)e^{-\frac{Q\alpha+R\psi}{D}}}{D\left(\alpha^3+3\alpha^2\psi+3\alpha\psi^2+\psi^3\right)} \\
& +cQe^{-\frac{\alpha(Q-R)}{D}}e^{-\frac{\psi R}{D}} \\
& +\frac{c\psi\left(e^{\frac{R(\alpha+\psi)}{D}}Q\alpha+e^{\frac{R(\alpha+\psi)}{D}}Q\psi-e^{\frac{R(\alpha+\psi)}{D}}R\alpha-e^{\frac{R(\alpha+\psi)}{D}}R\psi+De^{\frac{R(\alpha+\psi)}{D}}-Q\alpha-Q\psi-D\right)e^{-\frac{Q\alpha+R\psi}{D}}}{\alpha^2+2\alpha\psi+\psi^2} \\
& \left.\left.+\frac{sD\alpha\left(e^{\frac{R(\alpha+\psi)}{D}}-1\right)e^{-\frac{Q\alpha+R\psi}{D}}}{(\alpha+\psi)\psi}\right)\right) \\
& +\frac{sD\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)e^{-\frac{\alpha Q}{D}}\left(\frac{1}{\psi}+\frac{e^{-\frac{\psi R}{D}}R\psi+De^{-\frac{\psi R}{D}}-D}{D\psi}\right)}{\lambda+\psi} \\
& +\frac{\lambda\left(e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\alpha+e^{\frac{\alpha(Q-R)}{D}}\lambda+e^{\frac{\alpha(Q-R)}{D}}\psi-\alpha-\lambda-\psi\right)e^{-\frac{\alpha(Q-R)}{D}}sD}{(\lambda+\psi)(\alpha+\lambda+\psi)\psi} \\
& -\frac{hQ\left(e^{-\frac{\alpha(Q-R)}{D}}Q\alpha-e^{-\frac{\alpha(Q-R)}{D}}R\alpha+De^{-\frac{\alpha(Q-R)}{D}}-D\right)}{\alpha D} \\
& +\frac{1}{2}\,\frac{h\left(e^{-\frac{\alpha(Q-R)}{D}}Q^2\alpha^2-2e^{-\frac{\alpha(Q-R)}{D}}QR\alpha^2+e^{-\frac{\alpha(Q-R)}{D}}R^2\alpha^2+2De^{-\frac{\alpha(Q-R)}{D}}Q\alpha-2De^{-\frac{\alpha(Q-R)}{D}}R\alpha+2D^2e^{-\frac{\alpha(Q-R)}{D}}-2D^2\right)}{\alpha^2 D} \\
& +\frac{1}{2}\,\frac{h\left(Q^2-R^2\right)e^{-\frac{\alpha(Q-R)}{D}}}{D}+cQ\left(1-e^{-\frac{\alpha(Q-R)}{D}}\right)+\left(1-\frac{\lambda\left(1-e^{-\frac{(Q-R)(\lambda+\psi)}{D}}\right)}{\lambda+\psi}\right)c(Q-R)e^{-\frac{\alpha(Q-R)}{D}}
\end{aligned}
$$

3.4 EOQ with Lost Sales

EOQ with lost sales is an extension of classical EOQ model as shown in Figure 3.11. T_l is a decision variable represents the time with zero inventory, therefore the demand is lost during this period. When inventory level drops to zero, we wait for T_l until new order arrives.

Average total cost per unit time for EOQ with lost is given below.

$$C(Q,T) = \frac{h\frac{Q^2}{2D} + sT_lD + K}{Q/D + T_l}$$

$$\lim_{T_l -> \infty} C(Q,T_l) = sD$$

$$\frac{dC(Q,T_l)}{dT_l} = -h\frac{Q^2}{2D} + (s-c)Q - K$$

$$\frac{dC(Q,T_l)}{dQ} = h\frac{Q^2}{2D} + hT_lQ + (c-s)T_lD - K$$

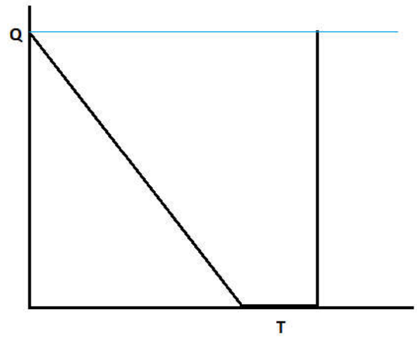

Figure 3.11: A Retailer Cycle of EOQ with Lost Sales

We try to find the optimal values of Q and T_l in this model. We show that if unit lost sales cost is small enough, optimal T_l goes to infinity and optimal Q is zero. If lost sales cost is reasonably large, optimal order quantity is the same with optimal order quantity of the classical EOQ model and optimal T_l is zero. For a given T_l value, $C(Q,T_l)$ is a convex function of Q.

Proposition 3.4.1 *$C(Q,T_l)$ is a convex function of Q.*

Proof h, T_l and D are all positive constants,

$\frac{d^2C(Q,T_l)}{d^2(Q)} = \frac{hQ}{D} + hT_l$ and

$\frac{d^2C(Q,T_l)}{d^2(Q)} > 0$ for all Q

Chapter 4

Solution Method

In previous chapter, we show that expected cycle time is increasing and concave function of Q-R. However, we cannot know the structure of the expected average cost per unit time function E[G] and expected cost of a retailer cycle E[C]. Therefore, we scan the whole space for the all integer values of ordering quantity and reorder point to understand the structure of functions as shown at Figure 4.1. R is less than equal to Q since Q is order-up to level. We use two set instances for experimental results. First set is generated by Sargut and Qi (2012) which will be described next chapter in detail. Second set is modified version of Qi et al.(2009) (shown in Table 4.1). Since the second parameter set includes more instances, we only use the second instances for the solution method. We use c programming for the solution algorithm.

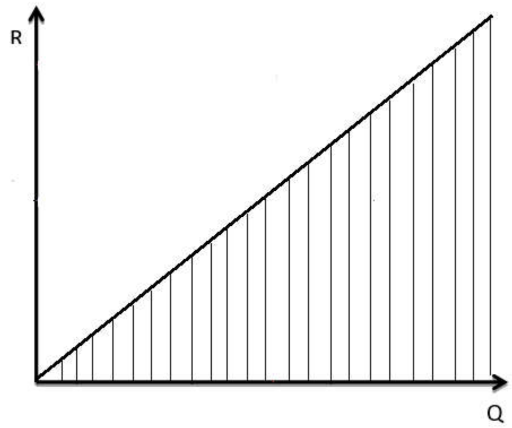

Figure 4.1: Scanned Space for Functions

Table 4.1: Second Parameter Set

Parameters	*Description*	*Values*
α		$0.01, 0.1, 1, 10$
λ		$0.01, 0.1, 1, 10$
ψ		$6, 12, 24, 48, 96$
s	Shortage Cost	$2c$
F	Fixed Cost Cost	$10, 50, 100$
h	Holding Cost	$0.01, 0.5, 1, 2$
D	Annual Demand Rate	$1, 10, 100, 500$
c	Unit Ordering Cost	$1, 2, 4, 8$

First, we start from E[C] to search. As it is shown in Figure 4.1, we investigate all expected average cost for all single instances. Totally, we have 15360 instances and we develop a search method to understand the structure of the function. The search method can be seen in Algorithm 1. Generally, as ordering quantity increases total expected cost increases. Hence, we check if there is a decreasing in any point. We notice a difference for only three instances where we can see at Table 4.2. In these cases, we see that only fixed cost can change depends on the instances and the other variables are the same. The main differences of these instances is total expected cost decreases one or more than one time when ordering quantity increases. While α, λ, holding cost, fixed cost are at their higher value, ψ takes the smallest.

Algorithm 1

```
int decrease=0
for All parameters do
    for R = 0 to 100 do
        for Q = R to 100 do
            if previous cost< current cost then
                decrease + +
if decrease > 0 then
    Not an increasing function
```

Table 4.2: The Instances which decrease any Level

Parameters	*Description*	*Instance* 1	*Instance* 2	*Instance* 3
α		10	10	10
λ		10	10	10
ψ		6	6	6
s	Shortage Cost	2	2	2
F	Fixed Cost	10	50	100
h	Holding Cost	2	2	2
D	Annual Demand Rate	100	100	100
c	Unit Ordering Cost	1	1	1

As it seen in the Figure 4.2, we show the total cost function Instance 1 (given in Table 4.2). For a single instance, we start from reorder point equal to 0 and evaluate for reorder values less than or equal to ordering quantity. In this instance, we first obtain a slight decrease on the total cost function when reorder point is 35. The cost function increases as ordering quantity increases until the 48 unit of ordering quantity then it continues to decrease until the 53 unit of order. After Q is 53, it again increases.

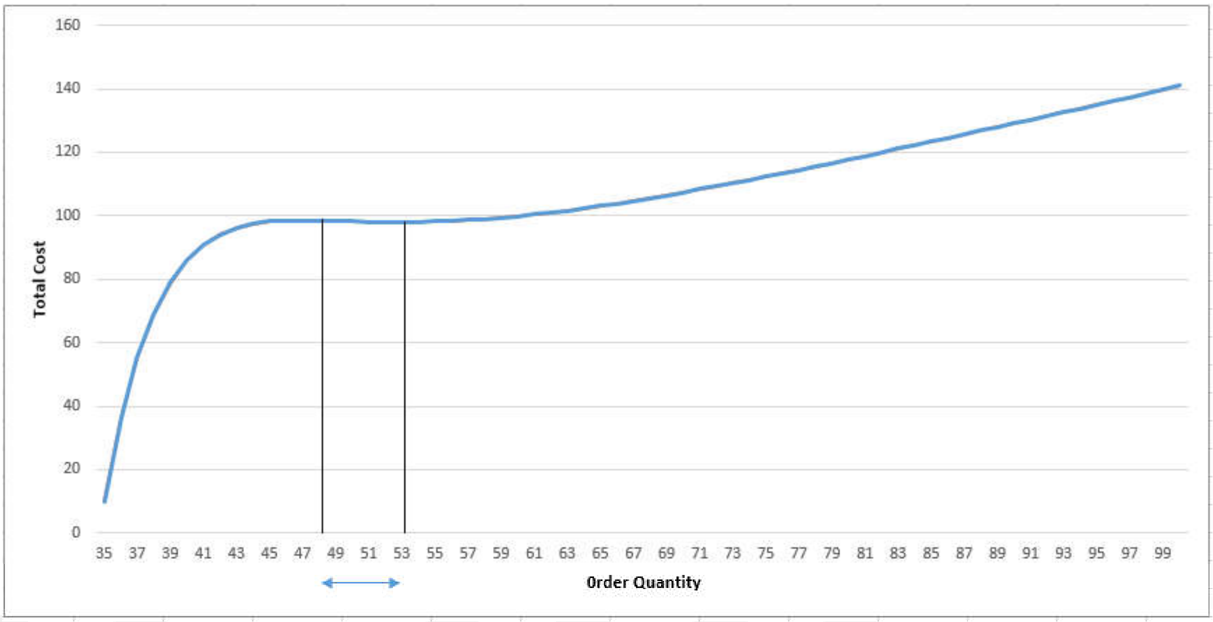

Figure 4.2: Total Cost Function of Instance 1, R=35

Total cost function of the other instances (except the three instances shown before in Table 4.2) can be seen in Figure 4.3. This instance depicted is $\alpha = 1$, $\lambda = 10$, $\psi = 6$, $s = 2$, $F = 10$, $h = 2$, $D = 100$, $c = 1$. All other cost function of Q these instances have the same properties. They are all increasing function. So, we can not say the cost function is concave like the expected time function. We can only say the total cost function is increasing function in general.

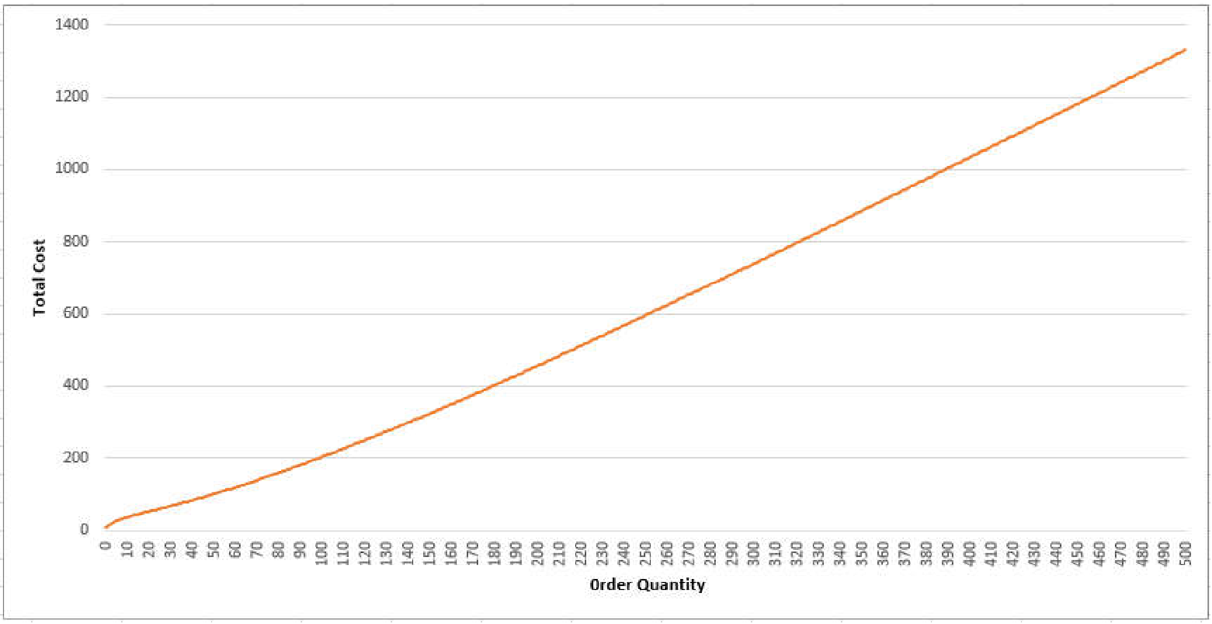

Figure 4.3: Total Cost Function for an Instance

We check 15360 cases when q is between r and 100, r is between 0 and 100. There is total (q+1)(q+2)/2*15360 points are checked. Out of 79.119.360 points, only 2.070 of them an unit increase of q decreases the $E[C]$. Total decrease is equal to 2358. Likewise, if we extend our q and r values to 500 out of 1.931.535.360, 119.812 points decreases the $E[C]$. We see the decreasing percentage is very low in these two tested performed.

We also analyze $E[G(Q,R)]$ with the same method. We again check 79.119.360 points and we show that it is a unimodal function. Unimodal function has only one local maximum or minimum extremum. While one side of the function is monotonically decreasing, the other side is monotonically increasing as it seen in Figure 4.4. In Figure 4.4 we only show on one instance that the function is unimodal but we know that when we do a search, all instances have same property. To use binary search is effective method to find the minimum point of unimodal functions. Also, binary search decreases the searching time. After define these function as a unimodal function we also check the E[G] if it is convex. To understand the function is convex or not we do a search as it shown in Algorithm 2. We conclude that the E[G] function is unimodal function but not a convex function.

Figure 4.4: Total Average Expected Cost Function

Algorithm 2

int notconvex=0
for All parameters **do**
    for $R = 0$ to 100 **do**
        for $Q = R$ to 100 **do**
            if previous difference $<$ current difference **then**
                $convex++$
if $notconvex > 0$ **then**
    Not a convex function

Chapter 5

Experimental Results

In this chapter, we use two different set of instances. We investigate the sensitivity of optimal order quantity (Q^*), reorder point (R^*) and expected average cost per unit time (C^*). Besides, we also compare our solution with the optimal economic order quantity level (Q_E). While finding Q^*, R^* and C^*, we again use the search method. We scan all order quantities up to 3000 and all reorder points which are less than or equal to current order quantity for all instances. With this search, we find optimal order quantity and optimal reorder point that minimizes expected average cost per unit time. Besides, we calculate Q_E for all single instance with the formula of classical EOQ which is given 1.0.1. First instance set is given from Sargut and Qi (2012). The second instance set is generated by inspired from Qi et al. (2009). We make minor changes at the data set to provide more appropriate result for our problem.

5.1 First Set of Instances

To analyze our policy and its sensitivity to problem parameters, we design numerical experiments. We use 1728 different problem instances given in Sargut and Qi(2012) also shown in Table 5.1. They define vulnerability of supplier (S2), which is calculated as the ratio of supplier disruption rate to its recovery rate. We conclude that reorder point is always zero when we use these instances. To see the effect of fixed cost, unit ordering cost and demand rate, several figures are shown. Since the trend is same for all instances for the variables, we took only a few instances among them.

Table 5.1: Parameter Set of Sargut and Qi(2012)

Parameters	*Description*	*Values*
α		10
$\lambda + \psi$		$10, 20$
$S2$	Vulnerability of Supplier	$0.25, 0.5, 1, 2, 5, 10$
$s - c$	Difference between lost sale and unit ordering costs	$2, 5, 7$
F	Fixed Cost	$100, 200, 400, 800$
h	Holding Cost	$0.5, 1, 2, 4$
D	Annual Demand Rate	$250, 500, 1000$

We use the same instance except one parameter of interest. Other parameters are kept constant.

The effect of unit ordering cost: At Figure 5.1, we see that when unit ordering cost increases total expected cost also increases. On the other hand, optimal ordering quantity decreases if unit ordering cost increases as it seen at Figure 5.2.

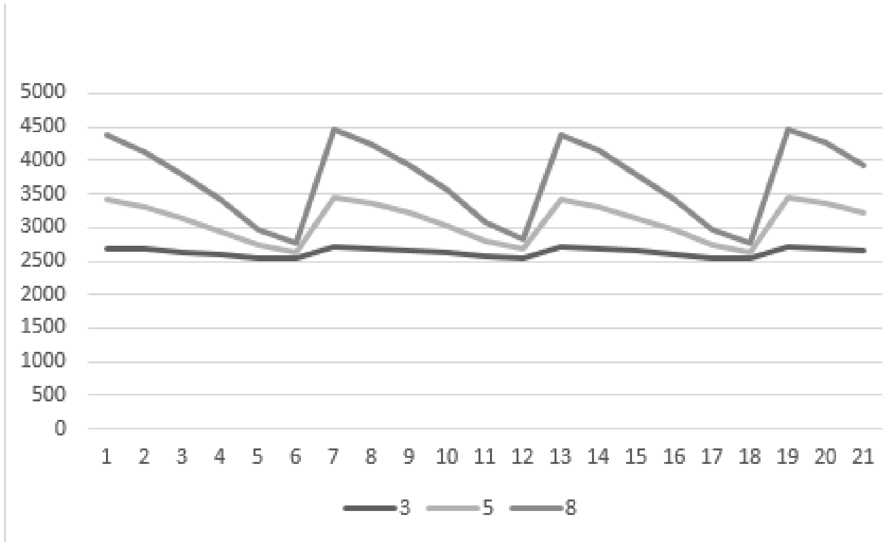

Figure 5.1: Effect of Unit Ordering Cost on Total Expected Cost

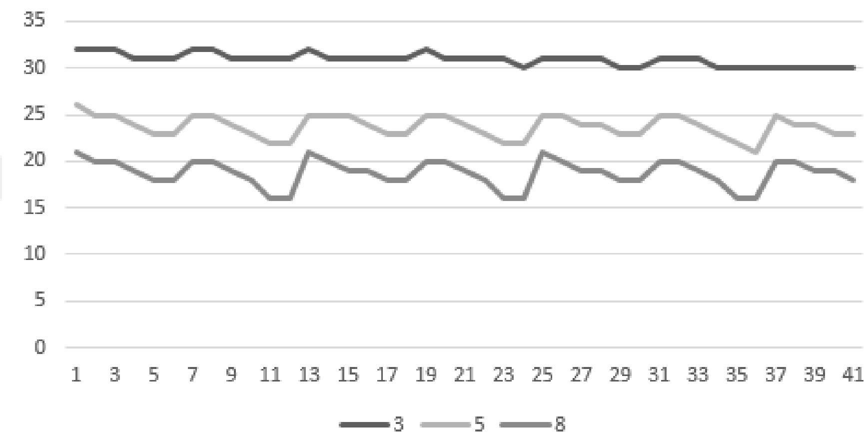

Figure 5.2: Effect of Unit Ordering Cost on Ordering Quantity

The effect of the holding cost: The order quantity is not very sensitive to the holding cost. Ordering quantities are almost the same for all different holding cost values. Although there is not a big difference between the ordering quantities, smallest holding cost always gives the highest ordering quantity.

The effect of fixed ordering cost: Total expected cost and optimal ordering quantity are also directly proportional to the fixed cost which can be seen at Figures 5.3 and 5.4. When fixed cost increases, ordering quantity and total expected cost also increases.

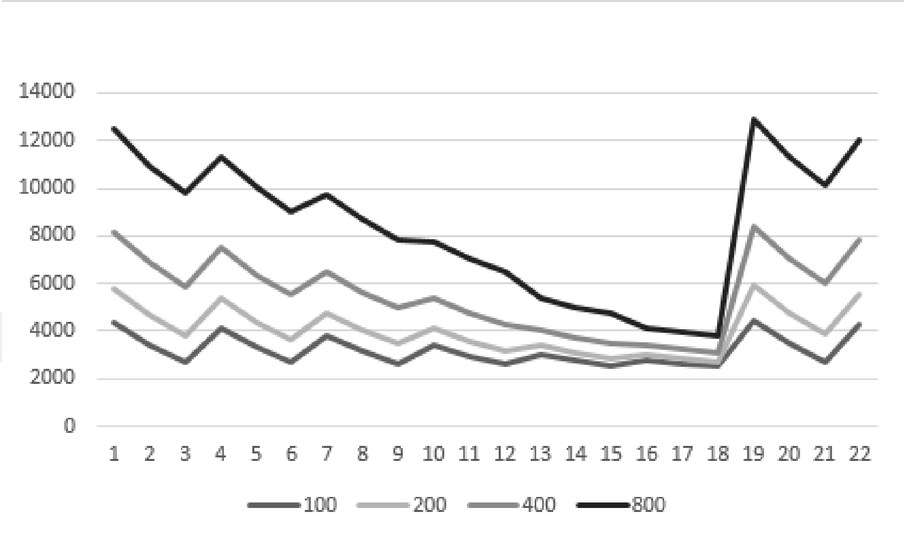

Figure 5.3: Effect of Fixed Cost on Total Expected Cost

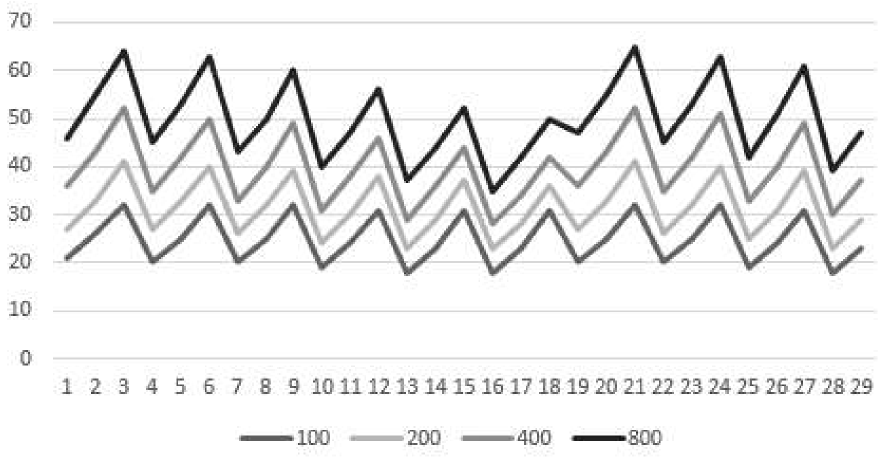

Figure 5.4: Effect of Fixed Cost on Order Quantity

The effect of demand rate. Total expected cost and optimal ordering quantity are directly related to demand rate. When demand rate increases total expected cost and optimal ordering quantity also increase. If reorder point is positive, it again increases as demand rate increases.

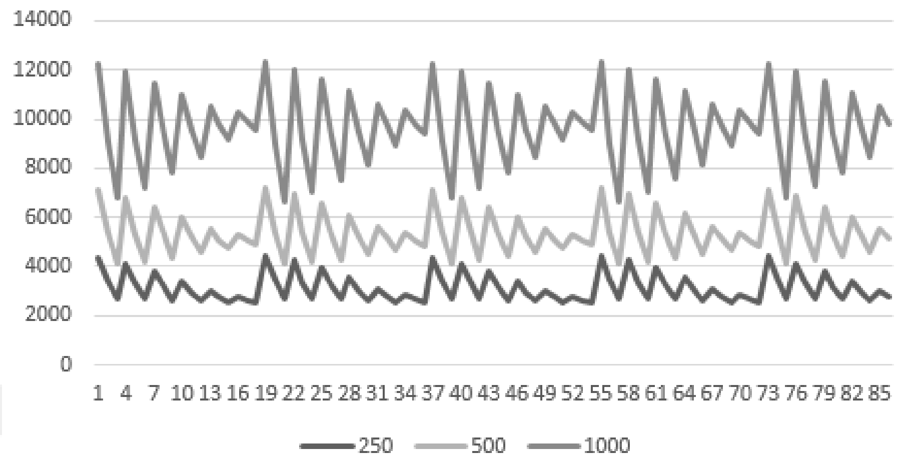

Figure 5.5: Effect of Demand Rate on Total Expected Cost

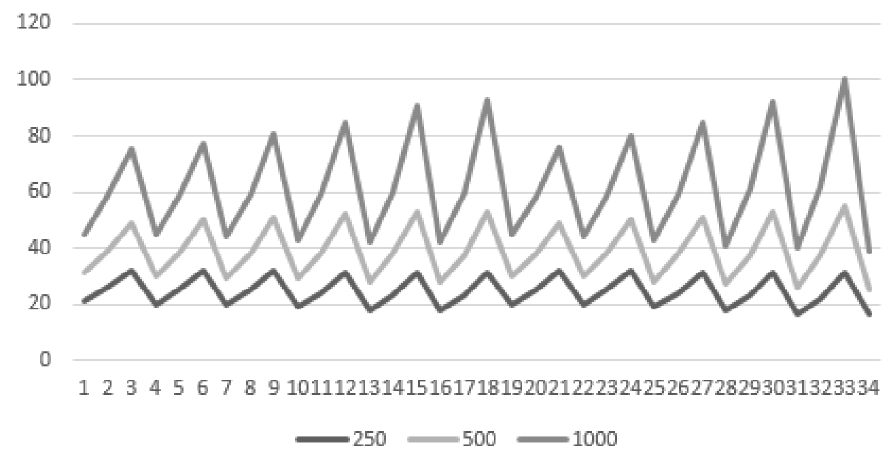

Figure 5.6: Effect of Demand Rate on Order Quantity

5.2 Second Set of Instances

Second part in the experimental results, we use the instances which are shown in Table 4.1. There are 15360 different instances. Unlike the first part of experiments, we observe that adding reorder point to the model reduces the expected cost per unit time. In total, there are 150 instances out of 15360 with non-negative reorder points where the optimal cost is achieved.

Reorder point gives a positive value and using reorder point is useful in these cases which are stated below:

- The ratio of retailer and supplier disruptions are always less than one. It means that supplier is disrupted more frequently than the retailer. In this situation reorder point can be used. As the contrary to that case, if the retailer is disrupted more frequently than supplier, retailer's inventory will tend to be destroyed, then it is unreliable and more costly to keep inventory on hand. Hence, it depends both retailer and supplier disruption rates. In other words, if the supplier is disrupted frequently and the supplier disruption occurs rarely, it is necessary to use reorder point for reducing cost.

- Supplier recovery rate also affects the reorder point, if the supplier is disrupted frequently and also recovered at the same rate, reorder point is useless. However, the value of reorder point increases as recover time of the supplier increases.

We also investigate the number of instances according to a parameter values when reorder point is positive. As it shown in Table 5.4, when α is 0.01, which is the smallest value of this variable takes positive reorder point values 117 times out of 150 instances. Since α is the retailer disruption rate, it proves that if the retailer not disrupted frequently, it is less costly to keep inventory on hand and order when inventory reaches reorder point. As a consequence, when α is 1 and 10, all instances have zero reorder point. When we look supplier disruption rate λ, it is seen that if the supplier disrupted frequently, adding positive reorder point is necessary to reduce cost. As long as supplier recovery rate ψ decreases, number of instances of which take positive reorder point decreases. It means, if the recovery of disruption takes too much time, it is meaningful to keep inventory on hand to avoid shortage cost and total it also decreases total cost.

Table 5.2: Number of Instances where reorder point is positive for the variable α

α	Number of Instances
0.01	117
0.1	33
1	0
10	0

Table 5.3: Number of Instances where reorder point is positive for the variable λ

λ	Number of Instances
0.01	0
0.1	0
1	10
10	140

Table 5.4: Number of Instances where reorder point is positive for the variable ψ

ψ	Number of Instances
6	56
12	44
24	30
48	15
96	5

In Table 5.5, we see that holding cost results in positive more than the other variables. If the holding cost is smaller it is less costly to keep inventory on hand. 123 instances takes positive reorder point when h is 0.01. Secondly, as demand rate increases number of instances with positive reorder point also increases as shown in Table 5.7. As demand rate increases, expected cycle length decreases. Since $s = 2 \times c$ number of instances with positive reorder point are the same for all the values (see Table 5.6 and Table 5.8). If the unit ordering cost increases it is more likely to see positive reorder point to avoid high unit ordering cost. Likewise, when the retailer out of inventory, shortage cost occurs. In this situation reorder point reduce the total cost if shortage cost is high. Lastly, as can be seen from the Table 5.9 as the fixed ordering cost F decreases, number of instances with positive reorder point increase.

Table 5.5: Number of Instances where reorder point is positive based on value h

h	Number of Instances
0.01	123
0.5	20
1	7
2	0

Table 5.6: Number of Instances where reorder point is positive based on value c

c	Number of Instances
1	14
2	24
4	40
8	72

Table 5.7: Number of Instances where reorder point is positive based on value D

D	Number of Instances
1	0
10	7
100	47
500	96

Table 5.8: Number of Instances where reorder point is positive based on value s

s	Number of Instances
2	14
4	24
8	40
16	72

Table 5.9: Number of Instances where reorder point is positive based on value F

F	Number of Instances
10	88
50	39
100	25

Independent from reorder point, we observe that total expected cost function and optimal ordering quantity are very sensitive to α value. As it shown in Figure 5.7 and 5.8. These two figures are opposite each other. As α increasing, total expected cost also increases whereas, optimal ordering quantity decreases. Although this relationship is seen for all instance for α, it is not valid for ψ and λ separately. We found that $\frac{\lambda}{\psi}$ effect the optimal ordering quantity as we can see at Figure 5.9. It can be interpret as $\frac{\lambda}{\psi}$ increases optimal ordering quantity also increases. In other words, if the supplier disrupted frequently and recovered slowly retailer should order more goods.

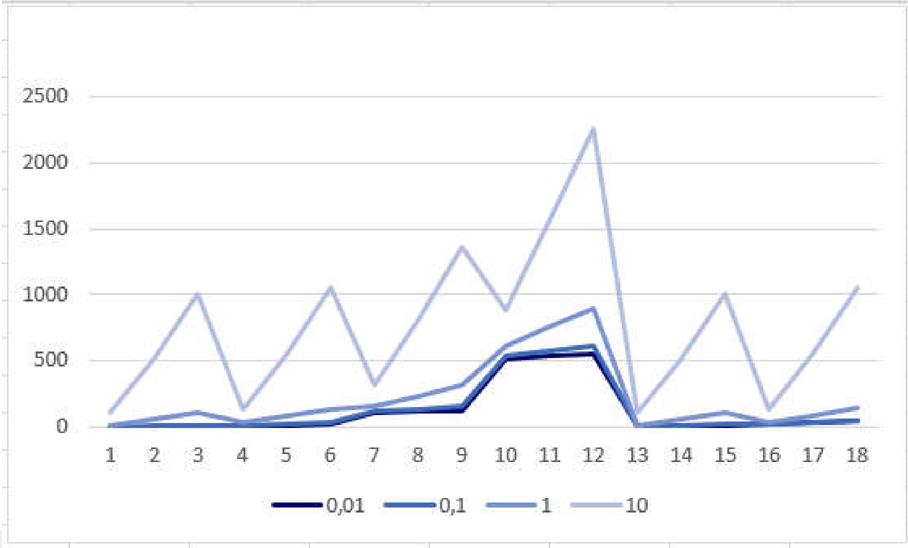

Figure 5.7: Effect of α on Total Expected Cost

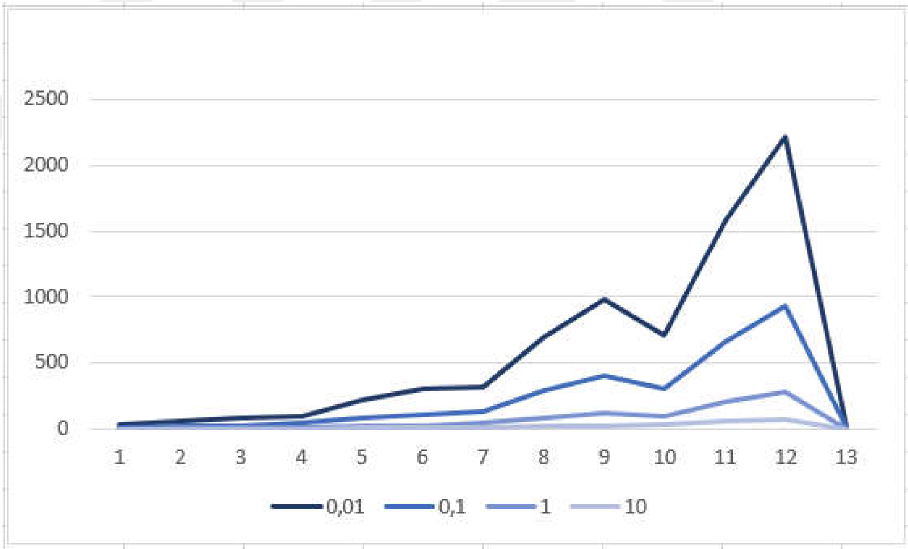

Figure 5.8: Effect of α on Optimal Ordering Quantity

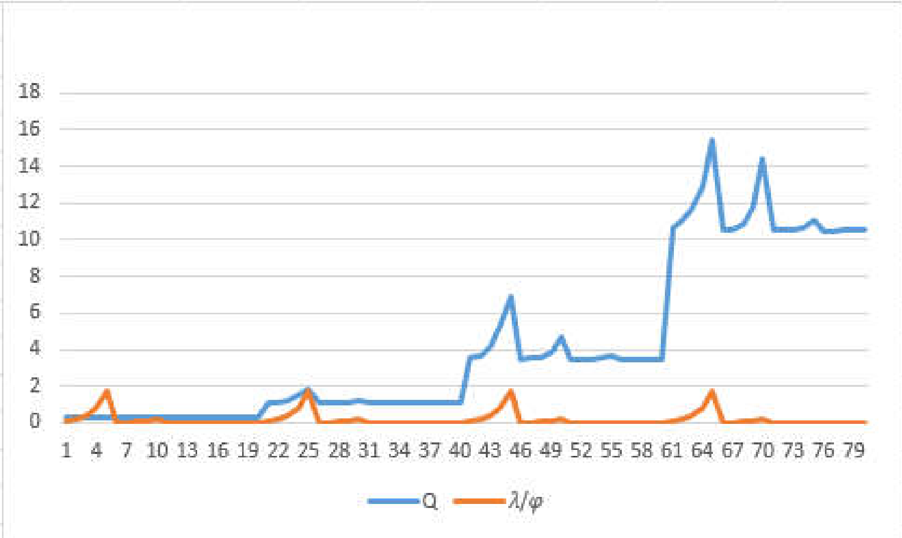

Figure 5.9: Effect of $\frac{\lambda}{\psi}$ on Optimal Ordering Quantity

However, when we look only ψ, we see that Q^* is insensitive to ψ. While all other parameters kept constant, change in ψ does not affect Q^*. In Figure 5.10, there are some instances and we see that while ψ increases, ordering quantity is same if the other variables keep constant.

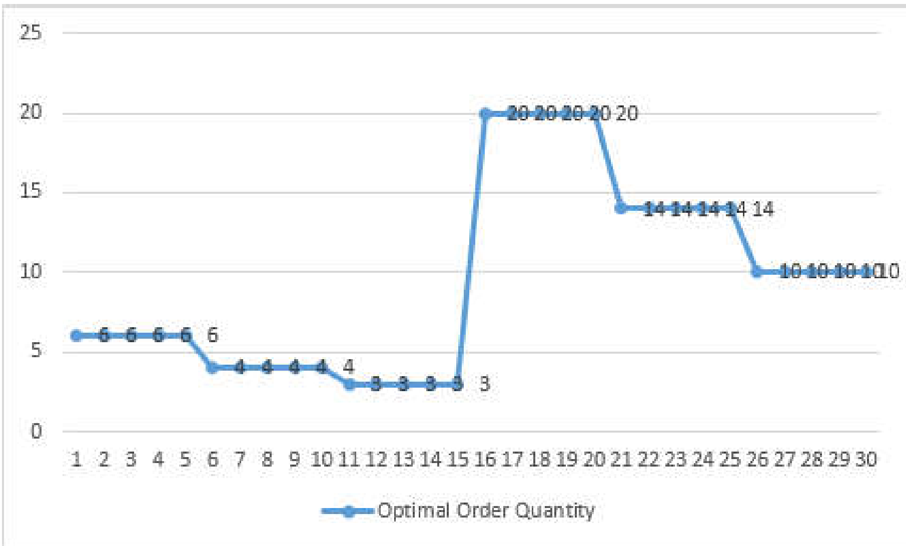

Figure 5.10: Effect of ψ on Ordering Quantity

5.3 Comparison with Economic Order Quantity

To see the value of our inventory problem with disruptions, we calculate EOQ value and put this ordering quantity to our expected average cost per unit time for all instances. We included all instances when we compare two expected average cost per unit time. As a result of our experiments, we found 150 instances which have positive reorder point and and 15210 instances with zero reorder point. Hence, we use the all 15360 instances to find the improvements on EOQ.

Lets define Q_E as the optimal ordering quantity with using EOQ model and $E[G(Q_E, 0)]$ is the expected average cost per unit time with using Q_E and $R = 0$. While calculating the percentage improvement from EOQ model we use the equation below:

$$Imp = \frac{E[G(Q_E, 0)] - E[G(Q^*, R)]}{[EG(Q_E, 0)]} \times 100 \tag{5.3.1}$$

Improvements are changes between 0% and 98%. Our average expected cost function gives always better or equally same results. In Figure 5.11, a histogram which gives the number of instances according to their improvements is shown. We see that almost 7700 instances are improved between 0% and 10%. Although, there are instances without any improvement, up to 98% improvement is achieved from some instances when we compare two models. In the Table 5.11, we show some instances according to their improvement order. 3814 of the instances are not improved where we can also say the optimal ordering quantities are the same in two problems. The remaining 11546 instances are improved. In Table 5.10, we can see some different instances which is not improved.

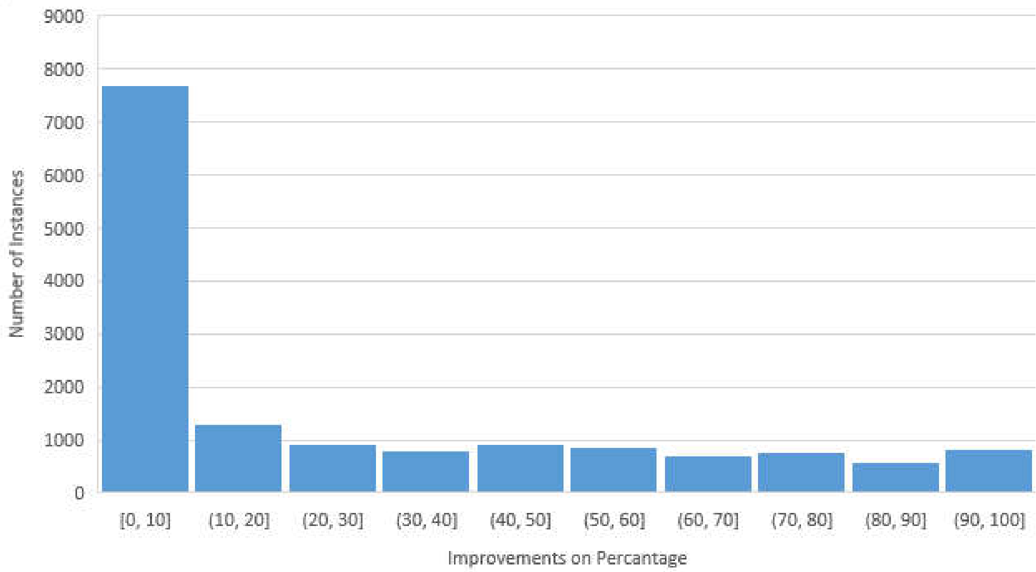

Figure 5.11: Improvement for all Instances

Table 5.10: Instances Samples without any Improvement

Parameters	*Description*	*Instance* 1	*Instance* 2	*Instance* 3
α		0.01	0.1	0.01
λ		0.01	0.01	0.01
ψ		12	6	96
s	Shortage Cost	8	8	16
F	Fixed Cost Cost	10	10	50
h	Holding Cost	2	0.5	2
D	Annual Demand Rate	1	1	100
c	Unit Ordering Cost	4	4	8

Common properties of the instances with highest improvements are listed below:

- When the improvement is between 95-98% (highest improvements), $\alpha = 10$

for all instances. It means as the retailer disruption rate is more frequent, improvement of percentage increases.

- When the improvement is between 95-98%, $h = 0.01$ for all instances. If the holding cost takes the lowest value, improvement is higher.

- Generally, instances with highest *shortage cost unit ordering cost* values, give the highest improvements.

- Demand rate does not affect the improvement considerably. Almost all values of demand rates exist in highest improved instances.

Also Q^*/Q_E decreasing function of α as it seen in Figure 5.12. The effect of other variables decreases as α increases. For instances, when $\alpha = 0.01$ there is a huge difference between Q^* and Q_E in some instances. However, when $\alpha = 10$ there is no significant difference between Q^* and Q_E relatively.

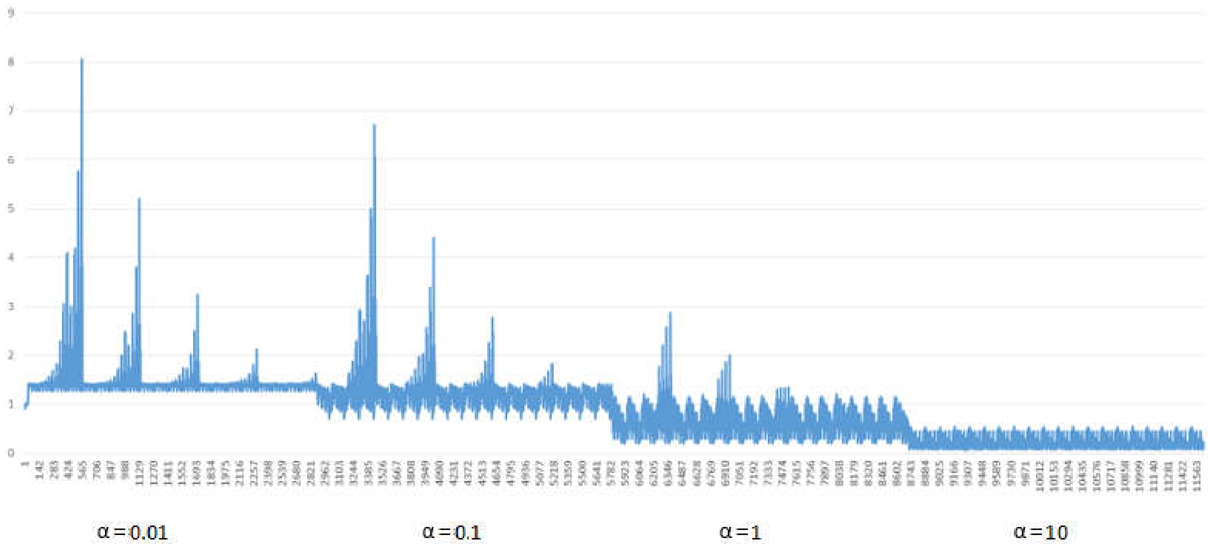

Figure 5.12: Effect of α on Q^*/Q_E

At Figure 5.13, λ does not affect Q^*/Q_E but the effect of other variables increase as λ increases.

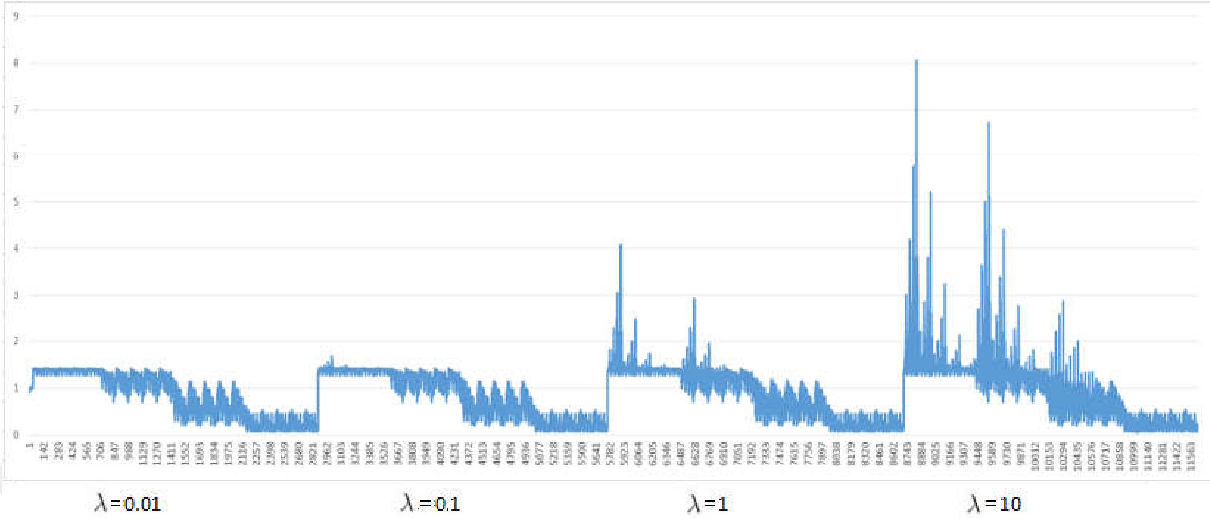

Figure 5.13: Effect of λ on Q^*/Q_E

As ψ increases the effect of other parameters decrease, in general Q^*/Q_E has the same structure repeating itself as it seen at Figure 5.14.

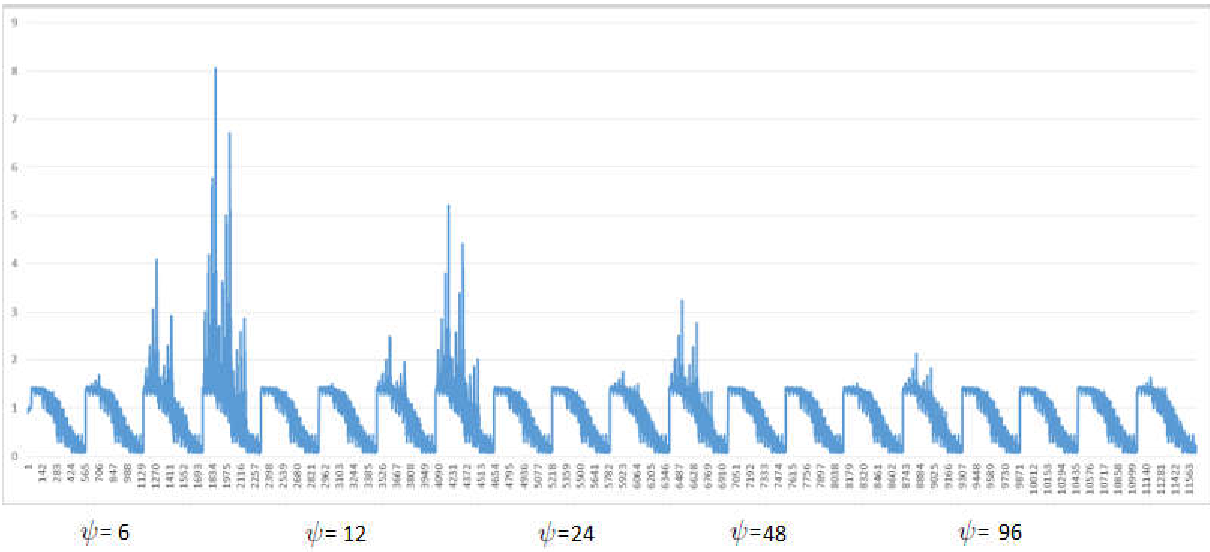

Figure 5.14: Effect of ψ on Q^*/Q_E

α	λ	ψ	F	c	h	s	D	$Imp\%$
10	0, 01	48	10	8	0, 01	16	10	98
10	0, 01	96	10	8	0, 01	16	10	98
10	0, 01	24	10	8	0, 01	16	10	98
10	0, 01	12	10	8	0, 01	16	10	98
10	0, 01	6	10	8	0, 01	16	10	98
10	0, 01	96	100	8	0, 01	16	100	98
10	0, 01	48	100	8	0, 01	16	100	98
10	0, 01	24	100	8	0, 01	16	100	98
10	0, 01	12	100	8	0, 01	16	100	98
10	0, 01	6	100	8	0, 01	16	100	98
10	0, 01	96	100	8	0, 01	16	500	97
10	0, 01	48	100	8	0, 01	16	500	97
10	0, 01	24	100	8	0, 01	16	500	97
10	0, 01	12	100	8	0, 01	16	500	97
10	0, 01	6	100	8	0, 01	16	500	97
10	0, 01	96	50	8	0, 01	16	100	97
10	0, 01	48	50	8	0, 01	16	100	97
10	0, 01	24	50	8	0, 01	16	100	97
10	0, 01	12	50	8	0, 01	16	100	97
10	0, 01	6	50	8	0, 01	16	100	97
10	0, 01	96	50	8	0, 01	16	10	97
10	0, 01	48	50	8	0, 01	16	10	97
10	0, 01	24	50	8	0, 01	16	10	97
10	0, 01	12	50	8	0, 01	16	10	97
10	0, 01	6	50	8	0, 01	16	10	97
10	0, 01	96	50	8	0, 01	16	500	96
10	0, 01	48	50	8	0, 01	16	500	96
10	0, 01	24	50	8	0, 01	16	500	96
10	0, 01	12	50	8	0, 01	16	500	96
10	0, 01	6	50	8	0, 01	16	500	96
10	0, 01	96	10	8	0, 01	16	100	96
10	0, 01	48	10	8	0, 01	16	100	96
10	0, 01	24	10	8	0, 01	16	100	96
10	0, 01	12	10	8	0, 01	16	100	96
10	0, 01	6	10	8	0, 01	16	100	96
10	0, 01	96	100	8	0, 01	16	10	96
10	0, 01	48	100	8	0, 01	16	10	96
10	0, 01	12	100	4	0, 01	8	100	96
10	0, 01	24	100	8	0, 01	16	10	96
10	0, 01	6	100	8	0, 01	16	10	96
10	0, 01	12	100	8	0, 01	16	10	96
10	0, 01	24	10	8	0, 01	16	1	95
10	0, 01	48	10	8	0, 01	16	1	95
10	0, 01	96	10	8	0, 01	16	1	95
10	0, 01	12	10	8	0, 01	16	1	95
10	0, 01	6	10	8	0, 01	16	1	95

Table 5.11: Instances Samples with Higher Improvement

5.4 Approximation for Q^*

In this chapter, we try to find an approximate equation for optimal ordering quantity using our test results. We only consider cases with zero optimal reorder point.

Theorem 5.4.1 *Let us define K as a constant and f as an arbitrary function. When we consider cases with $h \geq 0.5$*

$$Q^* \approx K * \sqrt{FD} * f(\lambda, \alpha, \psi, s, c, h)$$

Proof When all λ, α, ψ, c, h, and D are constant, Q^* is directly proportional to $\sqrt{F}$. When all λ, α, ψ, c, h, and F are constant, Q^* is directly proportional to $\sqrt{D}$. Q^* is an increasing function of $1/\sqrt{h}$.

Chapter 6

Conclusion and Future Study

In this study, a two-echelon continuous review inventory problem where supplier and retailer are subject to random disruptions is considered. We try to mitigate supplier and retailer disruptions by using inventory control policy. Supplier have ON and OFF periods. However, when retailer is disrupted in a retailer cycle, it recovers immediately. We investigate the structure of expected time function and cost function separately and we conclude that expected time function is a concave function. We modify Qi et al. (2009) model by adding reorder point as a decision variable like ordering quantity and we use two different test cases to analyze. We aim to show in which conditions reorder point helps to mitigate supply disruptions. Besides, we also analyze the optimal ordering quantity and optimal cost of a retailer and we compare the optimal ordering quantity with the classical EOQ. We deduce that, our model gives a solution with a better cost than EOQ.

As a future study, it can be assumed that there is a correlation between retailer and supplier disruptions. Also, we want to consider backordering and partial backordering cases. Moreover, instead of deterministic demand, stochastic demand can be considered. Another extension can be inclusion of perishability instead of destroying all inventory on hand.

Appendices

Appendix A

Search Code

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <string.h>

double cost(double D, double Q, double R, double l,
   double p, double a,double s, double h, double c,
   double Fx) {
return Fx+(l / (l + p) * (0.1e1 - exp(-(Q - R) / D * (l
    + p))) * (-h * a * (-0.2e1 * D * exp(R * (a + p) /
   D) * R * a - 0.2e1 * D * exp(R * (a + p) / D) * R *
   p + 0.2e1 * pow(D, 0.2e1) * exp(R * (a + p) / D) + R
    * R * a * a + 0.2e1 * R * R * a * p + R * R * p * p
    - 0.2e1 * pow(D, 0.2e1)) / (pow(a, 0.3e1) + 0.3e1 *
    a * a * p + 0.3e1 * a * p * p + pow(p, 0.3e1)) *
   exp(-(a * Q + p * R) / D) / 0.2e1 - h * p * (-0.2e1
   * D * exp(R * (a + p) / D) * R * a - 0.2e1 * D * exp
   (R * (a + p) / D) * R * p + 0.2e1 * pow(D, 0.2e1) *
   exp(R * (a + p) / D) + R * R * a * a + 0.2e1 * R * R
    * a * p + R * R * p * p - 0.2e1 * pow(D, 0.2e1)) /
   D * exp(-(a * Q + p * R) / D) / (pow(a, 0.3e1) + 0.3
   e1 * a * a * p + 0.3e1 * a * p * p + pow(p, 0.3e1))
   / 0.2e1 + c * Q * exp(-a * (Q - R) / D) * exp(-p * R
    / D) + c * p * (exp(R * (a + p) / D) * Q * a + exp(
   R * (a + p) / D) * Q * p - exp(R * (a + p) / D) * R
   * a - exp(R * (a + p) / D) * R * p + D * exp(R * (a
   + p) / D) - a * Q - Q * p - D) * exp(-(a * Q + p * R
   ) / D) / (a * a + 0.2e1 * a * p + p * p) + s * D * a
    * (exp(R * (a + p) / D) - 0.1e1) * exp(-(a * Q + p
   * R) / D) / (a + p) / p) + s * D * l / (l + p) *
   (0.1e1 - exp(-(Q - R) / D * (l + p))) * exp(-a * Q /
    D) * (0.1e1 / p + (exp(-p * R / D) * R * p + D *
   exp(-p * R / D) - D) / D / p) + l * (exp(-(Q - R) /
```

```
    D * (l + p)) * a + exp(a * (Q - R) / D) * l + exp(a
    * (Q - R) / D) * p - a - l - p) / (l + p) * exp(-a *
     (Q - R) / D) / (a + l + p) * s * D / p - h / a * Q
    * (exp(-a * (Q - R) / D) * Q * a - exp(-a * (Q - R)
    / D) * R * a + D * exp(-a * (Q - R) / D) - D) / D +
    h * pow(a, -0.2e1) / D * (exp(-a * (Q - R) / D) * Q
    * Q * a * a - 0.2e1 * exp(-a * (Q - R) / D) * Q * R
    * a * a + exp(-a * (Q - R) / D) * R * R * a * a +
    0.2e1 * D * exp(-a * (Q - R) / D) * Q * a - 0.2e1 *
    D * exp(-a * (Q - R) / D) * R * a + 0.2e1 * pow(D,
    0.2e1) * exp(-a * (Q - R) / D) - 0.2e1 * pow(D, 0.2
    e1)) / 0.2e1 + h * (Q * Q - R * R) / D * exp(-a * (Q
     - R) / D) / 0.2e1 + c * Q * (0.1e1 - exp(-a * (Q -
    R) / D)) + (0.1e1 - l / (l + p) * (0.1e1 - exp(-(Q -
     R) / D * (l + p)))) * c * (Q - R) * exp(-a * (Q - R
    ) / D));
}
double time(double D, double Q, double R, double l,
    double p, double a){
return -(exp(-(Q - R) * (a + l + p) / D) - 1) * l / (a
    + l + p) / p - (exp(-a * (Q - R) / D) - 1) / a;
}
double eoq(double D, double h, double c, double Fx){
return sqrt((2*Fx*D/h));
}

        void deney(){
        double k;       int v;  int vv;
        double k_ls;
        double minall=99999999999999999999;
        double min=99999000000000000000;
        double a=10;
        double l=5;
        double p=12;
    double D=1000;
        double Fx=60;
        double c=2;
        double s=10;
    double h=0.2;
    double b=999999999;
        int q=100;
        int r=50;
        int count=0;
        double lambda[4]={0.01,0.1,1,10};
        double psi[5]={6,12,24,48,96};
        double holding[4]={0.01,0.5,1,2};
        double fixedcost[4]={10,50,100};
        double demand[4]={1,10,100,1000};
```

```
    double unitcost[4]={1,2,4,8};
    double alpha[4]={0.01,0.1,1,10};
    int r_temp;
    int q_temp;
    int qls_temp;
    float q_eoq;
    FILE *finput;
    finput=fopen("analiz.txt","w");


    for(int t=0; t<=0; t++){
    D=demand[t];
    printf("demand:␣%f\n",D);
    for(int j=0; j<=2; j++){
    Fx=fixedcost[j];
    printf("fixed␣cost:␣%f\n",Fx);
    for(int i=0; i<=3; i++){
    h=holding[i];
    printf("holding:␣%f\n",h);
    for(int i_d=0; i_d<=3; i_d++){
l=lambda[i_d];
    printf("lambda:␣%f\n",l);
    for(int z=0; z<=3; z++){
    a=alpha[z];
    for(int w=0; w<=4; w++){
    p=psi[w];
    for(int ti=0; ti<=3; ti++){
    c=unitcost[ti];
for(int ss=0; ss<=0; ss++){
    s=unitcost[ti]*2;
    fprintf(finput,"%0.2f␣%0.2f␣%0.2f␣%0.0f␣%0.0f␣
       %0.2f␣%0.0f␣%0.0f␣",a,l,p,Fx,c,h,s,D);
    min=99999999999999999999;
    q_eoq=eoq(D,h,c,Fx);
    for(int q=0;q<=3000;q++){
    for(int r=0;r<q;r++){
    k=cost(D,q,r,l,p,a,s,h,c,Fx)/time(D,q,r,l,p,a);
    if(k<min){
    min=k;
    r_temp=r;
    q_temp=q;
    }}}
    double eoq_cost;
    eoq_cost=cost(D,q_eoq,0,l,p,a,s,h,c,Fx)/time(D,
       q_eoq,0,l,p,a);
    fprintf(finput,"%2d␣%2d␣%0.0f␣%0.0f␣%0.0f\n␣",
       q_temp, r_temp, min, q_eoq, eoq_cost);
    }}}}}}}}
```

```
        fclose(finput);
        }
int main()
{
        double a=1;
        double l=0.1;
        double p=12;
    double D=1000;
        double F=6;
        double s=0;
    double h=0.2;
    double b=99999999999999999999999999;
    double c=2;
        int Q=100;
        int R=0;
        double cost;
        deney();
        system("pause");
}
```

Chapter 7

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