

RANKING ALTERNATIVES USING DATA ENVELOPMENT ANALYSIS:
A DUAL APPROACH

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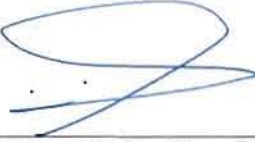
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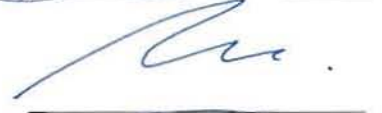

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ABSTRACT

RANKING ALTERNATIVES USING DATA ENVELOPMENT ANALYSIS: A DUAL APPROACH

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This thesis addresses the problem of ranking alternatives evaluated with multiple criteria. Köksalan and Tuncer (2009) propose a mathematical programming approach based on Data Envelopment Analysis (DEA). Their approach requires binary and continuous decision variables and removes alternatives until the alternative under consideration becomes efficient. In this study, we propose a dual DEA based approach that myopically removes alternatives, but does not require integer or binary decision variables. We test the performance of our method and its competitor on randomly generated instances. We show that our method is significantly faster and obtains nearly the same rankings. Finally, we use both methods to rank 160 countries based on Logistics Performance Index data.

Keywords : ranking alternatives, data envelopment analysis, dual approach

ÖZ

VERİ ZARFLAMA ANALİZİ KULLANARAK ALTERNATİFLERİ SIRALAMA: DUAL YAKLAŞIM

Yılmaz, Onur Can

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Bu tez çalışması, çok kriterli alternatiflerin sıralanması üzerinedir. Köksalan ve Tuncer (2009) tarafından veri zarflama analizi tabanlı bir matematiksel programlama modeli önerilmiştir. Önerilen model ikili ve sürekli karar değişkenleri gerektirmektedir ve değerlendirilen alternatif etkin oluncaya kadar diğer alternatifleri göz ardı eder. Bu çalışmada, alternatifleri miyopik olarak göz ardı eden ancak tamsayı veya ikili karar değişkenlerine ihtiyaç duymayan eşlenik (dual) veri zarflama analizi tabanlı bir yaklaşım önerilmiştir. Önerdiğimiz metot ile rakibinin performansı rassal üretilmiş örnekler üzerinde test edilmiştir. Önerdiğimiz metodun daha hızlı olduğu ve seçeneklerin sıralanmasında diğer yöntemle yakın sonuçlar verdiği gösterilmiştir. Son olarak, her iki yöntem 160 ülkeyi Lojistik Performans İndeksi verisi ile sıralamak için kullanılmıştır.

Anahtar Kelimeler: alternatifleri sıralama, veri zarflama analizi, eşlenik yaklaşım

to my family...

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LIST OF ABBREVIATIONS

AES	: Area of the Efficiency Score
CCR	: Charnes Cooper Rhodes
DEA	: Data Envelopment Analysis
DM	: Decision Maker
DMU	: Decision Making Units
KT	: Koksalan Tuncer (method)
LPI	: Logistics Performance Index
MCDM	: Multi - Criteria Decision Making
OY	: Özpeynirci Yılmaz (method)

CHAPTER 1

INTRODUCTION

The ranking alternatives (or units) evaluated with multiple criteria is a problem faced by the decision makers (DM) in real life applications. The existence of multiple criteria not only affects the decision making process, but also increases the complexity of the ranking problem. There are several methodologies and techniques available in the literature for ranking alternatives in the presence of multiple criteria.

Data Envelopment Analysis (DEA) is a technique that has been used widely in the recent years for evaluating the efficiency of alternatives and benchmarking between independent decision making units. In this study, we propose a model that ranks the alternatives based on the data envelopment analysis.

Researchers have started to categorize units with their own efficiency values through DEA. The DEA is a method which receives a growing interest in the problem area of evaluating the so called "Decision Making Units" (DMUs) which convert multiple inputs into multiple outputs.

The DEA method enables the researcher to rank performance/efficiency benchmarking of DMUs with multiple inputs and outputs. DEA considers each DMU separately and finds the maximum efficiency value of the DMU under consideration by comparing it with the other DMUs.

Köksalan and Tuncer (2009) propose a DEA based method for ranking alternatives. Their method considers each DMU one by one. For a given DMU, it solves a DEA model with additional binary decision variables. If the DMU under consideration is

not efficient, then they allow the model to remove one or more DMUs from the DMU list. This procedure is complete when the DMU under consideration becomes efficient. We propose a dual approach for ranking alternatives. Our approach also considers removing DMUs until the DMU under consideration becomes efficient. However, our approach does not require any binary or integer decision variables.

After this introductory chapter, we present background information and review DEA literature in the next chapter. In Chapter 3, we mention a DEA based approach to ranking multiple criteria alternatives proposed by Köksalan and Tuncer (2009). In Chapter 4, we introduce our dual approach to ranking multi-criteria alternatives. In Chapter 5, we test the performance of Köksalan and Tuncer's (2009) method and our method on randomly generated instances. This chapter also includes a comparison of the methods on a data set obtained from Logistic Performance Index of 160 countries. We conclude the thesis and discuss further research directions in the final chapter.

CHAPTER 2

LITERATURE REVIEW

In this part, we talk about the Data Envelopment Analysis literature and ranking methods.

2.1 Background Information

Data Envelopment Analysis (DEA) was known as "program follow through" in the early years. DEA was originated by Farrell in 1957 and it was called "The measurement of productive efficiency". The point of origin of DEA is relevant to productivity which was defined in his classical paper by Farrell (1957) as,

"The problem of measuring the productive efficiency of an industry is important to both the economic theorist and the economic theorist maker. If the theoretical arguments as to the relative efficiency of different economic systems are to be subjected to empirical testing, it is essential to be able to make some actual measurements of efficiency. Equally, if economic planning is to concern itself with particular industries, it is important to know how far a given industry can be expected to increase its output by simple increasing its efficiency, without absorbing further resources."

The focus is to evaluate the performance of DMUs while consuming some varying amount of its inputs for producing some varying amount of its outputs. Charnes (1978) uses linear programming models for assessing the productivity of DMUs which have multiple inputs and outputs. The initial framework was developed by

Charnes (1978), and implemented and defined as Data Envelopment Analysis. Charnes (1978) described DEA as a;

"Mathematical programming model applied to observational data which provides a new way of obtaining empirical estimates of relations - such as the production functions and/or efficient production possibility surfaces that are cornerstone of modern economics."

Many theoretical extensions are developed based on the DEA method, which is designed a method to measure the efficiency of similar Decision Making Units using the ratio of its weighted outputs to its weighted inputs that provides for ranking/sorting alternatives to Decision Makers (DM).

The scope of DMU is wide, flexible and generic. DMUs may represent several decision making units, varying from countries to cities, from universities to schools, from firms to academic departments, from agriculture to the industry, from restaurants to retail stores, besides the real life applications for modeling operational processes for the performance evaluation. This wide concept makes DEA a very active research tool used for efficiency and productivity analysis.

Besides the first model by Charnes, Cooper and Rhodes (1978), which is called the CCR model, there are several related DEA models in the literature. One can see Cooper, Seiford, Tone (2000) for different DEA models including CCR, Additive DEA models and Slack-Based Measurement models. Seiford and Thrall (1990), Li and Reeves (1999), Cooper and Seiford (2004), Ahn, Charnes and Cooper (2006), Cook and Seiford (2009) and also provide a review on CCR models.

2.2 Data Envelopment Analysis

There are several statistical methods for ranking alternatives in the literature. In this section, we focus on the main basic DEA models. The classical method is shown below, that assigns an efficiency score to each DMU between 0 (worst) and 1 (best). If the score is equal to 1, DMU is said to be efficient, and inefficient if otherwise. The non-linear classical DEA is simply modeled as the ratio of value of the multiple

outputs to that of multiple inputs. Before explaining this model, we look at the indices, parameters and the decision variables;

Indices:

$n =$ the number of DMU's in the process, $k=1, \dots, n$

$m =$ the number of inputs considered, $j=1, \dots, m$

$s =$ the number of outputs considered, $i=1, \dots, s$

Parameters:

$Y_{ik} =$ the value of output i for DMU k ($i=1, \dots, s ; k = 1, \dots, n$)

$X_{jk} =$ the value of input j for DMU k ($j=1, \dots, m ; k = 1, \dots, n$)

Decision Variables:

$U_i =$ the weight for output i ($i = 1, \dots, s$)

$V_j =$ the weight for input j ($j = 1, \dots, m$)

Objective function :

$$\max Z = \frac{\sum_{i=1}^s U_i Y_{io}}{\sum_{j=1}^m V_j X_{jo}}$$

This non-linear model's aim is to maximize the ratio of weighted of its inputs to weighted of its outputs for a DMU 0 . In the study, the DMU in consideration is referred to as DMU 0 .

subject to :

$$\frac{\sum_{i=1}^s U_i Y_{ik}}{\sum_{j=1}^m V_j X_{jk}} \leq 1 \quad \forall k$$

Obviously, without constraints, the objective function value is unbounded, so the solution will be infeasible. With these constraints, the ratio of virtual output to that virtual input is expected to be smaller than 1 for each DMU, so the efficiency score

of each DMU is equal to 1 or below. The efficiency ratio is ranged from 0 to 1, thus each unit will choose weights so as to maximize self efficiency.

for $j = 1, 2, \dots, n$,

$$u_i, v_j \geq 0 \text{ for all } i = 1, \dots, s; j = 1, \dots, m$$

Below we show the linear version of the above model. This model is also known as the CCR (Charnes, Cooper and Rhodes) model.

objective function:

$$\text{Max } Z = \sum_{i=1}^s U_i Y_{io}$$

subject to:

$$\sum_{j=1}^m V_j X_{j0} = 1$$

$$\sum_{i=1}^s U_i Y_{ik} \leq \sum_{j=1}^m V_j X_{jk}$$

for $k = 1, \dots, n$

$$U_i, V_j \geq 0, \quad i = 1, \dots, s; \quad j = 1, \dots, m.$$

This model determines the efficiency of each DMU and is widely used in the literature.

These models can be developed as input or output oriented. Output oriented puts the outputs values to consume minimum inputs. Input oriented puts the inputs values to produce maximum outputs. We will employ input oriented CCR model to evaluate the performance of DMUs because of example set.

We will explain the duality of the CCR model in the following part. The dual method's aim is to produce a hypothetical DMU consuming the minimum amount of inputs, while producing no less than the DMU under consideration. If the DMU is inefficient, then a combination of other DMUs can produce the same amount of output with less amount of inputs.

Before explaining this model, we first provide the indices, parameters and the decision variables;

Indices:

n = the number of DMU's in the process,

m = the number of inputs considered,

s = the number of outputs considered,

Parameters:

Y_{ik} = the value of output i for DMU k ($i = 1, \dots, n$) ; $k = 1, \dots, n$)

X_{jk} = the value of input j for DMU k ($j = 1, \dots, m$; $k = 1, \dots, n$)

Decision Variables:

λ_k = is for constituting virtually DMU k , hybrid k of the DMUs ($i = 1, \dots, n$)

θ = rate weight ($0 \leq \theta \leq 1$)

objective function:

min θ

subject to:

$$\sum_{k=1}^n \lambda_k X_{jk} \leq \theta X_{j0} \quad \forall j$$

Input j of the hybrid DMU must be smaller than or equal to the input j of DMU under consideration.

$$\sum_{k=1}^n \lambda_k Y_{ik} \geq Y_{i0} \quad \forall i$$

Output i of hybrid DMU must greater than or equal to output i of DMU under consideration.

$$\lambda_k \geq 0 \text{ for all } k,$$

If a DMU is detected to be inefficient, the combination of the other efficient units can produce the same amount of output using fewer inputs. In this dual model, the hybrid of DMU is constituted as virtually using λ_k and θ . We will utilize and develop duality of CCR model in our method in the forth section.

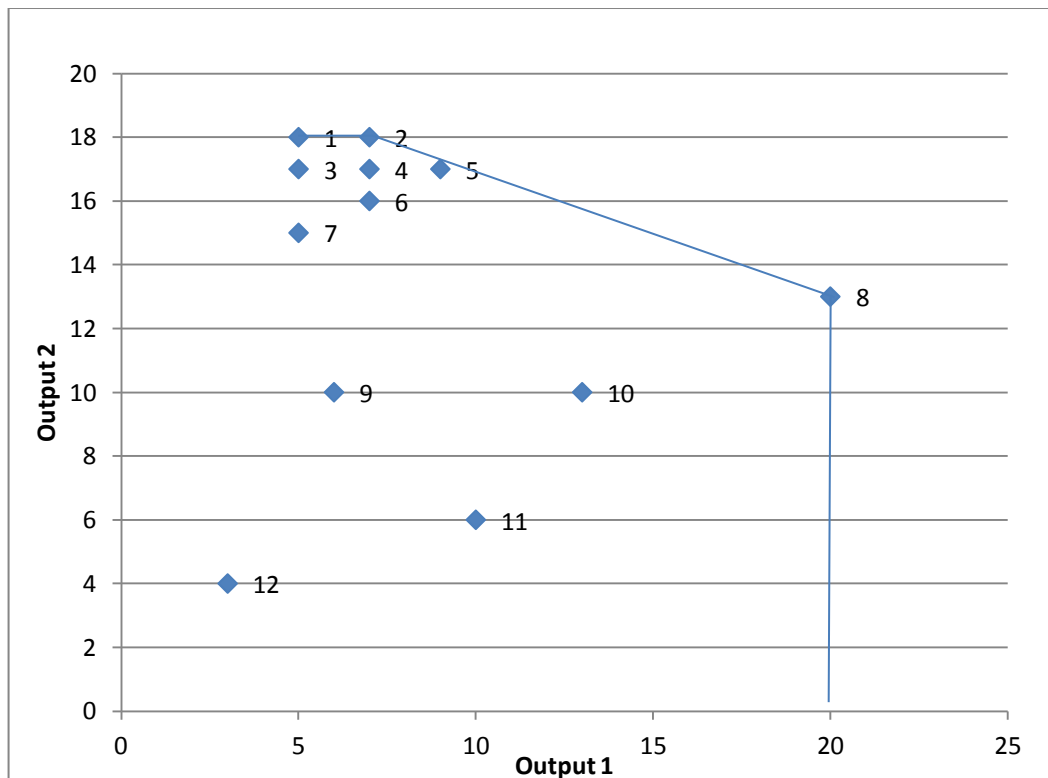
2.3 Ranking Methods

Several approaches (such as Cook and Kress, 1990; Sinuary-Stern, 1994; Landsdowne, 1996; Obata and Ishii, 2003; Foroughi and Tamiz, 2005; Büyükbaşaran, 2005; Köksalan et al., 2010) have adapted DEA models in practice. The determination of the in/efficient units is constituted with Pareto frontier. The classical DEA model (CCR) determines the in/efficient units for ranking alternatives. The DM can compute and determine the DMUs score for each unit's valuable or not. The efficiency or inefficiency score of the DMU's misguide the DM to eliminate alternatives according to the criteria in different situations. Köksalan and Tuncer (2009) illustrate the frontier and DMUs of a set graphically, using a small size data set example, which has 12 DMUs, one input and two outputs. The values of input and outputs are shown in Table 2.1.

Table 2.1 Inputs/outputs values for first example set
(Source: Köksalan and Tuncer, 2009)

DMU #	INPUT	OUTPUT1	OUTPUT2
1	1	5	18
2	1	7	18
3	1	5	17
4	1	7	17
5	1	9	17
6	1	7	16
7	1	5	15
8	1	20	13
9	1	6	10
10	1	13	10
11	1	10	6
12	1	3	4

Figure 2.1 Graphical representation of the first example
(Source : Köksalan and Tuncer, 2009)



Köksalan and Tuncer (2009) show the output 1 in the x-axis and output 2 in the y-axis. The frontier is formed by 1, 2 and 8 which has efficiency score of 1. We compute the efficiency scores of the DMU's using GAMS and show the results in Table 2.2.

Table 2.2 Efficiency scores for the first set.

# of DMUs	Efficiency Score
1	1
2	1
3	0.9444
4	0.9516
5	0.9888
6	0.9033
7	0.8333
8	1
9	0.5947
10	0.7249
11	0.5000
12	0.2490

Köksalan and Tuncer (2009) mention the ranking methods briefly. If we remove any DMUs from the set, the efficiency score of DMUs will change. If DMU 8 is removed from the set, the efficiency scores of several DMUs will increase, moreover some inefficient DMUs will become efficient.

Sinuary-Stern (1994) offer a measure to rank inefficient DMUs which counts the minimum number of inefficient units that should leave the set for each DMU in the set to become efficient. Köksalan and Tuncer (2009) propose a model uses the efficiency values for each number of DMUs (0 to $n-1$) leaving the reference set.

Below we present the model of Sinuary-Stern (1994)

objective function :

$$D_k = \text{Min } Z = \sum_{t=1}^n N_t$$

subject to :

$$\sum_{j=1}^m V_j X_{jk} = 1$$

$$\sum_{i=1}^s U_i Y_{ik} = 1$$

$$\sum_{i=1}^s U_i Y_{it} \leq \sum_{j=1}^m V_j X_{jt} + N_t * M$$

for all $t \neq k$

$$N_t = \begin{cases} 1, & \text{if DMU } t \text{ is deleted} \\ 0, & \text{otherwise} \end{cases}$$

$$U_i, V_j \geq 0, \quad i = 1, \dots, s; \quad j = 1, \dots, m$$

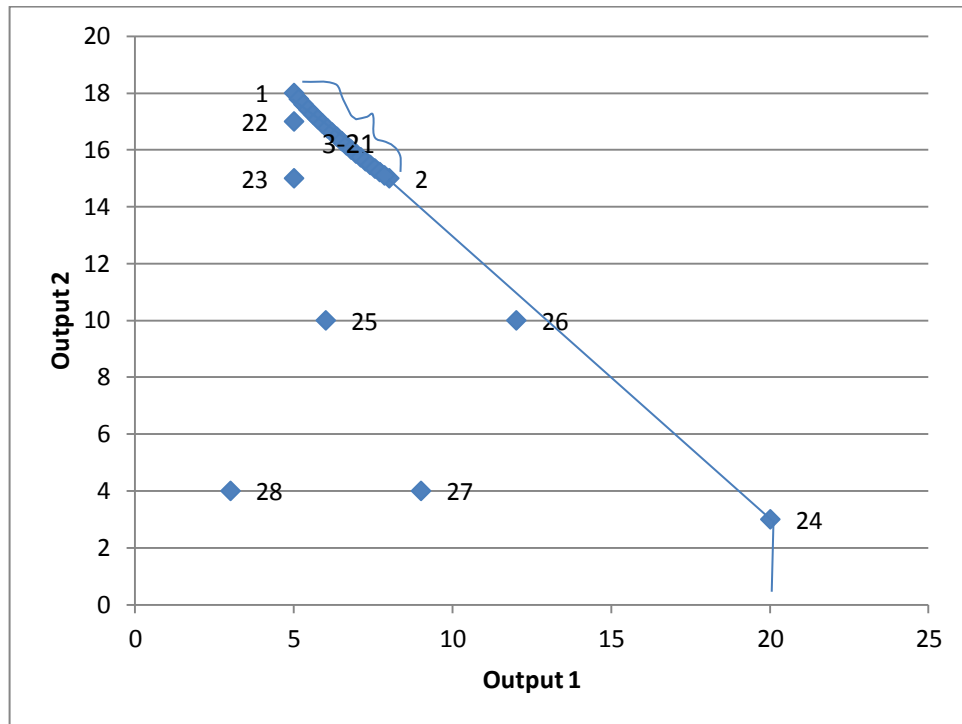
The objective function minimizes the numbers of DMUs that should be removed from the set so that the DMU under consideration (DMU k) becomes efficient. M is a sufficiently large constant that allows the weighted outputs of DMU t to be not larger than that of inputs if DMU t is not removed. On the other hand, if DMU t is removed from the set, the constraint becomes redundant.

Köksalan and Tuncer (2009) discuss that the model may have some problems in some special cases. They provide an example, of which the values of inputs/outputs are shown in Table 2.3, and the graphical representation in Figure 2.2.

Table 2.3 Input/Outputs Values, Efficiency scores and D_k for the second example
(Source: Köksalan and Tuncer, 2009).

# of DMUs	INPUT	OUTPUT1	OUTPUT2	Efficiency Score	D_k
1	1	5.00	18.00	1.0000	0
2	1	8.00	15.00	1.0000	0
3	1	5.15	17.79	0.9974	1
4	1	5.30	17.60	0.9957	2
5	1	5.45	17.42	0.9943	3
6	1	5.60	17.25	0.9935	4
7	1	5.75	17.08	0.9926	5
8	1	5.90	16.92	0.9922	6
9	1	6.05	16.76	0.9917	7
10	1	6.20	16.61	0.9917	8
11	1	6.35	16.46	0.9917	9
12	1	6.50	16.31	0.9917	10
13	1	6.65	16.16	0.9917	10
14	1	6.80	16.01	0.9917	9
15	1	6.95	15.87	0.9922	8
16	1	7.10	15.74	0.9930	7
17	1	7.25	15.60	0.9935	6
18	1	7.40	15.47	0.9943	5
19	1	7.55	15.34	0.9952	4
20	1	7.70	15.22	0.9965	3
21	1	7.85	15.10	0.9978	2
22	1	5.00	17.00	0.9565	6
23	1	5.00	15.00	0.8696	21
24	1	20.00	3.00	1.0000	0
25	1	6.00	10.00	0.6957	17
26	1	12.00	10.00	0.9565	1
27	1	9.00	4.00	0.5652	2
28	1	3.00	4.00	0.3043	25

Figure 2.2 Graphical Representation of Second Example
(Source: Köksalan and Tuncer, 2009)



Köksalan and Tuncer (2009) show the output 1 in the x-axis and output 2 in the y-axis in Figure 2.2. The frontier is formed by DMUs 1, 2 and 24, which has the efficiency score of 1. According to the graphic, DMUs 3-21 are located close to each other because of their output values. This is a very extreme situation for DM to be ranking/sorting DMUs in such a crowded region, because of problematic results. When values of input and outputs of DMU's are close to each alternative, it is not sufficient for DM for ranking/sorting or eliminating DMUs for a decision.

2.4 Other Methods

We can see many DEA based ranking methods in the literature. Adler, Friman and Sinuary-Stern (2002), Büyükbaşaran (2005) mention the detailed literature review of ranking methods. Many sorting/ranking methods are proposed to be used with DEA for the Decision Maker (DM) including efficiency method, cross efficiency method, super efficiency method, multivariate statistical techniques, and multiple criteria decision concepts.

Classical DEA methods compute the efficiency of the DMUs. However, when the aim is to rank these DMUs, the efficiency values may not be enough since there may be several efficient DMUs. In order to increase the differentiation among DMUs (especially among efficient ones), researchers propose different approaches. Super efficiency and cross efficiency are two such approaches.

In classical DEA, the efficiency scores of all DMUs are bounded by 100%. However, in super efficiency approach, the DMU under consideration is not used in efficient frontier construction. Hence it may be efficient more than 100% if it does not lie inside the efficient frontier. We refer to Zhu (2001) for a detailed discussion of super efficiency.

The next approach is cross efficiency. In classical DEA, each DMU selects the best input and output weights in order to maximize its efficiency and the rankings are based on these efficiency scores. The cross efficiency approach, on the other hand, uses the efficiency scores of DMUs when they are not under consideration. For each DMU, a model is solved and the efficiency scores of all DMUs (including that for the DMU under consideration) are recorded, hence for each DMU we collect n efficiency scores; one score when it is under consideration, and $n-1$ efficiency scores when other DMUs are under consideration. A function (for example average) of these n scores is used to compute the cross efficiency score of the DMU. We refer to Doyle and Green (1994) for a review of cross efficiency, including several derivations.

CHAPTER 3

DEA BASED APPROACH TO RANKING MULTI-CRITERIA ALTERNATIVES

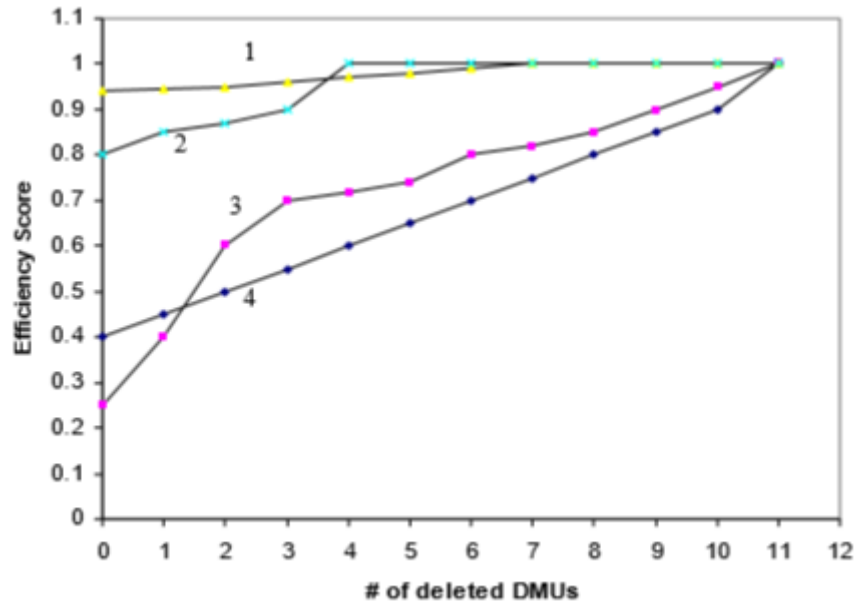
3.1 Motivation of Method

Köksalan and Tuncer (2009) develop a CCR based model Area of Efficiency Score (AES) for ranking alternatives. A detailed analysis of the method can be found in Tuncer (2006). It is a nonparametric method and measures the efficiency of units. We call this model proposed by Köksalan and Tuncer (2009) the KT model referring to the initials of the authors.

3.2. Area of the Efficiency Score

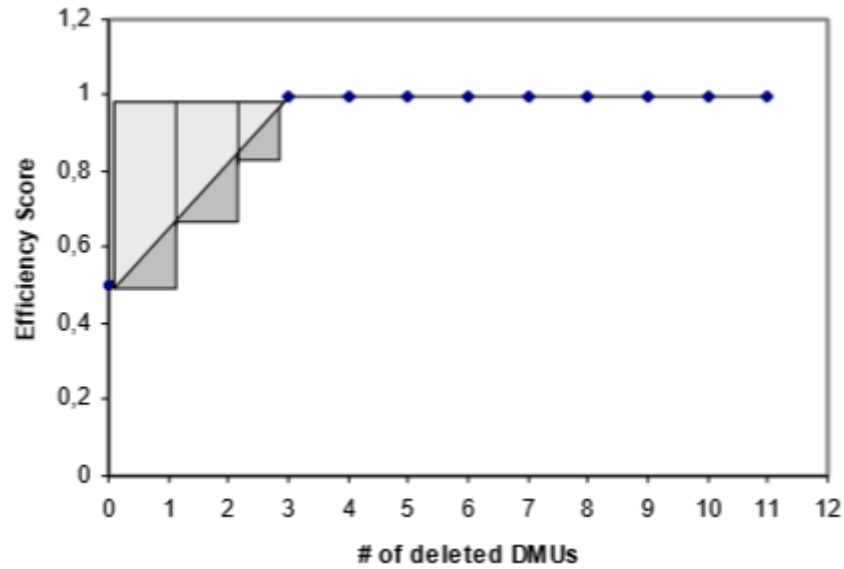
Köksalan and Tuncer (2009) use the area under the efficiency score curve. This curve is generated for each DMU separately. The curve shows the efficiency score of the DMU for each number of DMUs deleted from the DMU set. The efficient DMUs start at efficiency level while the inefficient DMUs start at their' beginning scores. The inefficient DMUs make their way up to (100% efficiency) from their initial efficiency scores by removing DMUs. This technique is called the Area of the Efficiency Score (AES) by Köksalan and Tuncer (2009). They illustrate this with four inefficient DMUs in Figure 3.1. They show the efficiency score of DMUs in the y-axis and number of deleted DMUs in the x-axis.

Figure 3.1 Efficiency score- # of deleted DMUs
 (Source : Köksalan and Tuncer, 2009)



This graph illustrates how the efficiency scores of DMUs change step by step as the number of deleted DMUs increase. DMU 1 starts with a high efficiency score but it can only reach the 1- level (100% efficient level) when 7 DMUs are deleted. DMU 2 reaches the efficiency line after deleting 4 DMUs. Although DEA score of DMU 3 is lower than that of DMU 4, DMU 3 improves quickly with the number of two deleted DMUs. The relationship between DMU 3 and DMU 4 shows the superiority of DMU 3 until the efficiency line. Köksalan and Tuncer (2009) investigate the area which is under the score curve for each DMUs. They offer two ways to measure the area. The first one is to measure the area between the score curves and the efficiency level line. The second approach measures the areas in discrete histogram-like bars. In Figure 3.2, the two approaches are shown, efficiency score of DMUs in y-axis and number of deleted DMUs in the x-axis.

Figure 3.2 The areas according to two measurement approaches
(Source: Köksalan and Tuncer, 2009)



The calculations of area of efficiency method is showed as a mathematically below. They proposed which they used second approach because of discrete measurement.

E_{kj} = the efficiency score of DMU k with deletion of j DMUs, $j = 0, \dots, D_k$

A_{kl} = area for DMU k computed with approach $l = 1, 2$

$$A_{k1} = \sum_{j=0}^{D_k-1} (1 - (E_{kj} + E_{kj+1})/2)/n - 1 \quad \text{is showed as a triangle in}$$

Figure 3.2. Since we compute the average of preceding E_{kj} pairs, there are $n-1$ terms in the above summation. By dividing this term to $n-1$, we compute the average area under the efficiency scores curve.

$$A_{k2} = \sum_{j=0}^{D_k} (1 - E_{kj}) /n \quad \text{is showed three bars in Figure 3.2}$$

They also offered third approach the efficiency measurement instead of inefficiency. The model is showed below.

$$A_{k3} = \sum_{j=0}^{n-1} E_{kj} / n$$

For instance, Table 3.1 show efficiency score of DMU k for each number of DMUs deleted from the five DMUs set.

Table 3.1 Efficiency scores of DMU k.

# of Deleted DMUs	Efficiency Score of DMU k
0	0.7
1	0.8
2	1

$$A_{k1} = \sum_{j=0}^{Dk-1} (1 - (E_{kj} + E_{kj+1}) / 2) / 4 = (1 - (0.7+0.8)/2) = 0.25$$

$$= (1 - (0.8+1) / 2) = 0.10$$

$$A_{k1} = 0.0875$$

$$A_{k2} = \sum_{j=0}^{Dk} (1 - E_{kj}) / 5 = (0.3+0.2+0+0+0) = 0.50/5 \quad A_{k2} = 0.10$$

$$A_{k3} = \sum_{j=0}^{n-1} E_{kj} / 5 = 0.7 + 0.8 + 1 + 1 + 1 = 4.5/5 \quad A_{k3} = 0.90$$

As is seen, A_{k1} and A_{k2} methods measure the area above the efficiency score curves of the DMUs. The third approach measures the area efficiency score with each deletion. Koksalan and Tuncer (2009) use the second method for the area of efficiency score.

3.3. Mathematical Model (KT)

This model is solved for each DMU k separately. We first introduce its indices, parameters and decision variables below;

Model KT(k):

Indices :

n = the number of DMUs in the process,

m = the number of inputs considered,

s = the number of outputs considered,

Parameters :

Y_{ik} = the value of output i for DMU k ($i=1, \dots, s$; $k = 1, \dots, n$)

X_{jk} = the value of input j for DMU k ($j=1, \dots, m$; $k = 1, \dots, n$)

M = A large number

Decision Variables :

U_i = the weight for output i ($i = 1, \dots, s$)

V_j = the weight for input j ($j = 1, \dots, m$)

$$N_t = \begin{cases} 1, & \text{if DMU } t \text{ is deleted} \\ 0, & \text{otherwise} \end{cases}$$

objective function :

$$\text{Max } Z = \sum_{i=1}^s U_i Y_{ik}$$

This model maximizes the efficiency of DMU by limiting the number of deleted DMUs by consecutive numbers between zero and D_k .

subject to :

$$\sum_{j=1}^m V_j X_{jk} = 1$$

This constraint sets the input value of DMU k as 1.

$$\sum_{i=1}^s U_i Y_{it} \leq \sum_{j=1}^m V_j X_{jt} + N_t * M$$

for all t ≠ k

The constraint ensures that the total output value of DMU t is less than or equal to its input value. However, if DMU t is deleted, then this constraint becomes redundant due to the M value on the right hand side.

$$\sum_{i=1}^s U_i Y_{ik} \leq 1$$

The total output value of DMU k (the efficiency of DMU k) should be at most 1.

$$\sum_{t=1}^n N_t \leq R_k$$

The above constraint ensures that at most R_k DMUs are deleted.

$$U_i, V_j \geq 0, \quad i = 1, \dots, s; \quad j = 1, \dots, m$$

Consequently the model gives the highest AES score to DMU_k when DM decide the how many DMUs are allowed to be removed as referred R_k .

3.4 KT Model Algorithm

Here we present the algorithm of Köksalan and Tuncer (2009). This algorithm considers each DMU separately, changes R_k values systematically and reports the AES at the end.

For each DMU k, the algorithm sets the R_k value as 0 and solves KT model. If the optimal objective function value is 1 (the DMU is efficient) then the algorithm computes the AES value and proceeds to next DMU. Otherwise, R_k value is increased by one until DMU k becomes efficient. The AES value is initially 0 for each DMU and it is increased by Z until DMU is efficient. Then AES value is

increased by $(n-R_k-1)$ units . Note that $(n-R_k-1)$ is the remaining number DMUs and the current DMU will be efficient if we delete these DMUs. Finally AES value is divided by n and we obtain a final score between 0 and 1.

KT model has n binary variables. The above algorithm requires to solve the KT model for at least n times one for each DMU. In case of inefficient DMUs, total number of models solved increases.

For each DMU k

Set $AES_k=0, R_k=0$

Solve KT model and let the optimal objective function value Z

Set $AES_k = Z$

While $Z < 1$

Set $R_k = R_k + 1$

Solve KT model and let the optimal objective function value Z

Set $AES_k = AES_k + Z$

End of while

Set $AES_k = AES_k + (n-R_k-1)$

Set $AES_k = AES_k / n$

next k

report AES and R vectors

CHAPTER 4

RANKING ALTERNATIVES USING DATA ENVELOPMENT ANALYSIS: A DUAL APPROACH

We develop a ranking method based on the CCR dual model, which is similar to the KT model. Originally, the dual technique yields the same efficiency score as the primal model. It provides another way of looking at the same problem. The dual model tries to construct a hypothetical composite unit out of the existing units which may outperform the given unit efficient or not. We refer to this method which is proposed by us, OY model.

4.1. Mathematical Model (OY)

Indices, parameters and the decision variables are mentioned at the second chapter. We add just one more constraint to this dual CCR to complete the same task KT model does. We know θ that represents the efficiency score of unit, changes from 0 to 1 ($0 < \theta \leq 1$). And λ_k represents the dual variables which identify benchmarks for inefficient units.

objective function :

$\min \theta$

subject to :

$$\sum_{k=1}^K \lambda_k X_{jk} \leq \theta X_{j0} \quad \forall j$$

$$\sum_{k=1}^K \lambda_k Y_{ik} \geq Y_{i0} \quad \forall i$$

$$\lambda_k \geq 0$$

$V_j : j = 1, \dots, m$ (for each input j)

$U_i : i = 1, \dots, s$ (for each output i)

We add just one constraint CCR dual model to generate our model that handles the removed DMUs.

$$\sum_{k=1}^n W_k \lambda_k = 0$$

W_k is a parameter for the model and initially $W_k = 0$ for all n . When DMU k is removed by the model, we update the $W_k = 1$ hence forcing the corresponding $\lambda_k = 0$ since λ_k are nonnegative decision variables.

4.2 OY Model Algorithm

We provide our algorithm below. This algorithm computes the OY score for each DMU. This algorithm uses OY model, which is a linear programming model.

The number of basic variables (that can be nonzero) of a linear programming model is equal to the number of constraints. In the OY model, there are $(I+J+1)$ constraints and at most $(I+J+1)$ out of n λ_k values can be positive at an optimal solution.

Consider an inefficient DMU 0 and let DMUs 1 and 2 are reference for this DMU, ie λ_1 and λ_2 are positive and other λ_k values are 0. In this case, DMU 0's efficiency value may increase only if DMU 1 or DMU 2 are removed from them DMU set. Removing any other DMU will not affect the efficiency of DMU 0. Based on this observation, we consider the reference DMUs and find the best reference DMU such that its removal will increase the efficiency score of DMU 0 most.

OY algorithm runs for each DMU k. If DMU k is efficient then it computes the OY_k score and proceeds to the next DMU. If it is inefficient, then it considers each reference DMU j with $\lambda_j > 0$. Temporarily sets w_j values to 1, solves OY model and sets w_j to 0 again. We identify the best DMU to be removed with this strategy. The removal of DMUs continue until DMU k becomes efficient.

For each DMU k

Set $OY_k=0$, $R_k=0$ and $w_j=0$ for all j

Solve OY model and let the optimal objective function value be Z

Set $OY_k = OY_k + Z$

while $Z < 1$ then

Solve OY model and let the optimal λ_j values be λ_j^*

Set $Z^* = 0$

For each reference DMU j (such that $\lambda_j^* > 0$)

Set $w_j = 1$

Solve OY model and let Z^* be the optimal obj. func. value

If $Z > Z^*$

Set $Z^* = Z$ and $j^*=j$

end

Set $w_j = 0$

next j

$OY_k = OY_k + Z^*$

Set $w_{j^*} = 1$

Set $R_k=R_k+1$

end (of while)

Set $OY_k = OY_k + (n-R_k-1)$

Set $OY_k = OY_k / n$

next k

report OY and R vectors

Note that this algorithm is myopic and detects the best DMU to be removed iteratively. Moreover, for a given R_k value, it sticks to the previous removal decisions and finds the DMU to be removed given the previous R_{k-1} removals. However, KT model considers the best subset (with cardinality of R_k) of DMUs to be removed.

Obviously, the removal decisions of KT and OY models differ. The AES and OY scores and total number of DMUs to be deleted may vary between the two models. We compute the correlations between these values in the next chapter.

CHAPTER 5

EXPERIMENTS

In this chapter, we present the computational experiments made for both OY and KT models. The differences of between the models are showed in this section.

We performed the computational experiments on computer with Intel R Xenon X5482 @ 3.20 GHz 2 processors, 10 GB RAM and Windows Server 2008 R2 Enterprise-64 Bit. We solve the mathematical models using GAMS 22.8.

5.1 Random Data Generation

We also benefit from data generation the following model, which is proposed by Özpeynirci and Köksalan (2007). The model is producing some random outputs with random weighted inputs for seeing differences between the methods.

$$Y_{rj} = H_j * \left(\sum_{i=1}^m W_{ir} * X_{ij} \right) + e_{rj}$$

W_{ir} = the weight of input i consumed for output r

e_{rj} = error term on output r of DMU j

H_j = the efficiency value of DMU j

X_{ij} = the amount of input i consumed by DMU j

Y_{rj} = the amount of output r produced by DMU j

$X_{ij} \sim \text{uniform}(10, 50)$

$W_{ir} \sim \text{uniform}(0.10, 0.50)$ and $\sum (W_{ir}) = 1$

$e_{rj} \sim \text{uniform}(0, 20)$

$H_j \sim \text{uniform}(60\%, 100\%)$

We can generate multiple output/input data set using this model for evaluating performance of DMUs. We produce data set which has 250 DMUs, 2 inputs and 6 outputs using this model, that is mentioned above. This set is used for evaluating the performance of the two different models. In Table 5.1, the results are shown only for 63 DMUs because of total time of KT model. Although our method found all the efficiency scores of DMUs in 1 hour 17 min, in contrast, the KT model took more than 50 hours to get same information. KT model finds the efficiency scores of only the first 63 DMUs at the end of a 50 hours run time. We present the results of the different methods for comparison two methods using data generation. We provide the details of Table 5.1 in Appendix A. Note that CPU and Total Time are given as in seconds.

Table 5.1 Comparison of between OY and KT Models with R_0 , EFF_0 , CPU_0 , $Duration TIME_0$ with using Data Generation

DMU	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
1	1	0	0.007	0.48	1	0	0.105	0.4	0	0.000000
2	1	0	0.015	0.58	1	0	0.164	0.5	0	0.000000
3	0.999764	4	0.751	2.34	0.999764	4	0.736	1.44	0	0.000000
4	0.861277	162	23.235	48.57	0.87837	156	16529.29	16543.09	6	-0.017093
5	0.986564	32	27.67	57.59	0.98743	32	16538.32	16554.29	0	-0.000866
6	0.993149	44	35.356	73.13	0.99424	38	16619.37	16638.02	6	-0.001091
7	0.958575	108	50.853	104.47	0.961286	102	24587.08	24613.29	6	-0.002711
8	0.878762	184	80.101	163.34	0.893367	147	48042.12	48080.14	37	-0.014605
9	0.998061	14	81.888	166.94	0.998146	13	48044.3	48083.19	1	-0.000085
10	0.947778	107	94.239	191.98	0.953126	97	49383.16	49430.21	10	-0.005348
11	0.981087	37	99.574	202.41	0.983066	33	49390.46	49439.94	4	-0.001979
12	0.984608	60	110.072	223.64	0.986621	58	50442.65	50496.57	2	-0.002013
13	0.993308	24	114.338	232.14	0.994079	22	50451.68	50507.26	2	-0.000771
14	0.996631	33	122.061	247.77	0.997118	27	50518.7	50576.54	6	-0.000487
15	0.99716	19	125.271	254.18	0.997557	16	50523.18	50582.48	3	-0.000397
16	1	0	125.314	254.28	1	0	50523.23	50582.59	0	0.000000
17	0.932982	130	150.83	305.79	0.944223	96	52786.13	52852.8	34	-0.011241
18	0.996288	24	154.726	313.59	0.997068	20	52790.7	52859.08	4	-0.000780
19	0.984088	35	159.047	322.2	0.986037	32	52799.81	52870.48	3	-0.001949
20	0.970658	61	168.05	340.43	0.975551	51	52831.84	52906.33	10	-0.004893
21	0.999735	5	168.754	341.83	0.999765	5	52832.7	52907.55	0	-0.000030

DMU	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
22	0.990573	19	171.397	347.33	0.991088	17	52835.23	52911.11	2	-0.000515
23	0.95134	76	182.17	369.47	0.955652	73	52965.7	53046.97	3	-0.004312
24	0.944513	119	195.424	396.40	0.94958	105	56751.26	56840.48	14	-0.005067
25	0.914119	119	210.943	427.52	0.921669	116	57690.69	57788.81	3	-0.007550
26	0.941818	123	231.714	469.7	0.948426	102	61766.36	61872.59	21	-0.006608
27	0.856687	203	256.86	520.73	0.872245	156	108044.9	108165.5	47	-0.015558
28	0.999761	4	257.71	522.44	0.999761	4	108045.6	108166.5	0	0.000000
29	0.999467	7	258.7	524.44	0.999467	7	108046.7	108168.2	0	0.000000
30	0.999626	4	259.279	525.64	0.999626	4	108047.4	108169.1	0	0.000000
31	0.981389	65	266.917	541.05	0.982448	62	108349.4	108476.2	3	-0.001059
32	0.989139	49	275.829	558.77	0.991273	39	108456.6	108586.3	10	-0.002134
33	0.974169	44	281.858	571.19	0.975283	43	108471.4	108604.1	1	-0.001114
34	0.839711	179	309.922	628.46	0.852353	177	132692.2	132841.1	2	-0.012642
35	0.930042	136	332.412	674.76	0.937163	114	146987.8	147145.9	22	-0.007121
36	0.928549	142	353.925	718.72	0.932467	133	169458.1	169626.8	9	-0.003918
37	0.999726	5	354.96	720.73	0.999726	5	169458.9	169628	0	0.000000
38	0.999039	15	357.184	725.43	0.999162	11	169461.2	169631	4	-0.000123
39	0.901807	113	374.49	760.9	0.913648	106	169713.7	169891.7	7	-0.011841
40	0.979057	66	383.054	778.02	0.982118	60	169973.7	170156.4	6	-0.003061
41	0.996021	29	388.055	788.03	0.996943	24	169988.1	170172.4	5	-0.000922
42	1	0	388.101	788.12	1	0	169988.1	170172.5	0	0.000000
43	0.912783	139	413.959	840.6	0.935093	98	170489.5	170681.5	41	-0.022310
44	0.992423	13	415.458	843.6	0.993061	13	170491.6	170684.4	0	-0.000638

DMU	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
45	0.872303	130	431.132	875.14	0.895833	117	170963.2	171165	13	-0.023530
46	0.986534	61	440.551	894.45	0.988809	51	171259.2	171464.7	10	-0.002275
47	0.980521	55	448.278	910.45	0.982997	52	171324.9	171534.7	3	-0.002476
48	0.960871	70	460.193	935.11	0.964834	57	171369.8	171584.2	13	-0.003963
49	0.975413	37	464.818	944.33	0.976778	37	171377.8	171595.1	0	-0.001365
50	0.901363	105	478.868	972.75	0.919213	89	171472	171696.3	16	-0.017850
51	0.959136	60	487	989.95	0.962488	55	171502	171730.8	5	-0.003352
52	0.988997	38	493.184	1002.67	0.990443	31	171519.5	171750.6	7	-0.001446
53	0.996917	26	497.305	1011.08	0.997291	22	171529.3	171761.9	4	-0.000374
54	0.96815	56	503.692	1024.29	0.972621	47	171546.9	171783	9	-0.004471
55	0.998947	6	504.147	1025.19	0.998947	6	171547.4	171784.2	0	0.000000
56	0.982897	25	507.02	1031	0.983069	25	171551.2	171789.9	0	-0.000172
57	0.952961	75	514.733	1046.51	0.962186	54	171574.2	171817.2	21	-0.009225
58	0.979836	41	521.243	1059.54	0.980925	39	171590.3	171836.3	2	-0.001089
59	0.99971	8	522.657	1062.35	0.999716	7	171591.6	171838.1	1	-0.000006
60	0.948718	88	535.22	1088.18	0.954973	82	172447.9	172700.4	6	-0.006255
61	0.988085	54	545.476	1109.11	0.98961	51	172934.6	173191.3	3	-0.001525
62	0.986128	29	549.504	1117.32	0.988492	26	172941	173199.6	3	-0.002364
63	0.98489	34	555.296	1128.52	0.987541	29	172949.9	173210.9	5	-0.002651

When we compare the two models which have 250 DMUs, 2 inputs and 6 outputs, we make the following observations:

1. CPU Time of OY model is much lower than that of KT model.
2. Duration Time of OY model is much lower than that of KT model.
3. DMU that is under consideration, $EFF_{OY} \leq EFF_{KT}$.
4. DMU that is under consideration, $R_{OY} \geq R_{KT}$.

We can observe the differences between $R_{OY} - R_{KT}$ and $EFF_{OY} - EFF_{KT}$ under consideration DMUs which the models make the same tasks. And we demonstrate our methods performance with examples.

We also make a correlation analysis between R_{OY} and R_{KT} values only for 63 DMUs. The correlation value is 0.988417. Number of DMUs deleted by OY model and KT model presented by graphically in Figure 5.1.

The correlation of efficiency scores (EFF_{OY} and EFF_{KT}) is 0.997647. The result of positive correlation analysis confirms that the results of our method is very similar to that of KT. The ratio of EFF_{OY} to EFF_{KT} is found nearly one, and we present it graphically in Figure 5.2.

In a detailed comparison between methods, each of DMUs choose their weights to become efficient in both models. DMUs which is are under consideration try to virtually delete. Consequently, KT model considers each DMU separately, needs n binary decision variables and searches all possible removal decisions for each DMU from the set to remove best R DMUs. This procedure continues until DMU becomes efficient. According to our method, OY model runs for each DMU and continues to compute efficiency score of DMUs with deleted DMUs (previously removed $R-1$ DMUs). The KT model aims is to remove best R DMUs, in contrast, our myopic model that does not need any binary or integer decision variables, removes the best DMU given the previously removed $R-1$ DMUs. This is the reason that the two methods are separated from each other. Although, the removal decisions are different in both models, our method finishes the process earlier for large number of DMUs.

Figure 5.1 Number of DMUs deleted by OY model and KT model

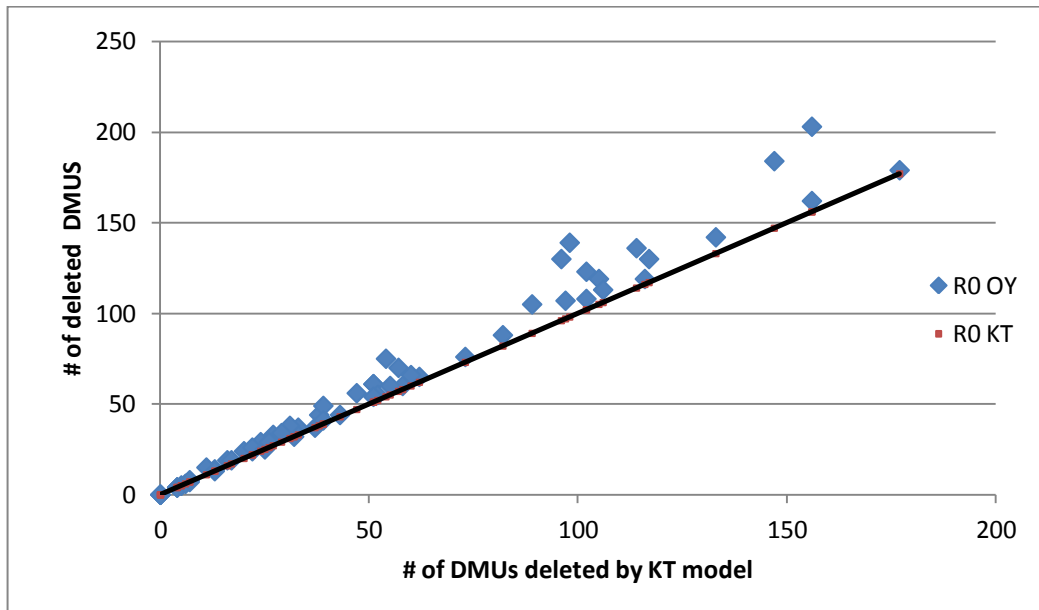
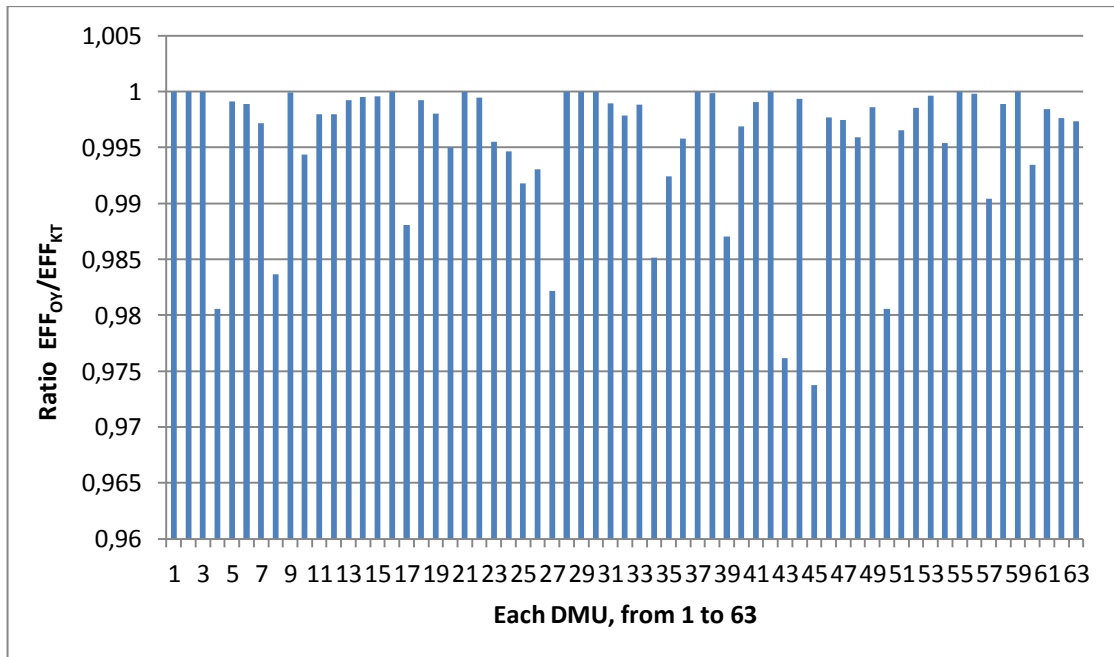


Figure 5.2 The Ratio of EFF_{OY} to EFF_{KT}



5.2 Logistics Performance Index Application

In this section, we apply KT and OY methods on Logistics Performance Index data set. Logistics Performance Index (LPI) is defined as "an interactive benchmarking tool created to help countries identify the challenges and opportunities they face in their performance on trade logistics and what they can do to improve their performance." The LPI data set for 2014 (LPI 2014) allows comparisons across 160 countries, calculated from two different perspectives: international and domestic on the six dimensions; the efficiency of customs and border management clearance "Customs", the quality of trade and transport infrastructure "Infrastructure", the ease of arranging competitively priced shipments (Shipments), the competence and quality of logistics services-trucking, forwarding, and customs brokerage (Quality of logistics services), the ability to track and trace consignments (Tracking and tracing), the frequency with which shipments reach consignees within scheduled or expected delivery times (Timeliness). We use the six dimensions above as outputs of the countries and one constant input which is value 1 in our method and compare the results with KT method.

The data which is presented in Table B.1 in Appendix B, is received from LPI for the comparison of two approaches.

We make a test for ranking countries as a logistics application. The results are in the Table C.1 in Appendix C.

Correlation Analysis is utilized efficiency score of between EFF_{OY} and EFF_{KT} models, the result found as 0.999778. The result of positive correlation analysis also confirms the sensitivity to another method. The ratio of EFF_{OY} to EFF_{KT} is analyzed and found to be nearly one in this application, that is presented by graphically in Figure 5.3.

On the other hand, we also make an correlation analysis between R_{OY} and R_{KT} for our application, which was found to be 0.999317. The number of DMUs deleted by OY model and KT model presented by graphically for LPI data set in Figure 5.4.

Figure 5.3 The Ratio of EFF_{OY} to EFF_{KT}

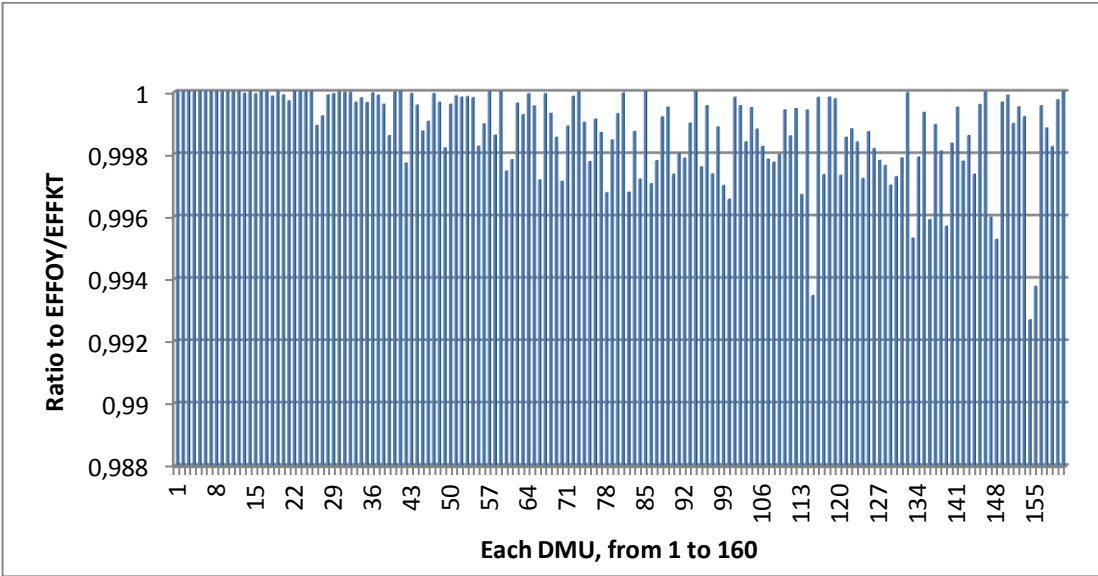
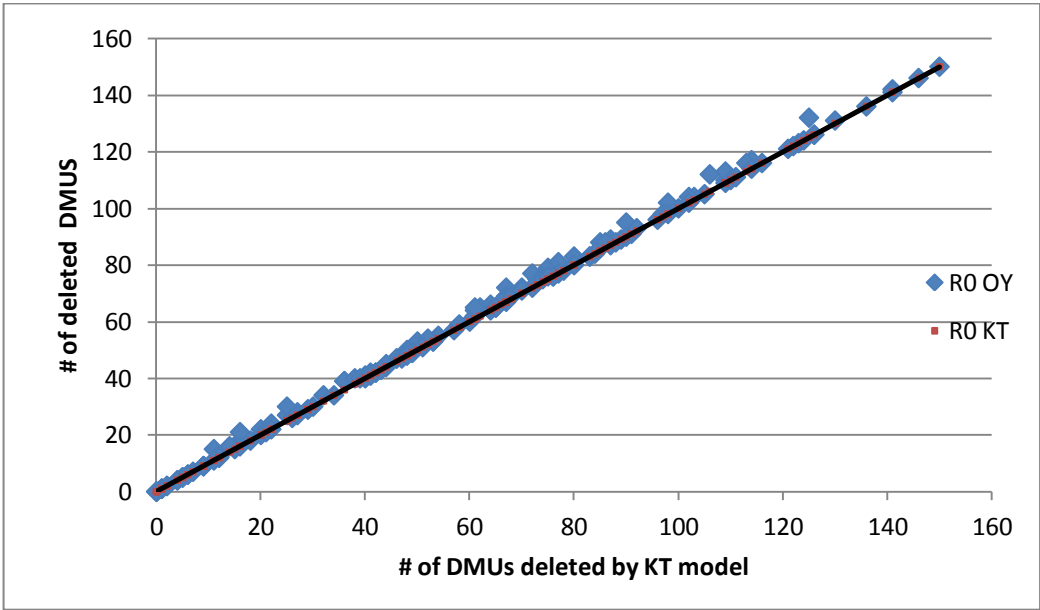


Figure 5.4 Number of DMUs deleted by OY model and KT model



5.3 Observations

In this section, we report our observations of the performances of the models on randomly generated and LPI data sets.

Table 5.2. Total Time (second) of Models related to # of DMUs, # of Inputs/Outputs

# of DMUs	2 inputs 3 outputs		2 inputs 6 outputs	
	OY Model	KT model	OY Model	KT model
25	34.54	26.69	35.13	25.84
50	146.79	129.54	161.68	116.82
75	327.17	333.82	333.57	345.55
100	602.87	929.94	684.66	993.87
150	1405.16	7304.9	1579	9857
250	4116.25	----	4657.5	----

The dashed line in Table 5.2 shows that the KT model could not find any solution in time, when we increase the number of DMUs, specifically 250 DMUs. We present precise total time measurement performance of models in Table 5.3.

Table 5.3 Total Time of the models related to numbers of DMUs and # of Inputs/Outputs

# of inputs and outputs	# of DMUs (by OY Method)			# of DMUs (by KT Method)		
	100	160 (LPI)	250	100	160 (LPI)	250
1 input+ 2 output	541.13	1491.44	3797.03	1152.72	2881.79	8890.09
1 input+ 4 output)	660.26	1713.08	4392.32	1048.51	2860.49	12482.24
1 input+ 6 output	719.13	1905.09	5001.68	1040.08	3122.10	11596.98

Table 5.4. Correlation Analysis between CPU_{OY} and CPU_{KT} , EFF_{OY} and EFF_{KT}

# of DMUs	CPU_{OY}	CPU_{KT}	CPU_0 Correlation Analysis	EFF_0 Correlation Analysis
25	14.578	15.185	0.996423	0.998249
50	87.874	74.863	0.999417	0.998515
75	178.734	261.500	0.995705	0.997141
100	350.310	836.230	0.993853	0.997684
150	870.135	9506.865	0.987685	0.997130
250	2244.771	---	----	----

On the other hand, we can be aware of differences between CPU_{OY} and CPU_{KT} in Table 5.4. From 25 DMUs to 150 DMUs, the degree of positive correlation decreases. In any case, the correlation of efficiency scores are very close and using OY method seems to be a reasonable sacrifice from the optimality in order to obtain a good result in less computation time.

If DM has many alternatives for ranking, the dual approach save time in the computational experiments. If the DM has a large set as numbers of alternatives/units, the dual form is more efficient computation compared to the primal model KT. This is the reason that the primal form KT model changes related to the number of DMUs when the OY model entails the number of inputs/outputs values.

Table 5.5 Average of R_O the models related to numbers of DMUs and # of Inputs/Outputs

# of inputs and outputs	Average of R_{OY}			Average of R_{KT}		
	100 DMUs	160 DMUs (LPI)	250 DMUs	100 DMUs	160 DMUs (LPI)	250 DMUs
1 input+ 2 output	21.53	71.58	121.62	19.84	71.29	119.59
1 input+ 4 output)	47.04	64.16	118.79	46.94	63.56	118.46
1 input+ 6 output	46.56	59.88	117.70	46.48	58.94	117.30

As is seen, the average of R_{OY} and R_{KT} are very close to each other in the same data set in Table 5.5.

CHAPTER 6

CONCLUSION

Data Envelopment Analysis (DEA) is a non-parametric technique that has been used for several purposes including the ranking alternatives in the literature. DEA is used for assessing the relative performance of units and benchmarking between the independent decision making units as we mentioned before. Especially, DEA does not limit the number of the inputs and outputs and not require to weight restrictions.

In the study, we propose a DEA-based CCR dual approach for ranking alternatives. Our new model considers the efficiency score of alternatives while virtually deleting DMUs in the creation of a ranking list. Mathematically, the dual approach seems more complex, but in fact it is much faster to solve, as it has only as many constraints as there are factors, as we have seen.

We use the data generation model and a data from 2014 published in the Logistics Performance Index (LPI). Using the different set in both models is showed in the computational experiments part. For comparison between KT and OY models, we observe that there are the approximately same results under different assumptions for ranking.

The KT and OY models are differed by the removal decisions. KT model considers all possibilities for each Decision Making Unit separately to remove best R DMUs. On the other hand, our model runs for each DMU to remove throughout previous removed DMUs in the process. Although the KT model is more effective in contrast to OY model, KT method could not give any solution in time for large number of

DMUs. Our OY model that does not need any binary or integer decision variables, review myopically on the DMUs and gives the results early.

In further research, If there would be many criteria with multiple inputs and outputs, KT model is useful for ranking in this situation. In other cases, our method is effective and beneficial for large number of DMUs for Decision Makers.

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APPENDIX A.

Table A.1 Comparison of OY and KT Model with using Data Generation (Continued)

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
1	1	0	0.007	0.48	1	0	0.105	0.4	0	0.000000
2	1	0	0.015	0.58	1	0	0.164	0.5	0	0.000000
3	0.999764	4	0.751	2.34	0.999764	4	0.736	1.44	0	0.000000
4	0.861277	162	23.235	48.57	0.87837	156	16529.29	16543.09	6	-0.017093
5	0.986564	32	27.67	57.59	0.98743	32	16538.32	16554.29	0	-0.000866
6	0.993149	44	35.356	73.13	0.99424	38	16619.37	16638.02	6	-0.001091
7	0.958575	108	50.853	104.47	0.961286	102	24587.08	24613.29	6	-0.002711
8	0.878762	184	80.101	163.34	0.893367	147	48042.12	48080.14	37	-0.014605
9	0.998061	14	81.888	166.94	0.998146	13	48044.3	48083.19	1	-0.000085
10	0.947778	107	94.239	191.98	0.953126	97	49383.16	49430.21	10	-0.005348
11	0.981087	37	99.574	202.41	0.983066	33	49390.46	49439.94	4	-0.001979
12	0.984608	60	110.072	223.64	0.986621	58	50442.65	50496.57	2	-0.002013
13	0.993308	24	114.338	232.14	0.994079	22	50451.68	50507.26	2	-0.000771
14	0.996631	33	122.061	247.77	0.997118	27	50518.7	50576.54	6	-0.000487
15	0.99716	19	125.271	254.18	0.997557	16	50523.18	50582.48	3	-0.000397
16	1	0	125.314	254.28	1	0	50523.23	50582.59	0	0.000000
17	0.932982	130	150.83	305.79	0.944223	96	52786.13	52852.8	34	-0.011241
18	0.996288	24	154.726	313.59	0.997068	20	52790.7	52859.08	4	-0.000780

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
19	0.984088	35	159.047	322.2	0.986037	32	52799.81	52870.48	3	-0.001949
20	0.970658	61	168.05	340.43	0.975551	51	52831.84	52906.33	10	-0.004893
21	0.999735	5	168.754	341.83	0.999765	5	52832.7	52907.55	0	-0.000030
22	0.990573	19	171.397	347.33	0.991088	17	52835.23	52911.11	2	-0.000515
23	0.95134	76	182.17	369.47	0.955652	73	52965.7	53046.97	3	-0.004312
24	0.944513	119	195.424	396.4	0.94958	105	56751.26	56840.48	14	-0.005067
25	0.914119	119	210.943	427.52	0.921669	116	57690.69	57788.81	3	-0.007550
26	0.941818	123	231.714	469.7	0.948426	102	61766.36	61872.59	21	-0.006608
27	0.856687	203	256.86	520.73	0.872245	156	108044.9	108165.5	47	-0.015558
28	0.999761	4	257.71	522.44	0.999761	4	108045.6	108166.5	0	0.000000
29	0.999467	7	258.7	524.44	0.999467	7	108046.7	108168.2	0	0.000000
30	0.999626	4	259.279	525.64	0.999626	4	108047.4	108169.1	0	0.000000
31	0.981389	65	266.917	541.05	0.982448	62	108349.4	108476.2	3	-0.001059
32	0.989139	49	275.829	558.77	0.991273	39	108456.6	108586.3	10	-0.002134
33	0.974169	44	281.858	571.19	0.975283	43	108471.4	108604.1	1	-0.001114
34	0.839711	179	309.922	628.46	0.852353	177	132692.2	132841.1	2	-0.012642
35	0.930042	136	332.412	674.76	0.937163	114	146987.8	147145.9	22	-0.007121
36	0.928549	142	353.925	718.72	0.932467	133	169458.1	169626.8	9	-0.003918
37	0.999726	5	354.96	720.73	0.999726	5	169458.9	169628	0	0.000000
38	0.999039	15	357.184	725.43	0.999162	11	169461.2	169631	4	-0.000123
39	0.901807	113	374.49	760.9	0.913648	106	169713.7	169891.7	7	-0.011841
40	0.979057	66	383.054	778.02	0.982118	60	169973.7	170156.4	6	-0.003061
41	0.996021	29	388.055	788.03	0.996943	24	169988.1	170172.4	5	-0.000922

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
42	1	0	388.101	788.12	1	0	169988.1	170172.5	0	0.000000
43	0.912783	139	413.959	840.6	0.935093	98	170489.5	170681.5	41	-0.022310
44	0.992423	13	415.458	843.6	0.993061	13	170491.6	170684.4	0	-0.000638
45	0.872303	130	431.132	875.14	0.895833	117	170963.2	171165	13	-0.023530
46	0.986534	61	440.551	894.45	0.988809	51	171259.2	171464.7	10	-0.002275
47	0.980521	55	448.278	910.45	0.982997	52	171324.9	171534.7	3	-0.002476
48	0.960871	70	460.193	935.11	0.964834	57	171369.8	171584.2	13	-0.003963
49	0.975413	37	464.818	944.33	0.976778	37	171377.8	171595.1	0	-0.001365
50	0.901363	105	478.868	972.75	0.919213	89	171472	171696.3	16	-0.017850
51	0.959136	60	487	989.95	0.962488	55	171502	171730.8	5	-0.003352
52	0.988997	38	493.184	1002.67	0.990443	31	171519.5	171750.6	7	-0.001446
53	0.996917	26	497.305	1011.08	0.997291	22	171529.3	171761.9	4	-0.000374
54	0.96815	56	503.692	1024.29	0.972621	47	171546.9	171783	9	-0.004471
55	0.998947	6	504.147	1025.19	0.998947	6	171547.4	171784.2	0	0.000000
56	0.982897	25	507.02	1031	0.983069	25	171551.2	171789.9	0	-0.000172
57	0.952961	75	514.733	1046.51	0.962186	54	171574.2	171817.2	21	-0.009225
58	0.979836	41	521.243	1059.54	0.980925	39	171590.3	171836.3	2	-0.001089
59	0.99971	8	522.657	1062.35	0.999716	7	171591.6	171838.1	1	-0.000006
60	0.948718	88	535.22	1088.18	0.954973	82	172447.9	172700.4	6	-0.006255
61	0.988085	54	545.476	1109.11	0.98961	51	172934.6	173191.3	3	-0.001525
62	0.986128	29	549.504	1117.32	0.988492	26	172941	173199.6	3	-0.002364
63	0.98489	34	555.296	1128.52	0.987541	29	172949.9	173210.9	5	-0.002651
64	0.865057	159	577.193	1174.18	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
65	1	0	577.247	1174.29	-	-	-	-	-	-
66	0.963132	129	590.311	1201.31	-	-	-	-	-	-
67	0.954029	78	601.68	1224.72	-	-	-	-	-	-
68	0.994841	24	604.7	1231.13	-	-	-	-	-	-
69	0.993896	30	608.634	1239.23	-	-	-	-	-	-
70	0.963778	60	617.085	1256.44	-	-	-	-	-	-
71	0.997526	17	620.119	1263.05	-	-	-	-	-	-
72	0.990137	15	621.847	1266.55	-	-	-	-	-	-
73	0.982609	24	624.267	1271.55	-	-	-	-	-	-
74	0.950798	62	633.458	1290.57	-	-	-	-	-	-
75	0.980994	60	643.606	1311	-	-	-	-	-	-
76	0.999891	2	644.098	1312.01	-	-	-	-	-	-
77	0.914999	128	656.589	1337.21	-	-	-	-	-	-
78	0.881182	179	683.152	1392.28	-	-	-	-	-	-
79	0.975495	45	688.926	1404.2	-	-	-	-	-	-
80	0.979386	73	702.34	1431.75	-	-	-	-	-	-
81	0.912821	148	727.382	1482.81	-	-	-	-	-	-
82	0.907548	118	744.592	1518.07	-	-	-	-	-	-
83	0.986047	42	751.89	1533.09	-	-	-	-	-	-
84	0.971401	66	760.572	1550.9	-	-	-	-	-	-
85	0.991822	13	761.65	1553.1	-	-	-	-	-	-
86	0.974947	68	774.363	1579.03	-	-	-	-	-	-
87	0.959159	104	792.783	1617.09	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
88	0.977421	82	807.091	1646.43	-	-	-	-	-	-
89	0.985384	28	810.562	1653.53	-	-	-	-	-	-
90	0.986638	19	812.594	1658.03	-	-	-	-	-	-
91	0.999947	1	812.806	1658.44	-	-	-	-	-	-
92	0.995432	28	818.452	1670.05	-	-	-	-	-	-
93	0.994121	40	822.556	1678.67	-	-	-	-	-	-
94	0.998924	13	826.177	1685.57	-	-	-	-	-	-
95	0.990273	33	831.19	1695.8	-	-	-	-	-	-
96	0.983104	57	840.903	1715.82	-	-	-	-	-	-
97	0.980348	64	851.257	1737.04	-	-	-	-	-	-
98	0.912269	155	872.829	1782.27	-	-	-	-	-	-
99	0.98959	48	881.316	1799.42	-	-	-	-	-	-
100	0.988034	55	889.553	1816.95	-	-	-	-	-	-
101	0.960204	99	906.126	1851.58	-	-	-	-	-	-
102	0.928248	118	922.188	1885.12	-	-	-	-	-	-
103	0.945635	108	936.266	1914.15	-	-	-	-	-	-
104	0.888911	138	956.731	1956.74	-	-	-	-	-	-
105	0.876183	188	993.023	2031.7	-	-	-	-	-	-
106	0.994765	40	999.027	2044.23	-	-	-	-	-	-
107	1	0	999.081	2044.33	-	-	-	-	-	-
108	1	0	999.131	2044.43	-	-	-	-	-	-
109	1	0	999.182	2044.53	-	-	-	-	-	-
110	0.871202	165	1029.732	2107.37	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
111	0.986988	36	1034.025	2116.28	-	-	-	-	-	-
112	0.945888	113	1050.478	2150.64	-	-	-	-	-	-
113	0.901535	101	1063.935	2179.06	-	-	-	-	-	-
114	0.936948	87	1074.418	2201.08	-	-	-	-	-	-
115	0.86934	196	1094.261	2241.85	-	-	-	-	-	-
116	0.999897	4	1094.941	2243.25	-	-	-	-	-	-
117	0.950936	109	1111.949	2278.52	-	-	-	-	-	-
118	0.876646	186	1139.78	2337.19	-	-	-	-	-	-
119	0.947506	113	1159.273	2377.04	-	-	-	-	-	-
120	1	0	1159.309	2377.14	-	-	-	-	-	-
121	0.991258	22	1162.917	2384.87	-	-	-	-	-	-
122	0.967565	78	1173.92	2407.41	-	-	-	-	-	-
123	0.953888	58	1180.094	2420.33	-	-	-	-	-	-
124	0.999232	10	1181.987	2424.13	-	-	-	-	-	-
125	0.915601	146	1204.508	2471.09	-	-	-	-	-	-
126	0.911968	158	1252.224	2542.83	-	-	-	-	-	-
127	0.995413	8	1253.074	2544.53	-	-	-	-	-	-
128	0.977737	39	1256.954	2552.44	-	-	-	-	-	-
129	0.958073	87	1270.211	2579.85	-	-	-	-	-	-
130	0.999903	2	1270.509	2580.46	-	-	-	-	-	-
131	0.98825	30	1275.316	2590.27	-	-	-	-	-	-
132	1	0	1275.368	2590.39	-	-	-	-	-	-
133	0.890191	149	1301.497	2644.94	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
134	0.990766	17	1303.654	2649.45	-	-	-	-	-	-
135	0.996266	19	1306.505	2655.47	-	-	-	-	-	-
136	0.987696	47	1315.241	2673.78	-	-	-	-	-	-
137	0.987602	41	1322.218	2688	-	-	-	-	-	-
138	0.998777	16	1324.758	2693.31	-	-	-	-	-	-
139	0.999843	3	1325.249	2694.41	-	-	-	-	-	-
140	0.997614	27	1329.299	2702.61	-	-	-	-	-	-
141	0.916641	99	1346.575	2739.38	-	-	-	-	-	-
142	0.998794	15	1349.957	2746.37	-	-	-	-	-	-
143	0.900023	161	1379.876	2808.25	-	-	-	-	-	-
144	0.995184	24	1383.779	2816.45	-	-	-	-	-	-
145	0.974139	83	1399.342	2848.92	-	-	-	-	-	-
146	0.949488	69	1407.938	2867.06	-	-	-	-	-	-
147	0.985914	22	1410.404	2872.26	-	-	-	-	-	-
148	1	0	1410.449	2872.36	-	-	-	-	-	-
149	0.958691	105	1425.392	2903.61	-	-	-	-	-	-
150	0.947598	79	1434.072	2921.62	-	-	-	-	-	-
151	0.838672	186	1458.823	2974.61	-	-	-	-	-	-
152	0.996878	23	1462.315	2982.12	-	-	-	-	-	-
153	0.917601	113	1474.192	3006.85	-	-	-	-	-	-
154	1	0	1474.235	3006.94	-	-	-	-	-	-
155	0.934613	74	1484.105	3027.66	-	-	-	-	-	-
156	0.999675	6	1485.372	3030.17	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
157	0.967794	91	1501.979	3064.93	-	-	-	-	-	-
158	0.997139	9	1502.546	3066.13	-	-	-	-	-	-
159	1	0	1502.592	3066.22	-	-	-	-	-	-
160	0.998148	15	1504.55	3070.43	-	-	-	-	-	-
161	0.959671	73	1515.536	3093.25	-	-	-	-	-	-
162	0.954737	108	1531.366	3126.81	-	-	-	-	-	-
163	0.861109	173	1553.006	3172.96	-	-	-	-	-	-
164	0.941785	94	1566.948	3202.39	-	-	-	-	-	-
165	0.997583	17	1569.873	3208.92	-	-	-	-	-	-
166	1	0	1569.906	3209.01	-	-	-	-	-	-
167	0.968351	64	1579.17	3228.64	-	-	-	-	-	-
168	0.961415	95	1589.351	3250.47	-	-	-	-	-	-
169	0.864282	193	1611.309	3296.94	-	-	-	-	-	-
170	0.998646	4	1611.756	3297.84	-	-	-	-	-	-
171	0.806721	204	1639.785	3358.1	-	-	-	-	-	-
172	0.997775	8	1640.269	3359.29	-	-	-	-	-	-
173	0.998172	12	1641.936	3362.7	-	-	-	-	-	-
174	0.820052	189	1666.765	3415.87	-	-	-	-	-	-
175	0.983764	42	1673.066	3429.48	-	-	-	-	-	-
176	0.998323	18	1675.579	3434.98	-	-	-	-	-	-
177	0.993106	12	1676.881	3437.69	-	-	-	-	-	-
178	0.868512	149	1696.092	3478.73	-	-	-	-	-	-
179	1	0	1696.141	3478.83	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
180	0.924775	106	1707.515	3503.56	-	-	-	-	-	-
181	1	0	1707.561	3503.66	-	-	-	-	-	-
182	0.870543	157	1729.834	3550.11	-	-	-	-	-	-
183	0.920277	141	1748.913	3590.97	-	-	-	-	-	-
184	0.998842	16	1751.73	3596.98	-	-	-	-	-	-
185	0.983545	35	1756.819	3608.1	-	-	-	-	-	-
186	0.964577	84	1765.661	3626.61	-	-	-	-	-	-
187	0.929623	117	1783.312	3664.36	-	-	-	-	-	-
188	0.992565	48	1789.333	3677.17	-	-	-	-	-	-
189	0.999947	2	1789.72	3677.97	-	-	-	-	-	-
190	0.887808	170	1813.564	3728.58	-	-	-	-	-	-
191	1	0	1813.617	3728.68	-	-	-	-	-	-
192	0.903573	126	1829.799	3763.2	-	-	-	-	-	-
193	0.886158	173	1857.174	3822.28	-	-	-	-	-	-
194	0.96376	92	1873.756	3857.04	-	-	-	-	-	-
195	0.9983	5	1874.199	3858.04	-	-	-	-	-	-
196	0.985739	21	1876.466	3862.95	-	-	-	-	-	-
197	0.994137	19	1879.523	3869.55	-	-	-	-	-	-
198	0.936277	130	1894.738	3903.08	-	-	-	-	-	-
199	0.847558	170	1914.995	3946.99	-	-	-	-	-	-
200	0.999835	3	1915.589	3948.31	-	-	-	-	-	-
201	0.999453	4	1916.169	3949.51	-	-	-	-	-	-
202	0.932849	101	1932.37	3983.67	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
203	0.999212	9	1934.16	3987.48	-	-	-	-	-	-
204	0.988549	17	1936.077	3991.28	-	-	-	-	-	-
205	0.810501	199	1956.251	4035.3	-	-	-	-	-	-
206	0.929839	138	1972.52	4070.33	-	-	-	-	-	-
207	0.991491	20	1974.891	4075.54	-	-	-	-	-	-
208	0.998558	11	1976.527	4078.94	-	-	-	-	-	-
209	0.969795	80	1990.038	4107.79	-	-	-	-	-	-
210	0.899955	152	2013.785	4158.55	-	-	-	-	-	-
211	0.985101	48	2020.66	4173.66	-	-	-	-	-	-
212	0.99993	1	2020.838	4174.06	-	-	-	-	-	-
213	0.984026	66	2033.578	4201.41	-	-	-	-	-	-
214	0.960586	102	2043.062	4222.12	-	-	-	-	-	-
215	0.924234	134	2068.803	4277.26	-	-	-	-	-	-
216	0.987339	43	2077.221	4295.21	-	-	-	-	-	-
217	0.998299	13	2079.011	4299.21	-	-	-	-	-	-
218	0.985712	42	2084.419	4310.32	-	-	-	-	-	-
219	0.996188	17	2086.679	4315.22	-	-	-	-	-	-
220	0.979412	73	2100.549	4344.89	-	-	-	-	-	-
221	0.990308	25	2104.023	4352.1	-	-	-	-	-	-
222	0.954025	105	2118.681	4383.54	-	-	-	-	-	-
223	0.999403	5	2119.084	4384.44	-	-	-	-	-	-
224	0.998837	12	2120.779	4388.24	-	-	-	-	-	-
225	0.995477	13	2122.328	4391.85	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
226	0.976308	56	2129.656	4408.07	-	-	-	-	-	-
227	0.983953	48	2136.609	4423.11	-	-	-	-	-	-
228	0.99126	13	2137.618	4425.61	-	-	-	-	-	-
229	0.979222	38	2144.143	4439.64	-	-	-	-	-	-
230	0.987384	19	2145.876	4443.45	-	-	-	-	-	-
231	0.968666	57	2153.372	4460.17	-	-	-	-	-	-
232	1	0	2153.426	4460.27	-	-	-	-	-	-
233	0.980638	31	2157.199	4468.68	-	-	-	-	-	-
234	0.991835	21	2159.622	4473.88	-	-	-	-	-	-
235	0.997684	23	2163.346	4481.7	-	-	-	-	-	-
236	0.996537	16	2166.016	4487.4	-	-	-	-	-	-
237	1	0	2166.07	4487.5	-	-	-	-	-	-
238	0.965618	86	2177.183	4511.24	-	-	-	-	-	-
239	0.999341	8	2178.534	4514.04	-	-	-	-	-	-
240	0.997727	16	2181.711	4520.75	-	-	-	-	-	-
241	0.994767	15	2183.466	4524.56	-	-	-	-	-	-
242	0.991386	42	2189.594	4537.87	-	-	-	-	-	-
243	0.988089	31	2193.359	4545.88	-	-	-	-	-	-
244	0.987227	55	2202.125	4564.6	-	-	-	-	-	-
245	0.98674	22	2204.9	4570.71	-	-	-	-	-	-
246	0.958046	91	2217.411	4597.35	-	-	-	-	-	-
247	0.964166	99	2230.24	4625.66	-	-	-	-	-	-
248	0.990876	28	2233.681	4633.16	-	-	-	-	-	-

DMUs	EFF_{OY}	R_{OY}	CPU_{OY}	$TIME_{OY}$	EFF_{KT}	R_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
249	0.971676	50	2240.652	4648.59	-	-	-	-	-	-
250	0.997424	25	2244.771	4657.5	-	-	-	-	-	-

APPENDIX B.

Table B.1 Logistics Performance Index Data

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
1	Germany	CODE1	4.10	4.32	3.74	4.12	4.17	4.36
2	Netherlands	CODE2	3.96	4.23	3.64	4.13	4.07	4.34
3	Belgium	CODE3	3.80	4.10	3.80	4.11	4.11	4.39
4	United Kingdom	CODE4	3.94	4.16	3.63	4.03	4.08	4.33
5	Singapore	CODE5	4.01	4.28	3.70	3.97	3.90	4.25
6	Sweden	CODE6	3.75	4.09	3.76	3.98	3.98	4.26
7	Norway	CODE7	4.21	4.19	3.42	4.19	3.50	4.36
8	Luxembourg	CODE8	3.82	3.91	3.82	3.78	3.68	4.71
9	United States	CODE9	3.73	4.18	3.45	3.97	4.14	4.14
10	Japan	CODE10	3.78	4.16	3.52	3.93	3.95	4.24
11	Ireland	CODE11	3.80	3.84	3.44	3.94	4.13	4.13
12	Canada	CODE12	3.61	4.05	3.46	3.94	3.97	4.18
13	France	CODE13	3.65	3.98	3.68	3.75	3.89	4.17
14	Switzerland	CODE14	3.92	4.04	3.58	3.75	3.79	4.06
15	Hong Kong SAR, China	CODE15	3.72	3.97	3.58	3.81	3.87	4.06
16	Australia	CODE16	3.85	4.00	3.52	3.75	3.81	4.00
17	Denmark	CODE17	3.79	3.82	3.65	3.74	3.36	4.39
18	Spain	CODE18	3.63	3.77	3.51	3.83	3.54	4.07
19	Taiwan	CODE19	3.55	3.64	3.71	3.60	3.79	4.02

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
20	Italy	CODE20	3.36	3.78	3.54	3.62	3.84	4.05
21	Korea, Rep.	CODE21	3.47	3.79	3.44	3.66	3.69	4.00
22	Austria	CODE22	3.53	3.64	3.26	3.56	3.93	4.04
23	New Zealand	CODE23	3.92	3.67	3.67	3.56	3.33	3.72
24	Finland	CODE24	3.89	3.52	3.52	3.72	3.31	3.80
25	Malaysia	CODE25	3.37	3.56	3.64	3.47	3.58	3.92
26	Portugal	CODE26	3.26	3.37	3.43	3.71	3.71	3.87
27	United Arab Emirates	CODE27	3.42	3.70	3.20	3.50	3.57	3.92
28	China	CODE28	3.21	3.67	3.50	3.46	3.50	3.87
29	Qatar	CODE29	3.21	3.44	3.55	3.55	3.47	3.87
30	Turkey	CODE30	3.23	3.53	3.18	3.64	3.77	3.68
31	Poland	CODE31	3.26	3.08	3.46	3.47	3.54	4.13
32	Czech Republic	CODE32	3.24	3.29	3.59	3.51	3.56	3.73
33	Hungary	CODE33	2.97	3.18	3.40	3.33	3.82	4.06
34	South Africa	CODE34	3.11	3.20	3.45	3.62	3.30	3.88
35	Thailand	CODE35	3.21	3.40	3.30	3.29	3.45	3.96
36	Latvia	CODE36	3.22	3.03	3.38	3.21	3.50	4.06
37	Iceland	CODE37	3.54	3.34	3.15	3.46	3.38	3.51
38	Slovenia	CODE38	3.11	3.35	3.05	3.51	3.51	3.82
39	Estonia	CODE39	3.40	3.34	3.34	3.27	3.20	3.55
40	Romania	CODE40	2.83	2.77	3.32	3.20	3.39	4.00
41	Israel	CODE41	3.10	3.11	2.71	3.35	3.20	4.18
42	Chile	CODE42	3.17	3.17	3.12	3.19	3.30	3.59
43	Slovak Republic	CODE43	2.89	3.22	3.30	3.16	3.02	3.94

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
44	Greece	CODE44	3.36	3.17	2.97	3.23	3.03	3.50
45	Panama	CODE45	3.15	3.00	3.18	2.87	3.34	3.63
46	Lithuania	CODE46	3.04	3.18	3.10	2.99	3.17	3.60
47	Bulgaria	CODE47	2.75	2.94	3.31	3.00	2.88	4.04
48	Vietnam	CODE48	2.81	3.11	3.22	3.09	3.19	3.49
49	Saudi Arabia	CODE49	2.86	3.34	2.93	3.11	3.15	3.55
50	Mexico	CODE50	2.69	3.04	3.19	3.12	3.14	3.57
51	Malta	CODE51	3.00	3.08	3.23	3.00	3.15	3.15
52	Bahrain	CODE52	3.29	3.04	3.04	3.04	3.29	2.80
53	Indonesia	CODE53	2.87	2.92	2.87	3.21	3.11	3.53
54	India	CODE54	2.72	2.88	3.20	3.03	3.11	3.51
55	Croatia	CODE55	2.95	2.92	2.98	3.00	3.11	3.37
56	Kuwait	CODE56	2.69	3.16	2.76	2.96	3.16	3.39
57	Philippines	CODE57	3.00	2.60	3.33	2.93	3.00	3.07
58	Cyprus	CODE58	2.88	2.87	3.01	2.92	3.00	3.31
59	Oman	CODE59	2.63	2.88	3.41	2.84	2.84	3.29
60	Argentina	CODE60	2.55	2.83	2.96	2.93	3.15	3.49
61	Ukraine	CODE61	2.69	2.65	2.95	2.84	3.20	3.51
62	Egypt, Arab Rep.	CODE62	2.85	2.86	2.87	2.99	3.23	2.99
63	Serbia	CODE63	2.37	2.73	3.12	3.02	2.94	3.55
64	El Salvador	CODE64	2.93	2.63	3.20	3.16	3.00	2.75
65	Brazil	CODE65	2.48	2.93	2.80	3.05	3.03	3.39
66	Bahamas, The	CODE66	3.00	2.74	2.96	2.92	2.64	3.19
67	Montenegro	CODE67	2.83	2.84	3.15	2.45	2.76	3.19

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
68	Jordan	CODE68	2.60	2.59	2.96	2.94	2.67	3.46
69	Dominican Republic	CODE69	2.58	2.61	2.93	2.91	2.91	3.18
70	Jamaica	CODE70	2.88	2.84	2.79	2.72	2.72	3.14
71	Peru	CODE71	2.47	2.72	2.94	2.78	2.81	3.30
72	Pakistan	CODE72	2.84	2.67	3.08	2.79	2.73	2.79
73	Malawi	CODE73	2.79	3.04	2.63	2.86	2.63	2.99
74	Kenya	CODE74	1.96	2.40	3.15	2.65	3.03	3.58
75	Nigeria	CODE75	2.35	2.56	2.63	2.70	3.16	3.46
76	Venezuela, RB	CODE76	2.39	2.61	2.94	2.76	2.92	3.18
77	Guatemala	CODE77	2.75	2.54	2.87	2.68	2.68	3.24
78	Paraguay	CODE78	2.49	2.46	2.83	2.76	2.89	3.22
79	Côte d'Ivoire	CODE79	2.33	2.41	2.87	2.62	2.97	3.31
80	Rwanda	CODE80	2.50	2.32	2.78	2.64	2.94	3.34
81	Bosnia and Herzegovina	CODE81	2.41	2.55	2.78	2.73	2.55	3.44
82	Maldives	CODE82	2.95	2.56	2.92	2.79	2.70	2.51
83	Cambodia	CODE83	2.67	2.58	2.83	2.67	2.92	2.75
84	Sao Tomé and Príncipe	CODE84	2.42	2.59	2.95	2.50	3.13	2.77
85	Lebanon	CODE85	2.29	2.53	2.53	2.89	3.22	2.89
86	Ecuador	CODE86	2.49	2.50	2.79	2.61	2.67	3.18
87	Costa Rica	CODE87	2.39	2.43	2.63	2.86	2.83	3.04
88	Kazakhstan	CODE88	2.33	2.38	2.68	2.72	2.83	3.24
89	Sri Lanka	CODE89	2.56	2.23	2.56	2.91	2.76	3.12
90	Russian Federation	CODE90	2.20	2.59	2.64	2.74	2.85	3.14
91	Uruguay	CODE91	2.39	2.51	2.64	2.58	2.89	3.06

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
92	Armenia	CODE92	2.63	2.38	2.75	2.75	2.50	3.00
93	Namibia	CODE93	2.27	2.57	2.70	2.69	2.56	3.15
94	Moldova	CODE94	2.46	2.55	3.14	2.44	2.35	2.89
95	Nicaragua	CODE95	2.66	2.20	2.69	2.58	2.58	3.17
96	Algeria	CODE96	2.71	2.54	2.54	2.54	2.54	3.04
97	Colombia	CODE97	2.59	2.44	2.72	2.64	2.55	2.87
98	Burkina Faso	CODE98	2.50	2.35	2.63	2.63	2.49	3.21
99	Belarus	CODE99	2.50	2.55	2.74	2.46	2.51	3.05
100	Ghana	CODE100	2.22	2.67	2.73	2.37	2.90	2.86
101	Senegal	CODE101	2.61	2.30	3.03	2.53	2.65	2.53
102	Liberia	CODE102	2.57	2.57	2.57	2.86	2.57	2.57
103	Honduras	CODE103	2.70	2.24	2.79	2.47	2.61	2.79
104	Ethiopia	CODE104	2.42	2.17	2.50	2.62	2.67	3.17
105	Nepal	CODE105	2.31	2.26	2.64	2.50	2.72	3.06
106	Solomon Islands	CODE106	2.49	2.46	2.22	2.72	2.72	2.96
107	Burundi	CODE107	2.60	2.40	2.60	2.51	2.51	2.76
108	Bangladesh	CODE108	2.09	2.11	2.82	2.64	2.45	3.18
109	Benin	CODE109	2.64	2.35	2.69	2.35	2.45	2.85
110	Tunisia	CODE110	2.02	2.30	2.91	2.42	2.42	3.16
111	Fiji	CODE111	2.40	2.47	2.72	2.22	2.47	2.97
112	Angola	CODE112	2.37	2.11	2.79	2.31	2.59	3.02
113	Chad	CODE113	2.46	2.33	2.33	2.34	2.71	3.02
114	Tajikistan	CODE114	2.35	2.36	2.73	2.47	2.47	2.74
115	Mauritius	CODE115	2.25	2.50	2.63	2.48	2.34	2.88

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
116	Georgia	CODE116	2.21	2.42	2.32	2.44	2.59	3.09
117	Macedonia, FYR	CODE117	2.35	2.50	2.38	2.51	2.46	2.81
118	Libya	CODE118	2.41	2.29	2.29	2.29	2.85	2.85
119	Mali	CODE119	2.08	2.20	2.80	2.20	2.70	2.90
120	Botswana	CODE120	2.38	2.23	2.42	2.58	2.40	2.94
121	Bolivia	CODE121	2.40	2.17	2.35	2.68	2.68	2.60
122	Guinea	CODE122	2.34	2.10	2.47	2.35	2.41	3.10
123	Zambia	CODE123	2.54	2.31	2.13	2.47	2.47	2.91
124	Guyana	CODE124	2.46	2.40	2.43	2.27	2.47	2.74
125	Azerbaijan	CODE125	2.57	2.71	2.57	2.14	2.14	2.57
126	Papua New Guinea	CODE126	2.40	2.23	2.47	2.47	2.27	2.73
127	Guinea-Bissau	CODE127	2.43	2.29	2.29	2.57	2.29	2.71
128	Comoros	CODE128	2.58	2.30	2.51	2.26	2.37	2.37
129	Uzbekistan	CODE129	1.80	2.01	2.23	2.37	2.87	3.08
130	Niger	CODE130	2.49	2.08	2.38	2.28	2.36	2.76
131	Lao PDR	CODE131	2.45	2.21	2.50	2.31	2.20	2.65
132	Madagascar	CODE132	2.06	2.15	2.38	2.33	2.29	3.07
133	Lesotho	CODE133	2.22	2.35	2.48	2.23	2.35	2.60
134	Central African Republic	CODE134	2.47	2.50	2.16	2.31	2.31	2.47
135	Mongolia	CODE135	2.20	2.29	2.62	2.33	2.13	2.51
136	Equatorial Guinea	CODE136	2.35	2.11	2.11	2.20	2.53	2.86
137	Zimbabwe	CODE137	1.89	2.25	2.25	2.50	2.22	2.93
138	Tanzania	CODE138	2.19	2.32	2.32	2.18	2.11	2.89
139	Togo	CODE139	2.09	2.07	2.47	2.14	2.49	2.60

SEQ	COUNTRY	CODE	CUSTOMS	INFRASTRUCTURE	INTERNATIONAL SHIPMENTS	LOGISTICS QUALITY AND COMPETENCE	TRACKING AND TRACING	TIMELINESS
140	Turkmenistan	CODE140	2.31	2.06	2.56	2.07	2.32	2.45
141	Iraq	CODE141	1.98	2.18	2.31	2.15	2.31	2.85
142	Cameroon	CODE142	1.86	1.85	2.20	2.52	2.52	2.80
143	Bhutan	CODE143	2.09	2.18	2.38	2.48	2.28	2.28
144	Haiti	CODE144	2.25	2.00	2.27	2.14	2.32	2.63
145	Myanmar	CODE145	1.97	2.14	2.14	2.07	2.36	2.83
146	Gambia, The	CODE146	2.06	2.00	2.67	2.22	2.00	2.46
147	Mozambique	CODE147	2.26	2.15	2.08	2.10	2.08	2.74
148	Mauritania	CODE148	1.93	2.40	2.07	2.06	2.23	2.75
149	Kyrgyz Republic	CODE149	2.03	2.05	2.43	2.13	2.20	2.36
150	Gabon	CODE150	2.00	2.08	2.58	2.25	1.92	2.31
151	Yemen, Rep.	CODE151	1.63	1.87	2.35	2.21	2.21	2.78
152	Cuba	CODE152	2.17	1.84	2.47	2.08	1.99	2.45
153	Sudan	CODE153	1.87	1.90	2.23	2.18	2.42	2.33
154	Djibouti	CODE154	2.20	2.00	1.80	2.21	2.00	2.74
155	Syrian Arab Republic	CODE155	2.07	2.08	2.15	1.82	1.90	2.53
156	Eritrea	CODE156	1.90	1.68	1.90	2.23	2.01	2.79
157	Congo, Rep.	CODE157	1.50	1.83	2.17	2.17	2.17	2.58
158	Afghanistan	CODE158	2.16	1.82	1.99	2.12	1.85	2.48
159	Congo, Dem. Rep.	CODE159	1.78	1.83	1.70	1.84	2.10	2.04
160	Somalia	CODE160	2.00	1.50	1.75	1.75	1.75	1.88

APPENDIX C.

Table C.1 Comparison of between OY and KT Models with R_0 , EFF_0 , CPU_0 , $TIME_0$ using LPI set

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
1	Germany	CODE1	0	1.000000	0.052	0.30	0	1.000000	0.163	0.40	0	0.000000
2	Netherlands	CODE2	1	0.999993	0.136	0.55	1	0.999993	0.396	0.81	0	0.000000
3	Belgium	CODE3	0	1.000000	0.143	0.70	0	1.000000	0.556	1.01	0	0.000000
4	United Kingdom	CODE4	1	0.999922	0.334	1.03	1	0.999922	0.682	1.22	0	0.000000
5	Singapore	CODE5	1	0.999935	0.389	1.21	1	0.999935	0.819	1.43	0	0.000000
6	Sweden	CODE6	2	0.999919	0.548	1.64	2	0.999919	1.217	1.95	0	0.000000
7	Norway	CODE7	0	1.000000	0.604	1.75	0	1.000000	1.267	2.15	0	0.000000
8	Luxembourg	CODE8	0	1.000000	0.611	1.84	0	1.000000	1.33	2.26	0	0.000000
9	United States	CODE9	1	0.999955	0.624	1.99	1	0.999955	1.557	2.58	0	0.000000
10	Japan	CODE10	5	0.999552	1.318	3.32	5	0.999552	2.168	3.44	0	0.000000
11	Ireland	CODE11	1	0.999936	1.333	3.46	1	0.999936	2.399	3.76	0	0.000000
12	Canada	CODE12	7	0.999107	1.981	5.49	7	0.999107	3.127	4.92	0	0.000000
13	France	CODE13	5	0.999402	2.421	6.39	5	0.999456	3.738	5.96	0	-0.000054
14	Switzerland	CODE14	5	0.999364	3.025	7.51	5	0.999364	4.137	6.62	0	0.000000
15	Hong Kong SAR, China	CODE15	9	0.998541	4.488	10.61	9	0.998608	5.241	8.32	0	-0.000068
16	Australia	CODE16	6	0.998827	5.257	12.02	6	0.998827	6.022	9.47	0	0.000000
17	Denmark	CODE17	2	0.999765	5.466	12.42	2	0.999765	6.335	9.99	0	0.000000
18	Spain	CODE18	15	0.997314	6.987	16.24	11	0.997467	7.575	12.08	4	-0.000153
19	Taiwan	CODE19	4	0.999553	7.111	16.95	4	0.999553	7.929	12.74	0	0.000000
20	Italy	CODE20	13	0.997631	7.964	19.91	12	0.997736	9.229	14.76	1	-0.000105

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
21	Korea, Rep.	CODE21	18	0.994869	11.156	25.77	16	0.995168	11.394	17.81	2	-0.000298
22	Austria	CODE22	9	0.998232	11.268	27.07	9	0.998232	12.439	19.79	0	0.000000
23	New Zealand	CODE23	4	0.999713	11.393	27.97	4	0.999713	12.772	20.51	0	0.000000
24	Finland	CODE24	6	0.999027	11.577	28.87	6	0.999027	13.729	21.86	0	0.000000
25	Malaysia	CODE25	9	0.998814	12.237	30.18	9	0.998814	14.825	23.61	0	0.000000
26	Portugal	CODE26	21	0.994352	14.894	35.60	16	0.995433	17.67	27.49	5	-0.001081
27	United Arab Emirates	CODE27	22	0.991218	17.574	41.03	20	0.991992	20.743	32.20	2	-0.000773
28	China	CODE28	20	0.995501	19.123	44.57	20	0.995609	23.815	36.93	0	-0.000108
29	Qatar	CODE29	15	0.996826	19.684	47.12	15	0.996895	26.184	40.47	0	-0.000069
30	Turkey	CODE30	16	0.995079	20.119	49.67	16	0.995079	28.092	43.80	0	0.000000
31	Poland	CODE31	12	0.996956	20.644	52.00	12	0.996982	30.086	46.58	0	-0.000026
32	Czech Republic	CODE32	12	0.997710	21.085	54.25	12	0.997738	31.761	49.14	0	-0.000028
33	Hungary	CODE33	16	0.996321	23.173	58.52	14	0.996667	33.957	52.50	2	-0.000346
34	South Africa	CODE34	24	0.993838	25.304	63.64	22	0.994036	37.536	57.30	2	-0.000198
35	Thailand	CODE35	27	0.991573	27.116	69.22	25	0.991925	41.554	62.81	2	-0.000352
36	Latvia	CODE36	18	0.995246	28.28	72.45	18	0.995286	44.32	66.98	0	-0.000040
37	Iceland	CODE37	21	0.990471	29.347	76.03	21	0.990587	47.688	71.96	0	-0.000116
38	Slovenia	CODE38	28	0.987953	32.231	82.54	27	0.988357	51.77	77.72	1	-0.000404
39	Estonia	CODE39	30	0.987878	34.025	88.21	25	0.989277	55.701	83.42	5	-0.001399
40	Romania	CODE40	26	0.993107	35.295	92.84	26	0.993129	60.094	89.63	0	-0.000022
41	Israel	CODE41	11	0.997393	35.602	94.00	11	0.997393	61.755	92.39	0	0.000000
42	Chile	CODE42	39	0.976615	41.924	106.61	36	0.978859	67.739	100.88	3	-0.002243
43	Slovak Republic	CODE43	29	0.990874	44.179	112.84	29	0.990935	72.61	107.97	0	-0.000061
44	Greece	CODE44	27	0.982956	46.129	117.57	27	0.983379	76.907	114.10	0	-0.000422
45	Panama	CODE45	39	0.979419	51.321	128.15	36	0.980660	83.806	123.30	3	-0.001241

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
46	Lithuania	CODE46	40	0.975241	56.607	139.58	38	0.976172	90.23	132.19	2	-0.000931
47	Bulgaria	CODE47	22	0.994265	57.237	141.91	22	0.994329	93.851	137.39	0	-0.000064
48	Vietnam	CODE48	41	0.979552	60.996	149.87	40	0.979889	100.895	146.89	1	-0.000337
49	Saudi Arabia	CODE49	34	0.975042	63.451	155.81	32	0.976803	106.008	154.21	2	-0.001761
50	Mexico	CODE50	42	0.977702	66.509	163.99	41	0.978098	113.607	164.64	1	-0.000396
51	Malta	CODE51	40	0.980490	68.905	171.12	39	0.980623	120.372	173.93	1	-0.000132
52	Bahrain	CODE52	29	0.980530	71.114	177.37	29	0.980703	125.293	180.97	0	-0.000172
53	Indonesia	CODE53	40	0.970585	75.022	185.74	40	0.970742	132.266	190.74	0	-0.000156
54	India	CODE54	42	0.977863	77.479	193.03	42	0.978059	139.252	200.36	0	-0.000196
55	Croatia	CODE55	47	0.962954	83.409	205.81	46	0.964645	147.799	211.76	1	-0.001691
56	Kuwait	CODE56	41	0.966860	86.735	213.98	41	0.967859	154.834	221.28	0	-0.001000
57	Philippines	CODE57	34	0.986998	87.497	217.56	34	0.986998	160.361	229.09	0	0.000000
58	Cyprus	CODE58	53	0.962272	94.556	232.65	50	0.963622	169.417	241.37	3	-0.001350
59	Oman	CODE59	30	0.991257	95.158	235.37	30	0.991257	174.427	248.63	0	0.000000
60	Argentina	CODE60	50	0.964515	100.877	247.74	48	0.966978	182.865	260.49	2	-0.002464
61	Ukraine	CODE61	43	0.967465	105.398	257.77	43	0.969579	190.432	270.83	0	-0.002114
62	Egypt, Arab Rep.	CODE62	42	0.968902	107.914	264.49	42	0.969259	197.6	280.69	0	-0.000356
63	Serbia	CODE63	47	0.971978	111.086	274.65	46	0.972695	206.507	292.67	1	-0.000716
64	El Salvador	CODE64	41	0.978200	113.119	281.37	41	0.978262	213.525	302.62	0	-0.000062
65	Brazil	CODE65	49	0.959443	118.348	294.07	49	0.959885	222.743	315.20	0	-0.000443
66	Bahamas, The	CODE66	44	0.962033	122.286	302.29	44	0.964766	230.994	326.50	0	-0.002733
67	Montenegro	CODE67	49	0.973333	125.67	309.14	49	0.973395	239.857	338.46	0	-0.000062
68	Jordan	CODE68	55	0.960889	130.387	321.46	54	0.961558	250.474	351.87	1	-0.000669
69	Dominican Republic	CODE69	64	0.952338	137.659	336.51	61	0.953737	261.571	366.74	3	-0.001399
70	Jamaica	CODE70	53	0.949545	143.211	348.54	53	0.952285	270.635	379.20	0	-0.002740

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
71	Peru	CODE71	65	0.952488	147.589	362.40	62	0.953546	282.905	395.69	3	-0.001058
72	Pakistan	CODE72	53	0.967337	150.185	370.11	53	0.967486	292.462	408.98	0	-0.000149
73	Malawi	CODE73	47	0.954741	151.848	376.80	47	0.954754	299.642	419.49	0	-0.000012
74	Kenya	CODE74	45	0.974174	154.058	385.63	44	0.975134	307.926	430.81	1	-0.000960
75	Nigeria	CODE75	49	0.963886	157.57	394.88	48	0.966053	316.963	443.51	1	-0.002167
76	Venezuela, RB	CODE76	66	0.952091	163.871	407.78	64	0.952932	330.649	461.84	2	-0.000841
77	Guatemala	CODE77	62	0.947845	169.583	422.01	61	0.949088	341.143	476.24	1	-0.001243
78	Paraguay	CODE78	72	0.940033	179.592	444.18	67	0.943087	355.377	494.42	5	-0.003054
79	Côte d'Ivoire	CODE79	65	0.949576	188.107	461.52	61	0.951050	369.179	511.97	4	-0.001474
80	Rwanda	CODE80	60	0.949186	191.895	472.96	60	0.949859	381.513	528.39	0	-0.000672
81	Bosnia and Herzegovina	CODE81	57	0.958398	193.533	479.58	57	0.958439	391.987	542.72	0	-0.000041
82	Maldives	CODE82	48	0.957737	196.631	488.49	48	0.960834	400.415	554.06	0	-0.003097
83	Cambodia	CODE83	64	0.943523	202.226	503.36	64	0.944731	413.008	570.78	0	-0.001208
84	Sao Tomé and Príncipe	CODE84	54	0.960726	204.024	513.76	52	0.963428	423.345	585.55	2	-0.002702
85	Lebanon	CODE85	43	0.968156	205.113	518.05	43	0.968156	430.707	595.73	0	0.000000
86	Ecuador	CODE86	79	0.933695	212.919	538.39	75	0.936462	447.26	616.87	4	-0.002767
87	Costa Rica	CODE87	68	0.935010	220.174	552.90	67	0.937087	458.781	632.76	1	-0.002078
88	Kazakhstan	CODE88	67	0.937896	224.686	565.96	67	0.938657	472.256	650.22	0	-0.000761
89	Sri Lanka	CODE89	65	0.940952	230.798	577.97	65	0.941420	483.055	664.89	0	-0.000468
90	Russian Federation	CODE90	75	0.931136	241.309	598.59	73	0.933604	499.529	686.16	2	-0.002468
91	Uruguay	CODE91	74	0.932242	249.503	615.43	73	0.934097	514.953	705.73	1	-0.001856
92	Armenia	CODE92	77	0.931386	256.968	633.67	72	0.933382	528.376	724.07	5	-0.001996
93	Namibia	CODE93	76	0.928602	264.23	651.32	74	0.929538	545.264	745.74	2	-0.000936
94	Moldova	CODE94	51	0.972299	265.18	656.33	51	0.972299	553.487	756.91	0	0.000000
95	Nicaragua	CODE95	69	0.933477	272.436	672.31	67	0.935738	566.889	774.93	2	-0.002260

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
96	Algeria	CODE96	65	0.932849	278.828	685.95	65	0.933269	579.149	791.13	0	-0.000420
97	Colombia	CODE97	78	0.924172	287.434	704.27	78	0.926620	595.792	812.53	0	-0.002448
98	Burkina Faso	CODE98	72	0.933845	291.648	717.12	70	0.934907	607.623	829.00	2	-0.001063
99	Belarus	CODE99	81	0.924971	300.772	738.15	77	0.927766	625.173	851.68	4	-0.002794
100	Ghana	CODE100	68	0.935161	305.151	752.50	67	0.938398	637.415	868.13	1	-0.003238
101	Senegal	CODE101	59	0.960896	306.954	759.92	58	0.961069	648.151	882.29	1	-0.000173
102	Liberia	CODE102	69	0.934820	309.418	770.77	68	0.935244	659.849	898.15	1	-0.000424
103	Honduras	CODE103	67	0.937264	315.538	783.54	66	0.938779	672.558	915.50	1	-0.001515
104	Ethiopia	CODE104	77	0.929431	320.459	796.82	77	0.929910	688.439	936.43	0	-0.000479
105	Nepal	CODE105	88	0.917457	332.919	821.24	85	0.918564	713.887	967.51	3	-0.001107
106	Solomon Islands	CODE106	83	0.917198	344.478	845.73	80	0.918814	741.426	1000.24	3	-0.001616
107	Burundi	CODE107	76	0.918424	351.228	860.69	76	0.920417	755.726	1019.42	0	-0.001993
108	Bangladesh	CODE108	75	0.936420	356.584	874.98	74	0.938548	771.02	1039.87	1	-0.002128
109	Benin	CODE109	72	0.927080	362.871	888.17	72	0.928959	783.618	1056.46	0	-0.001879
110	Tunisia	CODE110	71	0.947526	364.937	898.17	70	0.948085	796.352	1073.85	1	-0.000560
111	Fiji	CODE111	89	0.918979	373.679	918.28	89	0.920282	832.154	1115.73	0	-0.001304
112	Angola	CODE112	83	0.930203	378.853	932.74	83	0.930711	854.298	1143.51	0	-0.000508
113	Chad	CODE113	89	0.912208	388.701	954.02	87	0.915228	894.651	1189.71	2	-0.003019
114	Tajikistan	CODE114	91	0.918910	395.516	968.79	91	0.919456	916.898	1217.33	0	-0.000546
115	Mauritius	CODE115	95	0.903622	404.388	989.95	90	0.909571	940.129	1246.83	5	-0.005949
116	Georgia	CODE116	84	0.919230	408.534	1003.24	84	0.919400	957.076	1268.97	0	-0.000169
117	Macedonia, FYR	CODE117	90	0.894628	421.454	1028.21	90	0.897017	977.485	1295.32	0	-0.002389
118	Libya	CODE118	76	0.928182	426.21	1039.15	76	0.928347	990.662	1313.08	0	-0.000165
119	Mali	CODE119	80	0.932164	431.659	1052.04	80	0.932381	1007.247	1335.11	0	-0.000217
120	Botswana	CODE120	102	0.897946	442.833	1077.68	98	0.900362	1046.275	1380.76	4	-0.002416

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
121	Bolivia	CODE121	87	0.909641	450.237	1096.36	87	0.910973	1067.14	1406.68	0	-0.001332
122	Guinea	CODE122	86	0.918609	453.98	1109.03	85	0.919707	1084.067	1428.99	1	-0.001098
123	Zambia	CODE123	83	0.908597	460.903	1124.96	83	0.910065	1101.575	1451.95	0	-0.001468
124	Guyana	CODE124	96	0.893278	473.711	1150.36	96	0.895775	1133.697	1490.11	0	-0.002497
125	Azerbaijan	CODE125	65	0.924221	479.186	1163.30	65	0.925409	1144.536	1504.47	0	-0.001188
126	Papua New Guinea	CODE126	105	0.886391	491.646	1190.31	105	0.888014	1186.07	1552.83	0	-0.001623
127	Guinea-Bissau	CODE127	98	0.891637	501.925	1211.67	97	0.893606	1215.227	1587.72	1	-0.001968
128	Comoros	CODE128	80	0.912091	505.032	1224.03	80	0.914253	1229.189	1607.00	0	-0.002162
129	Uzbekistan	CODE129	76	0.928525	510.144	1236.05	75	0.931317	1242.631	1625.01	1	-0.002792
130	Niger	CODE130	93	0.895555	519.342	1255.52	92	0.898009	1262.435	1650.17	1	-0.002454
131	Lao PDR	CODE131	98	0.893819	529.458	1274.79	98	0.895724	1283.725	1677.72	0	-0.001905
132	Madagascar	CODE132	88	0.915597	531.709	1284.64	88	0.915628	1300.45	1699.83	0	-0.000031
133	Lesotho	CODE133	110	0.874045	543.371	1309.20	109	0.878173	1331.141	1737.24	1	-0.004128
134	Central African Republic	CODE134	88	0.895392	549.399	1324.92	86	0.897275	1347.114	1758.75	2	-0.001884
135	Mongolia	CODE135	109	0.897419	554.211	1341.56	109	0.898014	1380.076	1798.17	0	-0.000595
136	Equatorial Guinea	CODE136	113	0.881382	567.394	1367.86	109	0.885015	1458.84	1883.91	4	-0.003633
137	Zimbabwe	CODE137	104	0.893052	575.028	1387.57	103	0.893999	1487.044	1918.36	1	-0.000946
138	Tanzania	CODE138	112	0.884761	583.527	1410.03	106	0.886450	1526.74	1964.56	6	-0.001689
139	Togo	CODE139	117	0.868414	594.463	1433.67	114	0.872177	1567.132	2012.75	3	-0.003763
140	Turkmenistan	CODE140	116	0.883879	603.749	1454.81	113	0.885347	1604.415	2056.52	3	-0.001468
141	Iraq	CODE141	114	0.876111	609.727	1473.84	114	0.876548	1633.218	2092.43	0	-0.000437
142	Cameroon	CODE142	104	0.881313	620.407	1497.48	102	0.883284	1667.829	2133.68	2	-0.001971
143	Bhutan	CODE143	110	0.872401	628.06	1517.78	110	0.873635	1691.936	2164.41	0	-0.001234
144	Haiti	CODE144	126	0.844820	644.05	1549.27	126	0.847065	1748.928	2230.34	0	-0.002244
145	Myanmar	CODE145	116	0.872463	650.18	1567.31	116	0.872824	1776.369	2264.78	0	-0.000361

SEQ	COUNTRY	DMU	R_{OYS}	EFF_{OY}	CPU_{OY}	$TIME_{OY}$	R_{KT}	EFF_{KT}	CPU_{KT}	$TIME_{KT}$	$R_{OY} - R_{KT}$	$EFF_{OY} - EFF_{KT}$
146	Gambia, The	CODE146	100	0.907080	652.699	1578.03	100	0.907083	1801.824	2295.48	0	-0.000003
147	Mozambique	CODE147	132	0.854403	664.896	1605.37	125	0.857857	1874.007	2376.15	7	-0.003454
148	Mauritania	CODE148	102	0.874104	670.343	1622.54	102	0.878269	1903.399	2411.98	0	-0.004165
149	Kyrgyz Republic	CODE149	126	0.852366	675.114	1640.60	126	0.852654	1953.754	2470.52	0	-0.000288
150	Gabon	CODE150	111	0.889138	678.596	1653.61	111	0.889237	1982.932	2508.59	0	-0.000099
151	Yemen, Rep.	CODE151	123	0.860637	687.996	1676.08	123	0.861522	2021.076	2555.08	0	-0.000885
152	Cuba	CODE152	122	0.862961	691.77	1693.49	122	0.863383	2066.405	2608.18	0	-0.000422
153	Sudan	CODE153	124	0.843323	696.523	1713.04	124	0.844000	2104.249	2653.86	0	-0.000676
154	Djibouti	CODE154	131	0.848388	706.103	1738.23	130	0.854639	2170.735	2728.68	1	-0.006251
155	Syrian Arab Republic	CODE155	142	0.802288	727.621	1778.41	141	0.807327	2265.532	2832.24	1	-0.005039
156	Eritrea	CODE156	121	0.861959	731.409	1794.19	121	0.862350	2293.058	2867.35	0	-0.000392
157	Congo, Rep.	CODE157	141	0.812186	745.109	1825.74	141	0.813138	2366.618	2951.08	0	-0.000952
158	Afghanistan	CODE158	136	0.816476	756.351	1850.09	136	0.817920	2419.001	3013.17	0	-0.001443
159	Congo, Dem. Rep.	CODE159	150	0.751090	763.705	1871.65	150	0.751285	2468.662	3073.32	0	-0.000196
160	Somalia	CODE160	146	0.760860	767.052	1886.59	146	0.760860	2502.858	3116.91	0	0.000000